# USING THE BEHAVIOR ANTAGONISM AND THE BIMATRIX GAME THEORY IN THE IT PROJECT MANAGEMENT 

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#### Abstract

Context. The article proposes a method of analyzing the proposals of team members in order to avoid conflict situations at the stage of team formation.

Objective. The object of the study is the method of analyzing the proposals of team members while solving project tasks. The purpose of the work is to analyze the developed method of analysis of proposals of team members to avoid or resolve conflict situations at the stage of team activation

Method. The developed method is based on the theory of matrix games. Conflicts between individual team members mainly arise at the stage of team formation. For the project manager, it is important to identify the conflict situation in time and find a way out of it to satisfy both team members and without harming the teamwork as a whole. A team created to implement an IT project is often faced with a situation where two of its members have different visions of approaches to creating the final product. At the same time, each of them has experience in the development of similar software products or services by different teams. To effectively solve this situation, we suggest using approaches typical of bimatrix games, when each of these team members is considered as a player. This takes into account the fact that the bimatrix theory of games is based on a conflict between two players whose interests are opposite - an antagonistic zero-sum game is precisely the basis of the developed approach.

Results. The proposed method of analysis of proposals of team members contributes to the avoidance or resolution of conflict situations at the stage of their closer interaction. For efficient solution of the said situation, we propose to use approaches being typical for bimatrix games when each of these team members is treated as a player. At the same time, account is taken of the fact that the bimatrix game theory is based on a conflict of two players whose interests are opposite - an antagonistic game with a zero sum being that very element to constitute the basis of the approach developed.

Conclusions. The given calculation of the model example shows that the use of the proposed method allows the project manager to give a reasonable preference to another team member, since the expected average profit of this player is greater than that of the first player. In this case, the manager has an opportunity to simulate situations for the players (for the team) and promptly respond to probable deviations of their behavioral strategies from the optimal ones, establish healthy relationships between team members and choose the best proposals for solving project tasks.


KEYWORDS: bimatrix game theory, project manager, IT project, bimatrix games, project team.

## ABBREVIATIONS <br> IT - Information Technology; <br> LO - linear optimization.

## NOMENCLATURE

$\mathrm{A}, \mathrm{B}$ - players of the bimatrix game;
$A_{m}$ - pure player strategies $A$;
$B_{n}$ - pure player strategies $B$;
$C_{A}$ - the payoff matrix of the first player $A$;
$C_{B}$ - the payoff matrix of the second player $B$;
$i, j$ - the Nash equilibrium point;
$X_{a}-$ first player mixed strategy $A$;
$X_{b}-$ mixed strategy of the second player $B$;
$\mathrm{v}_{x}^{A}$ - the price of the game for the player $A$;
$v_{x}^{B}$ - the price of the game for the player $B$;
$\mathrm{a}_{\mathrm{A}}$ - the bottom price of the game for the player $A$;
$\beta_{\mathrm{A}}$ - the upper price of the game for the player $A$;
$\alpha_{\mathrm{B}}$ - the bottom price of the game for the player $B$;
$\beta_{\mathrm{B}}$ - the upper price of the game for the player $B$;
$M_{A}\left(X_{A}, X_{B}\right)$ - the player's mathematical expectation A;
$M_{B}\left(X_{A}, X_{B}\right)$ - the player's mathematical expectation $B$.

## INTRODUCTION

Development of each software product or service is implemented as a unique project requiring creation of a team of realizers. Creation of an efficient team requires time and work input for its development. Special software and project
management methods contribute to quick team formation. Implementation of these tools makes the team's interaction, communication and fulfilment of the task preset more efficient. Many researchers focus their attention on development of team formation algorithms and methods.

The purpose of the article is to develop a mathematical model for choosing optimal strategies for the behavior of IT project members and, on the basis of this model, to optimize the set of starting proposals of project participants in order to increase the effectiveness of its implementation

The objectives of the research are to study the behavior of the participants in a conflict situation using the apparatus of bimatrix game theory and to provide a compromise solution for choosing the proposals of IT project members. Existing approaches in research in this direction are normative and declarative. In view of this, the paper sets the goal of obtaining a specific content-algebraic model of equilibrium according to Nash, and at the second stage, to use the obtained results in project management.

## 1 PROBLEM STATEMENT

Let the task be set: from the point of view of the bimatrix game, it is necessary to analyze the behavior of participants A and B in a conflict situation and provide a compromise solution based on the recommendations for choosing the strategies of the players' behavior $A_{m}$ and Bn. Let's study a conflict between two players $A$ and $B$. The player $A$ may adhere to own pure strategies $\left\{A_{1}, A_{2}, \ldots, A_{m}\right\}$. The player $B$ may choose one arbitrary strategy of the following ones $\left\{B_{1}, B_{2}, \ldots, B_{n}\right\}$, and on the basis of this, make a choice of the situation in the game as a whole. To solve this problem, we use the concept of the optimal situation in the game for each player $X_{a}$ and $X_{b}$, which is called Nash equilibrium.

For solving the situation described, let us use the bimatrix game theory in which the equilibrium situation is called Nash equilibrium and consists in the following: status $\left(i^{*} j^{*}\right)$ of bimatrix game

$$
C_{A}=\left[a_{i j}\right]_{n \times n}, C_{B}=\left[b_{i j}\right]_{n \times n}
$$

is a Nash equilibrium point subject to meeting the condition for values of the of the first player $A$ proposal matrix

$$
a_{i j^{*}} \leq a_{i^{*} j^{*}}, i=1, \ldots, n
$$

as well as meeting the condition for values of the second player $B$ proposal matrix

$$
b_{i^{*} j} \leq b_{i^{*} j^{*}}, j=1, \ldots, n .
$$

Existing approaches in research in this direction are normative and declarative in nature. In this case, the goal of the work is to obtain a specific content-algebraic model of equilibrium according to Nash and, at the second stage, to use the obtained results in project management as a mathematical expectation of the players $M_{A}\left(X_{A}, X_{B}\right)$ та $M_{B}\left(X_{A}, X_{B}\right)$.

