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MODELING AND ESTIMATING THE MODEL ADEQUACY IN MUSCLE TISSUE ELECTRICAL STIMULATOR DESIGNING

*The research **object** is the mathematical modeling of human skeletal muscle electrical stimulation characteristics during therapy. The **subject** of research is mathematical models of electrical stimulation characteristics that relate muscle contraction amplitude to the amplitude, the rate of stimulating effects, and other parameters. The research **purpose** is to study such models, develop an algorithm for their correction and a method for estimating their adequacy. The **methods** used: mathematical modeling methods, methods of structural and parametric identification of models, optimization methods, methods for estimating the adequacy of models, and interval methods. The **results**: an algorithm for muscle electrical stimulation characteristics mathematical models correcting during several sessions in case of their change is proposed; a method for estimating the model adequacy area in the external variables space in order to control its adequacy is proposed; using the interval mathematics methods to construct the adequacy actual area is justified; an interval estimating of the error in modeling a certain output characteristic is introduced, that, in the case of characteristic monotonicity, allows checking the adequacy maintaining by checking some inequalities. The results can be used in the design of electrical stimulators and for determining the electrical stimulation effects of individual parameters during one session or a series of sessions. **The scientific originality**: the interval mathematics methods for approximating the mathematical model adequacy area in a hyperparallepiped and checking for nesting in the target area in the external parameters space in order to control the adequacy during the model correction in electrical stimulation is proposed and justified.*

Keywords: *electrical stimulation; skeletal muscle; mathematical modeling; area of adequacy; estimation; interval methods; electrical stimulators designing.*

1. Introduction

Recently, both in foreign and domestic scientific literature, much attention has been paid to the issues of electrical stimulation of human organs and tissues.

1.1. Motivation

Electrostimulation is a targeted impact of electrical signals with a certain set of parameters that affect contracted organs and tissues, replacing or supporting natural electrical impulses passing through nerve fibers [1, 2]. A special place is occupied by electromyostimulation aimed at striated (skeletal) muscles [3]. In this case, there are conditions for starting or accelerating metabolic processes, supplying tissues with blood and oxygen [4, 5]. It is used in the processes of therapeutic therapy and rehabilitation of patients, in the practice of sports training, cosmetic procedures, and prosthetics [6, 7]. Technical means of electrical stimulation (electrical stimulators) are usually focused on some standard modes that do not always ensure efficiency, considering the individual characteristics of patients [8]. Therefore, it is advisable to have a priori information about the electrical stimulation object that allows optimizing the parameters of electrical stimuli. For this purpose, it is necessary to adequately model the processes and technical means of electrical stimulation.

1.2. State of the Art

A number of works are devoted to electromyostimulation modeling issues [9]. For example, in [10, 11] it is proposed to approximate the amplitude and rate electrical stimulation characteristics by polynomials of a certain degree in order to find the optimal values of the stimulation parameters. In [12], an analytical description of the muscle strength-duration characteristic in normal and pathological conditions is obtained.

However, during therapy, the characteristics type can change, depending on the patient individual characteristics, a particular muscle group, or the stimulation technique used [13].

In some devices, a temporal algorithm for the formation of an adequate duration of stimuli is implemented. The duration of the stimulus is set in accordance with the measured value of current relaxation time in tissues under the electrodes. Periodic repetition of the processes of measuring and adjusting the duration of the stimulus allows you to maintain the effectiveness of the impact during prolonged anesthesia because changing the stimulation parameters helps to weaken the influence [14].

In this connection there is a need for a periodic correction of the model used.

Thus, the research purpose is to study such models, develop an algorithm for their correction and a method for estimating their adequacy. For this purpose, the use of methods of interval mathematics for approximating the adequacy region of a mathematical model in the form of a hyperparallepiped and checking for nesting in a given region in the space of external parameters in order to control the adequacy during the correction of the model during electrical stimulation is proposed and justified.

In general, the simulation requires the following operations:

– selecting the properties that the model should reflect;

– obtaining information about the properties selected;

– the model structural synthesis;

– the model parameters numerical values determination to minimize the error of a model with a given structure, i.e. min $\varepsilon_M(X)$, $X \in X_D$, where X – model parameters vector; X_D – admissible area for changing parameters; ε_M – model admissible error;

– estimating the adequacy and accuracy of the model.