## 2 REVIEW OF THE LITERATURE

The authors of article "A model for project team formation in complex engineering projects under uncertainty: A knowledge-sharing approach" [1] Mahdi Hosseini, Peyman Akhavan proposed a model based on knowledge sharing among separate team members for the aim of optimizing distribution of the load between them. Their approaches to project team formation are formulated as a fuzzy multipurpose model of integer programing $0-1$, they adopt the approach of fuzzy multipurpose programing with limited odds improving the efficiency of decision taking as to abilities of the candidates under conditions of uncertainty. The authors propose to solve the problems by combining the genetic non-dominated sorting algorithm II with algorithms of fuzzy modeling. The calculation results have been proved and the efficiency of such a combination clearly shown in provision of Paretooptimal solutions while creating the project team.

Margarita André Ampuero in her paper "Developing a Model and a Tool for the Formation of Project Team" [2] proposed an algorithm of a high-quality team formation for successful implementation of a project. The author carried out a deep analysis of models and tools for formation of project teams starting from detection of various factors having impact on these processes. She described the basic characteristics of the model and versions of a configured tool supporting the model and helping using it in various contexts enabling experiments with different solution algorithms and methods for determining those proposing the best results.
D.Strnad, N.Guid in the paper "A fuzzy-genetic decision support system for project team formation" [3] considered the problem of forming the optimum team because they believe that formation of the optimum team including several dozens of members requires suitable tools. In the researchers' opinion, the selection process is normally clearly defined, with formation of criteria for each team that concern necessary abilities of the team members. These abilities can be arbitrarily combined in the personnel and the target function becomes selfconflicting. This makes a team formation more difficult and requires a special software support. The research presents a fuzzy genetic analytical model for a project team formation based on quantity approaches but including the possibility of determining personnel's attributes from dynamic quantity data, modeling complicated attributes and processing the required excessive competence. This approach provides a specification of requirements using fuzzy descriptors and determining the formulated target function contributing to maximization. For optimizing the
selection of several project teams with probably conflicting requirements, the authors proposed special adaptation of an island genetic algorithm with a mixed crossover where the suitability of a joint solution is used for governing the selection within the islands.

Nataliia Dotsenko, Dmytro Chumachenko and Igor Chumachenko [4] in their paper "Project-oriented management of adaptive teams' formation resources in multiproject environment" took the processes of projectoriented management of resources for creation of adaptating teams in a multiproject environment. The researchers proposed models and methods of project-oriented management of resources for creation of adaptating teams in a multiproject environment.

They proposed a set of methods for determining human resources strategies in a multiproject environment providing for usage of stratified representation of resources management processes in a multiproject environment that will enable analyzing the human resources management processes. The paper has analyzed the specifics of the developed method based on the interested parties in management of human resources for projects in a multiproject environment.

The paper [5] "Formation of IT Project Implementation Team" provides an analysis of a project team formation criteria and the team effect on the project success. It is indicated that a team is selected for each project at the first stage of its implementation. The authors understand a project team as a group of like-minders motivated to implement the project. The paper an original order of forming a project group providing for selection of realizers under defined criteria based on weight factors assigned by experts. The assessment is based on competences acquired by graduates of IT faculties and represented in vocational training programs [6].

For solving such a problem, the authors proposed to create a mathematical support to construction of models for assessment and control of technical and manufacturing process values with use of artificial neural networks [7]. Their solution provides for proposing models enabling automatic selection of a subset of minimum-size specimens from the initial sample [8]. The approach proposed by the researchers is applicable to various forms of exclusion procedures used in games, particularly the iterative removal of strictly dominated strategies and removal of weakly dominated strategies.

However, for describing competitive situations with participation of two players, it makes more sense, in the researchers' opinion, to use matrix games as zero sum games for two players with finite sets of strategies with presumption of matrix games being interesting in terms of the analysis simplicity and structure specifics. Von Neumann and Morgenstern deemed it reasonable to use linear programming for solving these games. The described researches are based on analyzing matrix games in the context of pure strategies through such key notions and they show the pure strategic Nash equilibriums to be saddle points [9, 10]. The authors proposed the optimum strate-
gies and the game value through solving a linear programming problem [11].

The researchers proposed new interpretations to the known method as to the competition of individuals in pairwise interaction, they presented a class of models for situations including more than two persons [12].

The author [13] consider using the matrix game theory in case of the randomness matrix being winning. They present several model solutions for this type of game based on the win function characteristics and on the theory of optimization with probability boundaries.

## 3 MATERIALS AND METHODS

At the modern stage of the IT branch existence, with development of modern software products and services, special attention is paid to creation of the team with account taken of the project specifics. Creation of a professional team for each project is one of important commitments of the project manager who supervises the project in general, controls its basic parameters and coordinates activities of the team members. The project manager defines their number and professional competences. A professional team is created with the idea that it is not just a set of employees but a complicated social system with fixed interaction mechanisms the activities of which have effect on the project lifecycle processes.

Correctly selected team members guarantee the project to be successfully implemented in compliance with its period and budget. The experience of many IT projects showed that the team, as a rule, passes several stages in the project implementation project: formation, getting into closer interaction, functioning and dissolution. In this article, let us focus on the first two stages.

## 4 EXPERIMENTS

While forming a team, the project manager gathers a group of people uniting them by a common purpose. The team formation process specifics consist in the fact that the team members do not know each other and at this stage, they get to know each other, study the project product specifics, rules and regulations of interaction. This stage also includes setting tasks to each team member and determining the ways and methods of their fulfillment. Getting into closer interaction. Creation of a team for a new project also becomes more difficult due to the fact that operation of the team, as an integral system, is to be efficient and synchronized. Development of the corporate feeling and formation of the general intera ction rules inside such a team require some time. For successful project implementation, its team needs to get united before the beginning of an IT product development..

The well-coordinated work of a team fulfilling its joint tasks is to begin at this very stage. It is distinguished by increased probability of conflicts, which is caused by difference of team member characters, approaches and methods of task fulfillment. The team accommodates the growth of leaders, establishment of informal groups, determining functions of separate team members, formation of the group climate, internal culture, etc.