The following algorithm should be used; which diagram is shown in Fig. 1.

First, it is necessary to obtain the dependence of contraction amplitude on the amplitude, stimulating effect repetition rate, or their duration. It is discussed below. Based on this, structural and parametric identification of the model is carried out. At the stage of structural identification, possible approximating functions are determined, the polynomial degree is determined (with polynomial approximation), etc. At the stage of parametric identification, the model specific parameters (coefficients) are determined [15]. Further, specific optimal stimuli parameters are determined and stimulation sessions are conducted. If the stimulation characteristics is beyond control limits [16, 17], it is necessary to correct the model. First, by parametric identification, and if at this stage the required accuracy is not obtained, structural identification should be re-performed (as shown in Fig. 1).

Thus, before the start of each session, it is advisable to empirically determine the dependence of muscle contraction amplitude on the stimulating current (or voltage) amplitude and rate for each patient. Such dependencies based on power relations are qualitative. In addition, it is impossible to measure muscle contraction amplitude.

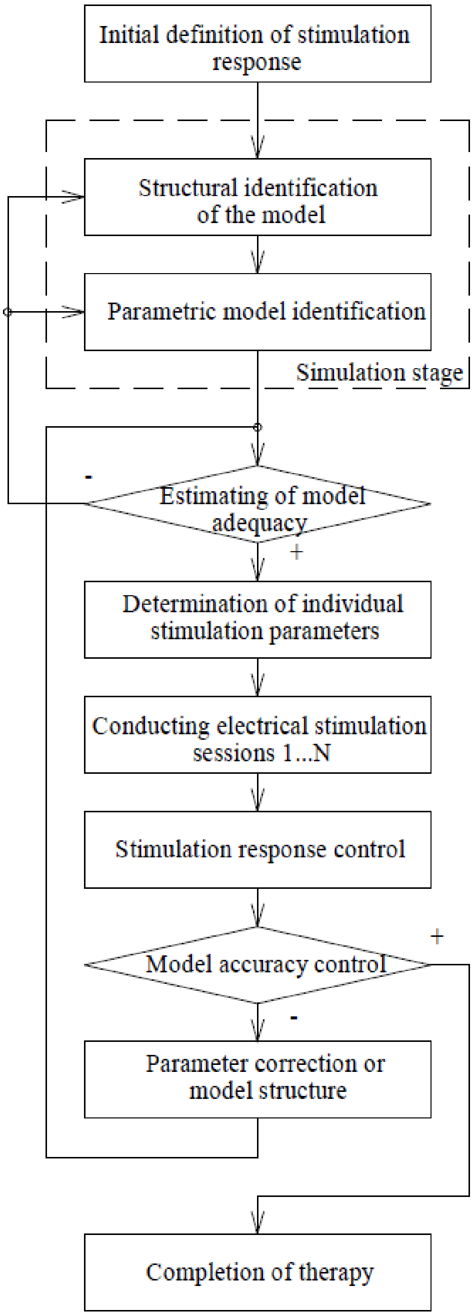

Fig. 1. The diagram of the model correction algorithm

2. Materials and methods of research

In this connection, an indirect method for determining the contraction amplitude using the electromyogram signal is proposed [18]. For this, stimulating effects are supplied to the examined muscle and a stimulation electromyogram is performed. For example, M-responses can be used (with the M-response threshold registration, maximum M-response at maximum and super-maximum stimulation), the H-reflex parameters analysis [8, 19] (Fig. 2).

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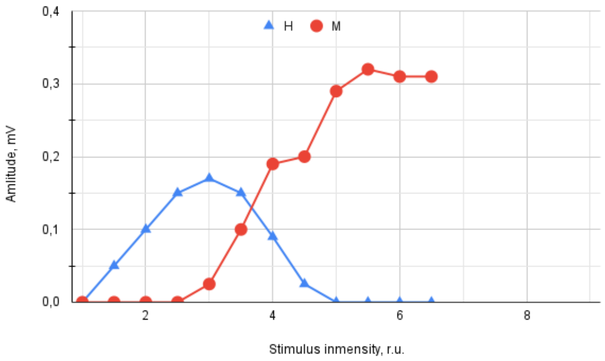

Fig. 2. M-response and H-reflex with increasing strength of stimulation

The model should be as universal as possible, adequate, and have sufficient accuracy and economy. Universality is determined by the model properties complete display. The model accuracy depends on matching experimental characteristics and parameters that are calculated using the mathematical model. Adequacy depends on the ability to display the object properties with accuracy not below the admissible specified. Adequacy that occurs only in a certain area of change in external parameters is the area of adequacy (OA):

$$OA = \{Q \mid \varepsilon_m \le \delta\}, \quad (1)$$

where δ>0 – value of model maximum admissible error; Q- vector of external parameters.

Such a region can be obtained using the methods [20, 21]. An adequacy region has a complex configuration, therefore check of accessory of the OA points requires sufficient computational burden.

Examining the model in various aspects of its use, there are several hierarchical levels of its properties. The first level is formed by the properties of its components, for example, the parameters (coefficients) of functional dependencies. The second, the properties of the model itself (the ability to display certain real processes, and convert signals). Third, the properties of the external environment (for example, the individual signals phase shifts impact, changes in the input signals rate, their amplitudes, etc.).

Quantitatively, these properties are characterized by the following parameters: internal, output and external, respectively. Parameters form vectors

$$X = (x_1,\dots,x_n)^t,\ Y = (y_1,\dots,y_m)^t,$$
$$Q = (q_1,\dots,q_k)^t$$

that are elements of internal QI, output QO and external QE parameter spaces, respectively.

3. Results

When constructing a model with a priori specified ranges of changes in external parameters at some i-th step, it becomes necessary to draw the resulting model OA and check for nesting of required OA in it, typically in the form of hyperparallelepiped

$$QP = \left\{Q \in QE / Q_{i\,min} \le q_i \le q_{i\,max},\ i = \overline{1,k}\right\}. \quad (2)$$

In addition to these inequalities, a point Q^* in the QE space is specified, where the model is constructed and optimized in the internal parameters space according to the minimum error criterion.

Designating the vector of output parameters determined experimentally as $Y_n = (y_{n1},\dots,y_{nm})^t$, the vector quantity

$$E = (\varepsilon_1,\dots,\varepsilon_m)^t, \quad (3)$$

where $\varepsilon_j = (y_j - y_{nj}) / y_{nj}$ – relative modeling error of the j-th parameter, is the model accuracy estimation.

Vector estimation can often be replaced by a scalar one:

$$\varepsilon_M = \|E\|, \quad (4)$$

where $\|E\|$ is the vector norm.

The model adequacy area is understood to be such a space area QE, for which the condition is satisfied $\varepsilon_M \le \delta$, where δ – the model maximum admissible error, i.e.

$$OA = \{Q \in QE \mid \varepsilon_M \le \delta\}. \quad (5)$$

In addition to the nominal OA (OAN) can be used as OA in sensitivity (OAS). It can be used in optimization tasks using first-order methods and is defined as

$$OAS = \{Q \in OAN \mid \| B - B_n \| \le \delta_1\}, \quad (6)$$

where

$$B = \left[\frac{\partial y_j}{\partial q_k} \cdot \frac{q_k^*}{y_j(Q^*)}\right], \quad B_n = \left[\frac{\partial y_{nj}}{\partial q_k} \cdot \frac{q_k^*}{y_{nj}(Q^*)}\right] \ –$$

relative sensitivity matrices.

The nesting of the given OA in the actual one means the possibility of further simplifying the model and increasing its computational efficiency within the admissible error.

Significant costs performing OA constructing stages and its approximation lead to the expediency of having OA estimated approximations that allow checking the match of the resulting OA with the target one. As such an approximation, a hyperparalepiped approximation based on interval methods is proposed.

Interval methods operate with scalar and vector quantities that are finite intervals of real numbers. As there are different approaches to interval mathematics, for certainty, basic interval definitions and properties [22, 22] are used.