This very stage, of team members getting into closer interaction, is associated with conflicts appearing between separate team members. For the project manager, it's extremely important to reveal a conflict situation in proper time and to find a way out of it to the satisfaction of the both team members and without harm to the teamwork in general. In our opinion, it is important for the project manager to use matrix games in such a situation.

The matrix game theory was formed on a conflict of two players with purely opposite interests - an antagonistic game with a zero sum. In the real life, conflict situations use to appear much more frequently, in which the interests of the players are not opposite any more, although not coinciding.

Lets study a conflict between two players $A$ and $B$. The player $A$ may adhere to own pure strategies

$$
\left\{A_{1}, A_{2}, \ldots, A_{m}\right\} .
$$

The player $B$ may choose one arbitrary strategy of the following ones

$$
\left\{B_{1}, B_{2}, \ldots, B_{n}\right\} .
$$

As they are participants to a team game where certain rules are introduced, their possible combinatory choice is subjected to a single-value estimation with a reward. Thus, if the player $A$ chose strategy $A_{i}$ and the player $B$ chose strategy $B_{j}$, the reward $A$ will be equal to $a_{i j}$. The win of another player $B$ under these conditions will be equal to $b_{s j}$. In this case

$$
a_{i j} \neq b_{i j}
$$

however, each player receives own win (winning situation). Such a serial search of strategies by the players determines two tables of wins (winning situations)
Table 1-Player A's reward table

| $C_{A}$ | $B_{1}$ | $B_{2}$ | $\cdots$ | $B_{n}$ |
| :--- | :--- | :--- | :--- | :--- |
| $A_{1}$ | $a_{11}$ | $a_{12}$ | $\cdots$ | $a_{1 n}$ |
| $A_{2}$ | $a_{21}$ | $a_{22}$ | $\cdots$ | $a_{2 n}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_{m}$ | $a_{m 1}$ | $a_{m 2}$ | $\cdots$ | $a_{m n}$ |


| $C_{B}$ | $B_{1}$ | $B_{2}$ | $\cdots$ | $B_{n}$ |
| :--- | :--- | :--- | :--- | :--- |
| $A_{1}$ | $b_{11}$ | $b_{12}$ | $\cdots$ | $b_{1 n}$ |
| $A_{2}$ | $b_{21}$ | $b_{22}$ | $\cdots$ | $b_{2 n}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_{m}$ | $b_{m 1}$ | $b_{m 2}$ | $\cdots$ | $b_{m n}$ |

A team created for implementation of an IT project often encounters a situation when two of its members have a different vision of approaches to the final product
creation. In this case, each of them has a big experience in developing similar software products or services in different teams. For efficient solution of the said situation, we propose to use approaches being typical for bimatrix games when each of these team members is treated as a player.

Using the first team member concept for creating a software product can be positively evaluated at $r(r>0)$ units, and the second team member concept is evaluated at $t(t>0)$ some conventional units.

Let us assume that the conditional benefit of using other proposals is equal to zero. The team of developers will gain additional positive benefit $s(s>0)$ subject to using the proposal of the first player enabling adoption of the experience of the previous team of developers who worked at the same company. Let us use the following calculations for taking the optimum decision and gaining the greatest benefit.

We have a typical bimatrix game. The first player $A-$ submitted the first proposal. The second player $B$ - the second proposal. Each of the players has two strategies using the first proposal $A_{1}, B_{1}$ or the second one $-A_{2}, B_{2}$. Therefore

$$
A=\left[A_{1}, A_{2}\right], B=\left[B_{1}, B_{2}\right] .
$$

According to the game rules, the matrixes of benefits are equal for the players:

Player $A$

| $C_{A}$ | $B_{1}$ | $B_{2}$ |
| :---: | :---: | :---: |
| $A_{1}$ | $r$ | $r+s$ |
| $A_{2}$ | $S$ | 0 |

Player $B$

| $C_{B}$ | $B_{1}$ | $B_{2}$ |
| :---: | :---: | :---: |
| $A_{1}$ | $t$ | $S$ |
| $A_{2}$ | $t+s$ | 0 |

or in a matrix form:

$$
C_{A}=\left[\begin{array}{cc}
r & r+s  \tag{1}\\
s & 0
\end{array}\right], C_{B}=\left[\begin{array}{cc}
t & s \\
t+s & 0
\end{array}\right] .
$$

For a bimatrix game, as well as for a matrix one, the created matrixes provide a quantity description of certain statuses in the game in which the interests of the players do not coincide. The main task of the bimatrix game theory is providing recommendations on players behavior with the aim to get the optimum result in a conflict situation of the game.

The optimum game result is reasonable behavior of the players and following the concept of balancing. As the interests of players are different, the project manager has to find a compromise solution that would satisfy as far as possible the both players and the team in general. In fact, this means that the recommended situation in the game is when departure from it would not increase the players'
wins but reduce them the other way round, or be a compromise in the best case. This status in the game is called equilibrium.

In the antagonistic matrix game theory, the equilibrium situation is associated with the saddle point. For solving the situation described, let us use the bimatrix game theory in which the equilibrium situation is called Nash equilibrium and consists in the following: status $\left(i^{*} j^{*}\right)$ of bimatrix game

$$
C_{A}=\left[a_{i j}\right]_{n \times n}, C_{B}=\left[b_{i j}\right]_{n \times n},
$$

is a Nash equilibrium point subject to meeting the condition for values of the of the first player $A$ proposal matrix

$$
a_{i j^{*}} \leq a_{i^{*} j^{*}}, i=1, \ldots, n
$$

as well as meeting the condition for values of the second player $B$ proposal matrix

$$
b_{i^{*} j} \leq b_{i^{*} j^{*}}, j=1, \ldots, n
$$

In other words, the locations of the matrixes accommodating at the same time the biggest values in columns for matrix $C_{A}$ of the first player $A$ and in rows for matrix $C_{B}$ of the second player $B$ - are the Nash equilibrium points.

The team assigns certain weight factors from 0 to 9 to the arguments in favor of the proposal of each player. If the bimatrix game is set by matrixes:

$$
C_{A}=\left[\begin{array}{ccc}
2 & 5 & 6  \tag{2}\\
6 & 7 & 1 \\
6 & 3 & 6
\end{array}\right], C_{B}=\left[\begin{array}{ccc}
3 & 7 & 8 \\
7 & 8 & 1 \\
8 & 4 & 4
\end{array}\right] .
$$

For finding possible equilibrium points in pure strategies for the first player $\boldsymbol{A}$, we need to choose the biggest values in columns of matrix $C_{A}$. For visual clarity, we mark these elements with circles. We need to choose the biggest values in rows to determine the candidates for the second player's equilibrium point. Let us encircle them as well for visual clarity.

The locations encircled in the both matrixes determine the Nash equilibrium points. In the first example, we have three such points - $\left(A_{1}, B_{3}\right) ;\left(A_{2}, B_{2}\right) ;\left(A_{3}, B_{1}\right)$. Each of these points sets the optimum dyad of the players' pure strategies:

$$
\begin{gathered}
\left(A_{1}, B_{3}\right) \rightarrow\left[\begin{array}{l}
X_{A_{1}}=[1,0,0] \\
X_{B_{3}}=[0,0,1]
\end{array},\left(A_{2}, B_{2}\right) \rightarrow\left[\begin{array}{l}
X_{A_{2}}=[0,1,0] \\
X_{B_{2}}=[0,1,0]
\end{array}\right.\right. \\
\left(A_{3}, B_{1}\right) \Rightarrow\left[\begin{array}{l}
X_{A_{3}}=[0,0,1] . \\
X_{B_{1}}=[1,0,0]
\end{array}\right.
\end{gathered}
$$

If the bimatrix game is set by matrixes

$$
C_{A}=\left[\begin{array}{lcc}
6 & 9 & 2 \\
3 & 2 & 7 \\
9 & 8 & 7
\end{array}\right], \quad C_{B}=\left[\begin{array}{ccc}
8 & 4 & 2 \\
1 & 7 & 9 \\
6 & 1 & 8
\end{array}\right] .
$$

In this example, we have one equilibrium point

$$
\left(A_{2}, B_{3}\right) \Rightarrow\left[\begin{array}{l}
X_{A_{2}}=[0,1,0] \\
X_{B_{3}}=[0,0,1]
\end{array}\right.
$$

If the bimatrix game is set by matrixes

$$
C_{A}=\left[\begin{array}{ccc}
9 & 7 & 1 \\
1 & 99 & 9 \\
9 & 6 & 7
\end{array}\right], \quad C_{B}=\left[\begin{array}{ccc}
1 & 8 & 4 \\
5 & 1 & 2 \\
4 & 1 & 8
\end{array}\right] .
$$

For this matrix game, there is no equilibrium in pure strategies. If the bimatrix game is set by matrixes

$$
C_{A}=\left[\begin{array}{cc}
-a & (a) \\
0 & 0
\end{array}\right] \quad C_{B}=\left[\begin{array}{cc}
0 & -a \\
0 & a
\end{array}\right] .
$$

For this couple of matrixes, a Nash equilibrium does not exist.

For bimatrix games, like in the matrix game theory, equilibrium points do not always exist like in variant 3 , which just confirms such a case. Therefore, it makes sense to move from pure strategies to mixed ones. In this case, the players repeat many times their pure strategies to search the equilibrium, however, with certain frequencies (probabilities).

Should the game be repeated many times with constant conditions, player $A$ applies own strategies $\left\{A_{1}, A_{2}, \ldots\right.$, $\left.A_{n}\right\}$ with respective frequencies $\left\{p_{1}, p_{2}, \ldots, p_{n}\right\}$,

$$
\begin{equation*}
\sum_{i=1}^{n} p_{i}=1, X_{A}=\left[p_{1}, p_{2}, \ldots, p_{n}\right] \tag{3}
\end{equation*}
$$

A move is carried out to mixed strategies by defining frequencies of using pure strategies for the first player.

Player $B$ applies frequencies

$$
\begin{equation*}
X_{B}=\left[q_{1}, q_{2}, \ldots, q_{n}\right], \sum_{i=1}^{n} q_{i}=1 \tag{4}
\end{equation*}
$$

of own strategies $\left\{B_{1}, B_{2}, \ldots, B_{n}\right\}$.
The mixed strategies allowed broadening the tasks of bimatrix games and finding possibilities of more reasonable distribution of wins in the game as average values -
mathematic expectations set by matrix distributions $\mathrm{C}_{\mathrm{A}}$ and $\mathrm{C}_{\mathrm{B}}$, and calculating by the formulae:

$$
\begin{gather*}
M_{A}\left(X_{A}, X_{B}\right)=\sum_{i=1}^{n} \sum_{j=1}^{n} a_{i j} p_{i} q_{j}  \tag{5}\\
M_{B}\left(X_{A}, X_{B}\right)=\sum_{i=1}^{n} \sum_{j=1}^{n} b_{i j} p_{i} q_{j} . \tag{6}
\end{gather*}
$$

Status $\left(X^{*}{ }_{A}, X^{*}{ }_{B}\right)$ in the bimatrix game system is given by frequencies

$$
\begin{equation*}
X_{A}^{*}=\left[p_{1}^{*}, p_{2}^{*}, \ldots, p_{n}^{*}\right] \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
X_{B}^{*}=\left[q_{1}^{*}, q_{2}^{*}, \ldots, q_{n}^{*}\right] \tag{8}
\end{equation*}
$$

as a Nash equilibrium in mixed strategies, if the following conditions are met for any $X_{A}$ and $X_{B}$ :

$$
\begin{align*}
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right) & \geq M_{A}\left(X_{A}, X_{B}^{*}\right),  \tag{9}\\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right) & \geq M_{B}\left(X_{A}^{*}, X_{B}\right) \tag{10}
\end{align*}
$$

These conditions of a bimatrix game equilibrium status are interpreted as follows: departure from the system status $\left(X^{*}{ }_{A}, X^{*}{ }_{B}\right) \quad$ by one of the players, subject to reserving their own choice by the others, does not enable this player to increase the win. In other words, changing the equilibrium status in the bimatrix game system makes no sense for any of the players.