If R – all real numbers are set, then the interval A=[a1,a2], a1≤a2, is a closed limited subset A of the set R type of $A=[a1,a2]=\{x/(x\in R)\wedge(a1\le x\le a2)\}$. All intervals set are I(R).

Two intervals A and B are equal when $a_1=b_1$, $a_2=b_2$. The order relation on the set I(R) is defined as follows: A<B when $a_2<b_1$.

The intersection A∩B of intervals A and B is empty if A<B or B<A, otherwise $A\cap B=[\max\{a_1,b_1\}, \min\{a_2,b_2\}]\in I(R)$. The width w(A) of interval A is the value $w(A)=a_2-a_1$. The midpoint m(A) – is the half-sum of the ends of the interval A: $m(A)=(a_1+a_2)/2$.

Interval addition and multiplication are associative and commutative. The roles of zero and one are played by the usual 0 and 1, which are identified with the degenerate intervals [0,0] and [1,1]. If an operand is a non-degenerate interval, the arithmetic operation result is also a non-degenerate interval. The exception is multiplication by 0=[0,0]. Therefore, for a nondegenerate interval A, there are no elements inverse to addition and multiplication, since if A+B=0, AC=1, then A, B, C must be degenerate. That is, subtraction is not inverse to addition, and division is not inverse to multiplication: A–A≠0, A/A≠1, when w(A>0). However, always $0\in A-A, 1\in A/A$.

The important property of interval arithmetic operations is the non-compliance of the distributive law is the non-compliance of the distributive law - the equality A(B+C)=AB+AC is not always the case. However, the inclusion $A(B+C)\subset AB+AC$, called subdistributivity, is always true. The main property of interval calculations is inclusion monotonicity.

The concept of combined and interval extensions of a function is used. Let f – a function defined for $x\in A=(A_1,\dots,A_n)$ with values in R or I(R). A function joint extension f(X) is a function $\bar{f}(X)=\bar{f}(X_1,...,X_n)$, $X_i\subset A_i$, $i=\overline{1,n}$, defined by the equality $\bar{f}(X)=\bigcup_{x\in X} f(x_1,\dots,x_n),\ i=\overline{1,n}.$

If f(X) – a continuous function, then $\bar{f}(X)\in I(R)$, for $X\subset A$. An important property of combined extensions is that $X^{(1)}\subset X^{(2)}$ follows $\bar{f}(X^{(1)})\subset\bar{f}(X^{(2)})$.

f(X) function interval extension is an interval-valued function F of interval variables $X_1,\dots,X_n$ such that:

$$F(X)=F(X_1,\dots,X_n)\supset\bar{f}(X_1,\dots,X_n)=$$
$$=\bar{f}(X)=\{f(X_1,\dots,X_n):(X_1,\dots,X_n)\in X\};$$

$$F(X_1,\dots,X_n)=f(X_1,\dots,X_n),\ x_i\in X_i, i=\overline{1,n}.$$

An interval extension of a continuous real function is inclusion monotone. For a real rational function, a natural interval extension can be constructed. It does so if all real variables are replaced by intervals, and real arithmetic operations are replaced by interval arithmetic. The natural interval extension includes the combined extension.

Then QP is considered an interval vector

$$QP=(Q_1,...,Q_к)^t, \tag{7}$$

where $Q_i=[q_{i\min},q_{i\max}],\ i=\overline{1,к}$.

Its width is:

$$w(QP)=\|w(Q_1),...,w(Q_к)\|, \tag{8}$$

where $w(Q_i)$ is the width of the i-th interval.

The model equations

$$F_n(X_n,Y,Q_n)=0; \tag{9}$$
$$F_M(X_M,Y,Q_M)=0, \tag{10}$$

where F_n, F_M – the model operators, respectively, and $Q_M\le Q_n$.

Replacing the vector components Q_n with the corresponding interval vector components QP, from (9) the interval equation, the solution of which by interval methods gives the output vector interval value Y^n with components

$$Y_i=[y_{li}^n, y_{vi}^n],\ i=\overline{1,m}, \tag{11}$$

where y_{li}^n, y_{vi}^n – the lower and upper limits of the i-th interval, respectively.

Equally, solving equations (10), the interval vector Y^M.