Let the player $A$ have pure strategies $\left\{A_{1}, A_{2}, \ldots A_{n}\right\}$. Each of them correlates with vector of frequencies $X_{A i}$, $i=1,2, \ldots, n$. Thus, pure strategy $\mathrm{A}_{1}$ corresponds to frequency vector

$$
\begin{equation*}
X_{A_{1}}=[1,0, \ldots, 0] \tag{11}
\end{equation*}
$$

Similarly, for the other player $B$

$$
\begin{equation*}
X_{B_{1}}=[1,0, \ldots, 0] \tag{12}
\end{equation*}
$$

Therefore, frequency vectors $X_{A i}$ and $X_{B i}, i=1,2, \ldots, n$. have been put to correspond to each pure strategy of the players.

In this case, the system of conditions describing point $\left(X^{*}{ }_{A}, \quad X^{*}{ }_{B}\right)$ of the bimatrix game equilibrium can be recorded as follows:

$$
\left\{\begin{array}{c}
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{1}}, X_{B}^{*}\right) \geq 0 \\
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{2}}, X_{B}^{*}\right) \geq 0 \\
\vdots  \tag{13}\\
\vdots \\
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{n}}, X_{B}^{*}\right) \geq 0 \\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{1}}\right) \geq 0 \\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{2}}\right) \geq 0 \\
\vdots \\
\vdots \\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{n}}\right) \geq 0
\end{array}\right.
$$

Take note that the relation symbols between the parts of conditions of system (13) can be rechecked following the principle of limit solutions in the theory of linear (in our case relative to products $p_{i} q_{i}$ ) inequalities (complementary slackness) [6]. Thus, if at equilibrium point ( $X^{*}{ }_{A}$, $X^{*}{ }_{B}$ ) component $p_{i}^{*}>0$, the respective inequality is solved as an equation (so-called active correlation)

$$
\begin{equation*}
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{i}}, X_{B}^{*}\right)=0 \tag{14}
\end{equation*}
$$

and if $q^{*} \gg 0-$ we have equality

$$
\begin{equation*}
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{j}}\right)=0 \tag{15}
\end{equation*}
$$

Let $\left(X_{A}^{*}, \quad X^{*}{ }_{B}\right)$ be an equilibrium point in mixed bimatrix game strategies. In this case, from the condition of the so-called passive correlation

$$
\begin{equation*}
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{i}}, X_{B}^{*}\right)>0 \tag{16}
\end{equation*}
$$

we get $\mathrm{p}^{*}>0 p_{i}^{*}=0$, and from condition

$$
\begin{equation*}
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{j}}\right)>0 \tag{17}
\end{equation*}
$$

we get $q^{*}>0$.
The system of conditions (13) lets us study a bimatrix game from the normative point of view and always detect equilibrium points in mixed strategies. This approach is a consequence of the known principle of limit solutions in the theory of linear inequalities. Each system of linear inequalities compatible with rang $>0$, above the real number field, contains at least one subsystem of the same rang and with a number of inequalities equal to this rang, each solution of which converts the inequality into equation and complies with the initial system. In our case, the system is linear relative to products $p_{i} q_{i}$.

The full solution of the problem will include a bimatrix game solution based on correlations of the condition of Nash equilibrium (13) and two couples of antagonistic matrix games with a zero sum.

Let us have a bimatrix game given by matrixes $\mathrm{C}_{\mathrm{A}}, \mathrm{C}_{\mathrm{B}}$ of two players $A$ and $B$.

$$
C_{A}=\left[\begin{array}{lll}
2 & 5 & 6 \\
6 & 7 & 1 \\
6 & 3 & 6
\end{array}\right], C_{B}=\left[\begin{array}{lll}
3 & 7 & 8 \\
7 & 8 & 1 \\
8 & 4 & 4
\end{array}\right] .
$$

The frequency vectors of mixed solutions of the first and second players are respectively equal to:

$$
\begin{align*}
X_{A} & =\left[p_{1}, p_{2}, p_{3}\right]=\left[p_{1}, p_{2}, 1-p_{1}-p_{2}\right],  \tag{18}\\
X_{B} & =\left[q_{1}, q_{2}, q_{3}\right]=\left[q_{1}, q_{2}, 1-q_{1}-q_{2}\right] . \tag{19}
\end{align*}
$$

In this case, the game value for player $A$

$$
\begin{gather*}
v_{x}^{A}=M_{A}\left(X_{A}, X_{B}\right)=\sum_{i=1}^{a} \sum_{j=1}^{a} a_{i j} p_{i} q_{j}=\sum_{i=1}^{3} \sum_{j=1}^{3} a_{i j} p_{i} q_{j}=  \tag{20}\\
=\left(-4 q_{1}+2 q_{2}\right) p_{1}+\left(5 q_{1}+9 q_{2}-5\right) p_{2}-3 q_{2}+6 .
\end{gather*}
$$

and for player $B$

$$
\begin{gather*}
v_{x}^{B}=M_{B}\left(X_{A}, X_{B}\right)=\sum_{i=1}^{3} \sum_{j=1}^{3} b_{i j} p_{i} q_{j}=  \tag{21}\\
=\left(-9 p_{1}+2 p_{2}+4\right) q_{1}+\left(-p_{1}+7 p_{2}\right) q_{2}+4 p_{1}-3 p_{2}+4 .
\end{gather*}
$$

For calculation of the problem solution, we compose system of conditions (13)

$$
\left\{\begin{array}{l}
\left(-4 q_{1}+2 q_{2}\right) p_{1}+\left(5 q_{1}+9 q_{2}-5\right) p_{2} \geq 0  \tag{22}\\
\left(4 q_{1}-2 q_{2}\right)\left(1-p_{1}\right)+\left(5 q_{1}+9 q_{2}-5\right) p_{2} \geq 0 \\
\left(-4 q_{1}+2 q_{2}\right) p_{1}+\left(5 q_{1}+9 q_{2}-5\right)\left(p_{2}-1\right) \geq 0 \\
\left(-9 p_{1}+2 p_{2}+4\right) q_{1}+\left(-p_{1}+7 p_{2}\right) q_{2} \geq 0 \\
\left(9 p_{1}-2 p_{2}-4\right)\left(1-q_{1}\right)+\left(-p_{1}+7 p_{2}\right) q_{2} \geq 0 \\
\left(-9 p_{1}+2 p_{2}+4\right) q_{1}+\left(p_{1}-7 p_{2}\right)\left(1-q_{2}\right) \geq 0
\end{array}\right.
$$

Solution of system (12) looks as follows:

$$
\begin{gathered}
X_{A}=\left[\frac{28}{61}, \frac{4}{61}, \frac{29}{61}\right] \approx[0.46,0.07,0.47] \Rightarrow[46 \%, 7 \%, 47 \%] \\
v_{A}^{x}=\frac{108}{23} \approx 4,70 \\
X_{B}=\left[\frac{5}{23}, \frac{10}{23}, \frac{8}{23}\right] \approx[0.22,0.43,0.35] \Rightarrow[22 \%, 43 \%, 35 \%]
\end{gathered}
$$

$$
v_{B}^{x}=\frac{344}{61} \approx 5.64
$$

We form the first matrix game

$$
\left\{\begin{array}{c}
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{1}}, X_{B}^{*}\right) \geq 0 \\
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{2}}, X_{B}^{*}\right) \geq 0 \\
\vdots \\
\vdots  \tag{23}\\
M_{A}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{A}\left(X_{A_{n}}, X_{B}^{*}\right) \geq 0 \\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{1}}\right) \geq 0 \\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{2}}\right) \geq 0 \\
\vdots \\
\vdots \\
M_{B}\left(X_{A}^{*}, X_{B}^{*}\right)-M_{B}\left(X_{A}^{*}, X_{B_{n}}\right) \geq 0
\end{array}\right.
$$

Solution of the problem by simplex method is given in Table 1.

From the last simplex table, we obtain:

$$
\begin{aligned}
& \quad x_{1}^{* A}=\frac{5}{72}, x_{2}^{* A}=\frac{1}{18}, x_{3}^{* A}=\frac{19}{216} \cdot \text { As } \\
& x_{1}^{* A}+x_{2}^{* A}+x_{3}^{* A}=\frac{1}{v_{A}^{x}} \text { and } x_{i}^{* A}=\frac{x_{i}^{A}}{v_{A}^{x}}, \\
& i=1,2,3 \text { we get: } \\
& X_{A}^{m}=\left[\frac{15}{46}, \frac{6}{23}, \frac{19}{46}\right] \approx[0.33,0.26,0.41] \Rightarrow[33 \%, 26 \%, 41 \%],
\end{aligned}
$$

$$
v_{A}^{x}=\frac{108}{23} \approx 4.70
$$

Let's compose the next matrix game.

Table 3 - Solving the problem by the simplex method

| Basis $C$ | $B$ | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ | $a_{5}$ | $a_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 1 | 1 | 0 | 0 | 0 |


| $a_{4}$ | 0 | -1 | -2 | -6 | -6 | 1 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{5}$ | 0 | -1 | -5 | -7 | -3 | 0 | 1 | 0 |
| $a_{6}$ | 0 | -1 | -6 | -1 | -6 | 0 | 0 | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{0}\right)=$ | 0 | -1 | -1 | -1 | 0 | 0 | 0 |


| $a_{1}$ | 1 | $1 / 2$ | 1 | 3 | 3 | $-1 / 2$ | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{5}$ | 0 | $3 / 2$ | 0 | 8 | 12 | $-5 / 2$ | 1 | 0 |
| $a_{6}$ | 0 | 2 | 0 | 17 | 12 | -3 | 0 | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{1}\right)=$ | $1 / 2$ | 0 | 2 | 2 | $-1 / 2$ | 0 | 0 |


| $a_{1}$ | 1 | $-1 / 16$ | 1 | 0 | $-3 / 2$ | $7 / 16$ | $-3 / 8$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $3 / 16$ | 0 | 1 | $3 / 2$ | $-5 / 16$ | $1 / 8$ | 0 |
| $a_{6}$ | 0 | $-19 / 16$ | 0 | 0 | $-27 / 2$ | $37 / 16$ | $-17 / 8$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{2}\right)=$ | $1 / 8$ | 0 | 0 | -1 | $1 / 8$ | $-1 / 4$ | 0 |


| $a_{1}$ | 1 | $5 / 72$ | 1 | 0 | 0 | $13 / 72$ | $-5 / 36$ | $-1 / 9$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $1 / 18$ | 0 | 1 | 0 | $-1 / 18$ | $-1 / 9$ | $1 / 9$ |
| $a_{3}$ | 1 | $19 / 216$ | 0 | 0 | 1 | $-37 / 216$ | $17 / 108$ | $-2 / 27$ |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{\text {opt }}\right)=$ | $23 / 108$ | 0 | 0 | 0 | $-5 / 108$ | $-5 / 54$ | $-2 / 27$ |

We solve the optimization problem by simplex method as per Table 4 below.

$$
\begin{gather*}
\mathrm{W}_{\mathrm{II}}^{* A}=y_{1}^{* A}+y_{2}^{* A}+y_{3}^{* A} \rightarrow \max , \\
\Omega_{\mathrm{II}}^{* A}:\left\{\begin{array}{l}
2 y_{1}^{* A}+5 y_{2}^{* A}+6 y_{3}^{* A} \leq 1, \\
6 y_{1}^{* A}+7 y_{2}^{* A}+y_{3}^{* A} \leq 1, \\
6 y_{1}^{* A}+3 y_{2}^{* A}+6 y_{3}^{* A} \leq 1,
\end{array}\right.  \tag{25}\\
y_{i}^{* A}=\frac{y_{i}^{A}}{v_{A}^{v}}, y_{i}^{A} \in[0,1], i=1,2,3 .
\end{gather*}
$$