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Let us introduce an interval estimate of the error in modeling the i-th output characteristic $\varepsilon_i = [\varepsilon_{li}, \varepsilon_{vi}]$, where $\varepsilon_{li} = (y^n_{li} - y^м_{li}) / y^n_{li}$, $\varepsilon_{vi} = (y^n_{vi} - y^м_{vi}) / y^n_{vi}$ – the admissible errors corresponding to the lower and upper limits of the interval of the i-th output characteristic. For monotonic characteristics, checking the adequacy of AM is reduced to the performance of the conditions

$$\varepsilon^t_{li} \geq \varepsilon^a_{li}, \varepsilon^t_{vi} \geq \varepsilon^a_{vi}. \quad (12)$$

Interval methods for solving linear equations both with interval coefficients and with the interval right part are currently known. These methods are also applicable to systems of differential equations [24-26]. At the same time, in order to increase the accuracy of the estimates, it is necessary to consider as interval only those parameters, changes in which lead to independent variations in the elements of the model matrix.

For example, in particular, estimating OA of a frequency model with monotonic characteristics, a hyperparallelpiped in QE, space is constructed, which does not contain the dimension corresponding to the input signal frequency. The solution of interval equations (9), (10) is performed for a fixed set from the range $[\omega_{min}, \omega_{max}]$. Taking into account the method proposed above, it was checked whether the condition for the adequacy of such a model is satisfied in the frequency range 0≤ω≤2000 Hz when changing two external parameters with an acceptable level of relative error of 3 %. Parameter 1 and parameter 2 are some equivalent capacitance and resistance, which simulate changes in the current relaxation time constant under the electrodes. Changes in the current relaxation time constant can occur with changes in the intensity of peripheral blood flow during therapy. Therefore, in this case, they will be external.

Using the usual interval arithmetic and interval expansion of functions, we obtain the interval values of the model gain for a set of frequencies in a given range (Table 1).

Table 1

Interval values for the set of rates in a target range

Frequency, Hz	$\lvert\dot{K}_u\rvert$, Parameter 1	$\lvert\dot{K}_u\rvert$, Parameter 2	ε, %
0	0.7576, 0.7937	0.7576, 0.7937	-0, 0
500	0.5757, 0.6031	0.5785, 0.6061	-0.5, 0.5
1000	0.3821, 0.7400	0.3861, 0.4045	-1, 1
1500	0.2745, 0.2876	0.2789, 0.2922	-1.6, 1.6
2000	0.2117, 0.2273	0.2164, 0.2268	-2.2, 2.2

Here, parameter 1 varies in the range 0.05 μF ≤ Ce ≤ 0.1 μF, and parameter 2 in the range 10 kΩ ≤ R_e≤ 20 kΩ, the gain intervals for each frequency value are given in columns 2 and 3, respectively, and the relative error intervals – in column 4. Obviously, for direct current, the error interval is zero; with increasing frequency, it increases due to changes in the relaxation time. But does not exceed the set value of 3%

The values of the interval error ε show that the actual OA is embedded in the given. Thus, the model remains adequate throughout the range.

4. Discussion

Constructing OA is a rather time-consuming procedure. As OA has a complex configuration, the test of points belonging to the adequacy area requires significant computational costs. Therefore, in fact, various OA approximations based on OA limit hypersurfaces simplicial approximation [27] and the hyperfigures fitting into a target area [28] are used.

In fact, the most appropriate OA (OAA) approximation is by a hyperparallelepiped, based on the «growth-motion» algorithm. However, this algorithm has too high computational costs and cannot be used. At the same time, approximation by a hyperparallelepiped, carried out according to the maximum criterion of the minimum approximating edge

$$\min_{OAA \subseteq OA} \max_{i \in [1,k]} (q_{i\,max} - q_{i\,min}) / q^*_i \quad (13)$$

does not guaranty a positive answer to the question about the nesting of the target OA in OAA, even if it is nested in the actual OA. For example, in the case of a two-dimensional space QE with coordinates U_{in} – input signal amplitude and f – input signal frequency, OA for $\varepsilon_м = \varepsilon^*$ looks like the area shown in Fig. 3, a. OAN approximation, performed according to criterion (13), gives OAA, represented by a shaded rectangle (Fig. 3, a). It also shows cases when the model loses its adequacy (Fig. 3, b) or is adequate in the entire target area OAT (Fig. 3, c). The nesting OAT in the actual OA (AOA) is checked according to the nesting conditions specified in the form of the inequality

$$q^t_{i\,max} \leq q^a_{i\,max}, \; q^t_{i\,min} \geq q^a_{i\,min}, i = \overline{1, m}, \quad (14)$$

where indexes «t» and «a» refer to target and actual limits of OA respectively, and m=2.