From the last simplex table, we obtain:
$y_{1}^{* A}=\frac{2}{27}, y_{2}^{*^{A}}=\frac{5}{54}, y_{3}^{*_{A}}=\frac{5}{108} . \mathrm{As}$
$y_{1}^{* A}+y_{2}^{* A}+y_{3}^{* A}=\frac{1}{v_{A}^{y}}$ and $y_{i}^{* A}=\frac{y_{i}^{A}}{v_{A}^{y}}$,
$I=1,2,3$ we get:
$Y_{A}^{m}=\left[\frac{5}{23}, \frac{10}{23}, \frac{8}{23}\right] \approx[0.22,0.43,0.35] \Rightarrow$
[ $22 \%, 43 \%, 35 \%]$.

$$
v_{A}^{y}=\frac{108}{23} \approx 4,70 .
$$

We evaluate the lower and the upper boundaries of the game result for matrix $C_{B}$ of the other player, being equal to $\alpha_{B}=\operatorname{maxmin} C_{B}=4$ and

$$
\begin{gather*}
\mathrm{W}_{\mathrm{I}}^{* B}=x_{1}^{{ }^{B B}}+x_{2}^{*^{B}}+x_{3}^{* B} \rightarrow \mathrm{~min}, \\
\Omega_{\mathrm{I}}^{{ }^{* B}}:\left\{\begin{array}{l}
3 x_{1}^{* B}+7 x_{2}^{{ }^{* B}}+8 x_{3}^{* B} \geq 1, \\
7 x_{1}^{* B}+8 x_{2}^{*_{B}}+4 x_{3}^{* B} \geq 1, \\
8 x_{1}^{* B}+x_{2}^{* B}+4 x_{3}^{*_{B} B} \geq 1,
\end{array}\right.  \tag{27}\\
x_{i}^{*_{B}}=\frac{x_{i}^{B}}{v_{B}^{x}}, x_{i}^{B} \in[0,1], i=1,2,3 .
\end{gather*}
$$

$\beta_{\mathrm{B}}=\operatorname{minmax} \mathrm{C}_{\mathrm{B}}=8$
The first one looks as follows.

Table 4 - Optimization problem by simplex method

| Basis $C$ | $B$ | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ | $a_{5}$ | $a_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 1 | 1 | 0 | 0 | 0 |


| $a_{4}$ | 0 | 1 | 2 | 5 | 6 | 1 | 0 | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{5}$ | 0 | 1 | 6 | 7 | 1 | 0 | 1 | 0 |
| $a_{6}$ | 0 | 1 | 6 | 3 | 6 | 0 | 0 | 1 |
| $\Delta_{j}$ | $\mathrm{~W}_{\mathrm{II}}\left(Y_{0}\right)=0$ |  | -1 | -1 | -1 | 0 | 0 | 0 |


| $a_{4}$ | 0 | $2 / 3$ | 0 | $8 / 3$ | $17 / 3$ | 1 | $-1 / 3$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{1}$ | 1 | $1 / 6$ | 1 | $7 / 6$ | $1 / 6$ | 0 | $1 / 6$ | 0 |
| $a_{6}$ | 0 | 0 | 0 | -4 | 5 | 0 | $-1 / 1$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\text {II }}\left(Y_{1}\right)=1 / 6$ | 0 | $1 / 6$ | $-5 / 6$ | 0 | $1 / 6$ | 0 |  |


| $a_{4}$ | 0 | $2 / 7$ | $-16 / 7$ | 0 | $37 / 7$ | 1 | $-5 / 7$ | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $1 / 7$ | $6 / 7$ | 1 | $1 / 7$ | 0 | $1 / 7$ | 0 |
| $a_{6}$ | 0 | $4 / 7$ | $24 / 7$ | 0 | $39 / 7$ | 0 | $-3 / 7$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{II}}\left(Y_{2}\right)=1 / 7$ | $-1 / 7$ | 0 | $-6 / 7$ | 0 | $1 / 7$ | 0 |  |


| $a_{1}$ | 1 | $2 / 37$ | $-16 / 37$ | 0 | 1 | $7 / 37$ | $-5 / 37$ | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $5 / 37$ | $34 / 37$ | 1 | 0 | $-1 / 37$ | $6 / 37$ | 0 |
| $a_{6}$ | 0 | $10 / 37$ | $216 / 37$ | 0 | 0 | $-39 / 37$ | $12 / 37$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{II}}\left(Y_{3}\right)=$ | $7 / 37$ | $-19 / 37$ | 0 | 0 | $6 / 37$ | $1 / 37$ | 0 |


| $a_{3}$ | 1 | $2 / 27$ | 0 | 0 | 1 | $1 / 9$ | $-1 / 9$ | $2 / 27$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $5 / 54$ | 0 | 1 | 0 | $5 / 36$ | $1 / 9$ | $-17 / 108$ |
| $a_{1}$ | 1 | $5 / 108$ | 1 | 0 | 0 | $-13 / 72$ | $1 / 18$ | $37 / 216$ |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\text {II }}\left(Y_{\text {opt }}\right)=$ | $23 / 108$ | 0 | 0 | 0 | $5 / 72$ | $1 / 18$ | $19 / 216$ |

We solve optimization problem (15) by simplex method as per Table 5.