An analytical mathematical model adequacy estimation method based on the method of interval estimates is suggested.

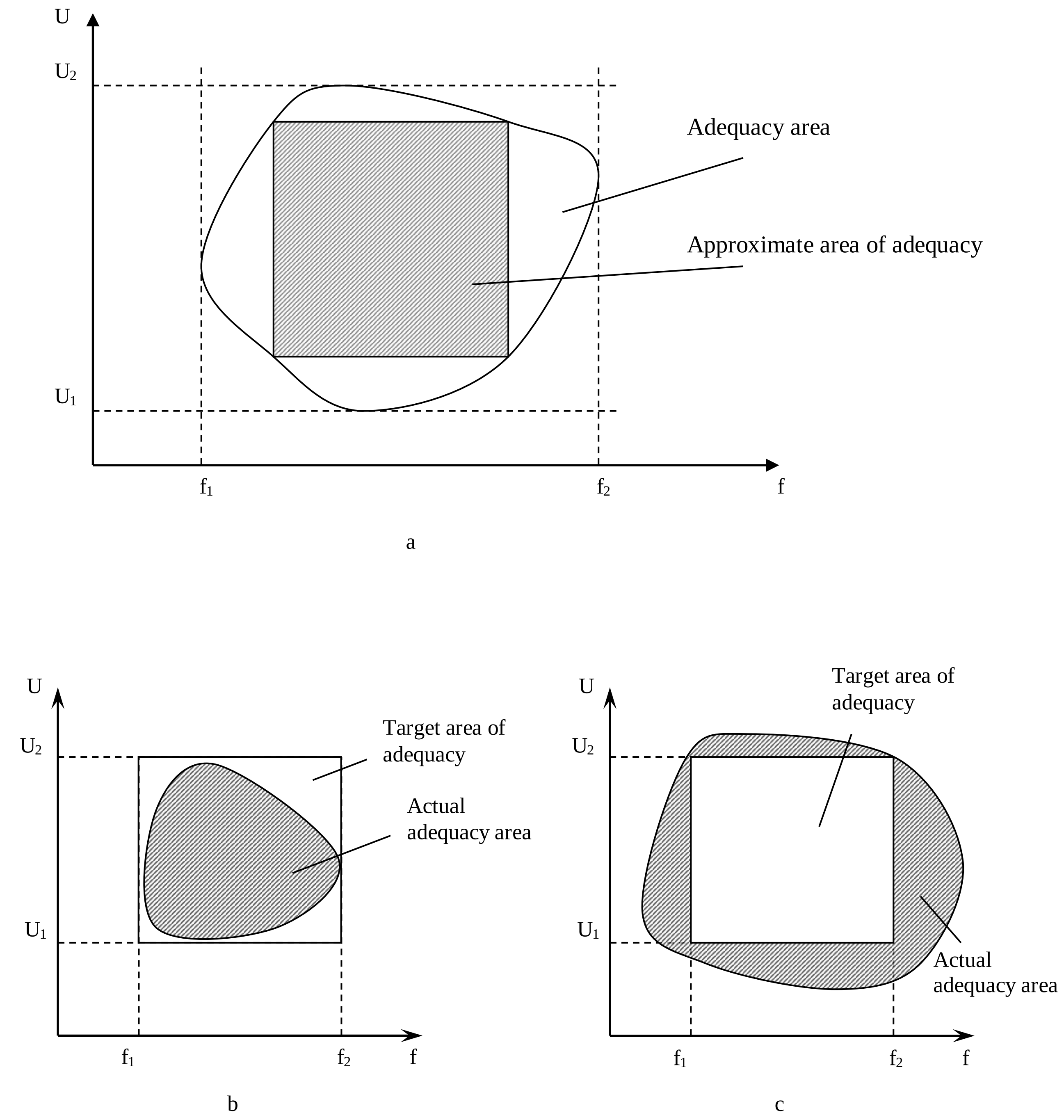

Fig. 3. Model adequacy area determine

This does not avoid multivariate analysis but significantly reduces the number of calculations.

Note that the accuracy of the considered approach decreases as the dimensionality of the space QE increases.

5. Conclusions

The construction of mathematical models depicting the processes during muscular electrostimulation makes it possible to determine the optimal set of stimulation parameters, considering the individual characteristics of the patients. Such models need to be corrected periodically, because during several sessions the electrostimulation characteristics may change significantly. The proposed modeling algorithm makes it possible to take such changes into account. It is especially important in modeling process control to ensure the model adequacy and the admissible level of error. For this purpose, an adequacy area approximation, based on the interval methods used is proposed. This eliminates the need for time-consuming procedures for constructing

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the entire adequacy area. The interval estimation of some modeling accuracy characteristics is introduced, which is reduced to check the inequality performance. As the use cases show, approximation error does not exceed 5 %, which is sufficient for most practical applications.

Main contribution. The proposed method of interval error estimation makes it possible to constantly monitor the adequacy of the model without the use of traditional complex and time-consuming calculations, which do not allow you to quickly assess the possibility of using the model without correcting it.

The results can be used in new electronic medical device automated design based on microcontrollers.

Further research may be related to a more in-depth consideration of nonlinear models, as well as an estimate of the accuracy with an increase in the dimension of the space of external parameters.

Contributions of authors: conceptualization, methodology, analytical review and analysis of information sources – **Igor Prasol**; development of a mathematical model and its structural and parametric identification, selection and use of software tools for modeling and presentation of results, analysis of results, visualization – **Igor Prasol, Olha Yeroshenko**; writing – original draft preparation, writing – review and editing – **Olha Yeroshenko**.

All the authors have read and agreed to the published version of the manuscript.

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МОДЕЛЮВАННЯ ТА ОЦІНКА АДЕКВАТНОСТІ МОДЕЛЕЙ ПРИ ПРОЄКТУВАННІ АПАРАТІВ ДЛЯ ЕЛЕКТРОСТИМУЛЯЦІЇ М'ЯЗОВИХ ТКАНИН

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Об'єкт дослідження – процес математичного моделювання характеристик електростимуляції скелетних м'язів людини під час терапії. Предмет дослідження – математичні моделі електростимуляційних характеристик, які пов'язують амплітуду скорочення м'язів із амплітудою, частотою стимулюючих впливів та ін. параметрами. Мета роботи – дослідження таких моделей, розробка алгоритму їх корекції та методу оцінки адекватності. Методи, що використовуються: методи математичного моделювання, методи структурної та параметричної ідентифікації моделей, методи оптимізації, методи оцінки адекватності моделей, інтервальні методи. Отримані результати: запропоновано алгоритм корекції математичних моделей характеристик електростимуляції м'язів протягом ряду сеансів у разі їх змін; запропоновано спосіб оцінки області адекватності моделі у просторі зовнішніх змінних з метою контролю її адекватності; обґрунтовано застосування методів інтервальної математики для побудови фактичної області адекватності; введено інтервальну оцінку похибки моделювання деякої вихідної характеристики, яка у разі монотонності характеристики дозволяє перевірити збереження адекватності шляхом перевірки виконання низки нерівностей. Результати можуть бути використані в процесі проєктування апаратів електростимуляції та визначення індивідуальних параметрів впливів електростимуляції протягом одного сеансу або ряду сеансів. Наукова новизна: запропоновано та обґрунтовано застосування методів інтервальної математики для апроксимації області адекватності математичної моделі у вигляді гіперпаралепіпеда та перевірки на вкладеність у задану область у просторі зовнішніх параметрів з метою контролю адекватності під час корекції моделі при електростимуляції.

Ключові слова: електростимуляція; скелетний м'яз; математичне моделювання; область адекватності; оцінка; інтервальні методи; проєктування апаратів.

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