From the last simplex table, we obtain:
$x_{1}^{{ }^{* B}}=\frac{7}{86}, \quad x_{2}^{{ }^{*} B}=\frac{1}{86}, \quad x_{3}^{* B}=\frac{29}{344} . \quad$ As
$x_{1}^{{ }^{B}}+x_{2}^{{ }^{B} B}+x_{3}^{{ }^{B} B}=\frac{1}{v_{B}^{x}} \quad$ and $\quad x_{i}^{{ }^{B}}=\frac{x_{i}^{B}}{v_{B}^{x}}$,
$i=1,2,3$ we finally obtain:
$X_{B}^{m}=\left[\frac{28}{61}, \frac{4}{61}, \frac{29}{61}\right] \approx[0.46,0.07,0.47] \Rightarrow[46 \%, 7 \%, 47 \%]$,

$$
\cdot v_{B}^{x}=\frac{344}{61} \approx 5.64 .
$$

Table 5 - Solution of the optimization problem (15) by the simplex method

| Basis $C$ | $B$ | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ | $a_{5}$ | $a_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 1 | 1 | 0 | 0 | 0 |


| $a_{4}$ | 0 | -1 | -3 | -7 | -8 | 1 | 0 | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{5}$ | 0 | -1 | -7 | -8 | -4 | 0 | 1 | 0 |
| $a_{6}$ | 0 | -1 | -8 | -1 | -4 | 0 | 0 | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{0}\right)=0$ | 0 | -1 | -1 | -1 | 0 | 0 | 0 |


| $a_{1}$ | 1 | $1 / 3$ | 1 | $7 / 3$ | $8 / 3$ | $-1 / 3$ | 0 | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{5}$ | 0 | $4 / 3$ | 0 | $25 / 3$ | $44 / 3$ | $-7 / 3$ | 1 | 0 |
| $a_{6}$ | 0 | $5 / 3$ | 0 | $53 / 3$ | $52 / 3$ | $-8 / 3$ | 0 | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{1}\right)=1 / 3$ | 0 | $4 / 3$ | $5 / 3$ | $-1 / 3$ | 0 | 0 |  |


| $a_{1}$ | 1 | $1 / 11$ | 1 | $9 / 11$ | 0 | $1 / 11$ | $-2 / 11$ | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{3}$ | 1 | $1 / 11$ | 0 | $25 / 44$ | 1 | $-7 / 44$ | $3 / 44$ | 0 |
| $a_{6}$ | 0 | $1 / 11$ | 0 | $86 / 11$ | 0 | $1 / 11$ | $-13 / 11$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{2}\right)=$ | $2 / 11$ | 0 | $17 / 44$ | 0 | $-3 / 44$ | $-5 / 44$ | 0 |


| $a_{1}$ | 1 | $7 / 86$ | 1 | 0 | 0 | $7 / 86$ | $-5 / 86$ | $-9 / 86$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{3}$ | 1 | $29 / 344$ | 0 | 0 | 1 | $-57 / 344$ | $53 / 344$ | $-25 / 344$ |
| $a_{2}$ | 1 | $1 / 86$ | 0 | 1 | 0 | $1 / 86$ | $-13 / 86$ | $11 / 86$ |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{I}}\left(X_{\text {opt }}\right)=$ | $61 / 344$ | 0 | 0 | 0 | $-25 / 344$ | $-19 / 344$ | $-17 / 344$ |

Table 6 - Solution of the optimization problem (16) by the simplex method

| Basis | C | B | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ | $a_{5}$ | $a_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 1 | 1 | 0 | 0 | 0 |


| $a_{4}$ | 0 | 1 | 3 | 7 | 8 | 1 | 0 | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{5}$ | 0 | 1 | 7 | 8 | 1 | 0 | 1 | 0 |
| $a_{6}$ | 0 | 1 | 8 | 4 | 4 | 0 | 0 | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{II}}\left(Y_{0}\right)=0$ |  | -1 | -1 | -1 | 0 | 0 | 0 |


| $a_{4}$ | 0 | $4 / 7$ | 0 | $25 / 7$ | $53 / 7$ | 1 | $-3 / 7$ | 0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{1}$ | 1 | $1 / 7$ | 1 | $8 / 7$ | $1 / 7$ | 0 | $1 / 7$ | 0 |
| $a_{6}$ | 0 | $-1 / 7$ | 0 | $-36 / 7$ | $20 / 7$ | 0 | $-8 / 7$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\text {II }}\left(Y_{1}\right)=1 / 7$ | 0 | $1 / 7$ | $-6 / 7$ | 0 | $1 / 7$ | 0 |  |

Table 6 continuation

| $a_{4}$ | 0 | $1 / 8$ | $-25 / 8$ | 0 | $57 / 8$ | 1 | $-7 / 8$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $1 / 8$ | $7 / 8$ | 1 | $1 / 8$ | 0 | $1 / 8$ | 0 |
| $a_{6}$ | 0 | $1 / 2$ | $9 / 2$ | 0 | $7 / 2$ | 0 | $-1 / 2$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\mathrm{II}}\left(Y_{2}\right)=$ | $1 / 8$ | $-1 / 8$ | 0 | $-7 / 8$ | 0 | $1 / 8$ | 0 |


| $a_{3}$ | 1 | $1 / 57$ | $-25 / 57$ | 0 | 1 | $8 / 57$ | $-7 / 57$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $7 / 57$ | $53 / 57$ | 1 | 0 | $-1 / 57$ | $8 / 57$ | 0 |
| $a_{6}$ | 0 | $25 / 57$ | $344 / 57$ | 0 | 0 | $-28 / 57$ | $-4 / 57$ | 1 |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\text {II }}\left(Y_{3}\right)=$ | $8 / 57$ | $-29 / 57$ | 0 | 0 | $7 / 57$ | $1 / 57$ | 0 |


| $a_{3}$ | 1 | $17 / 344$ | 0 | 0 | 1 | $9 / 86$ | $-11 / 86$ | $25 / 344$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{2}$ | 1 | $19 / 344$ | 0 | 1 | 0 | $5 / 86$ | $13 / 86$ | $-53 / 344$ |
| $a_{1}$ | 1 | $25 / 344$ | 1 | 0 | 0 | $-7 / 86$ | $-1 / 86$ | $57 / 344$ |
| $\Delta_{\mathrm{j}}$ | $\mathrm{W}_{\text {II }}\left(Y_{\text {opt }}\right)=$ | $61 / 344$ | 0 | 0 | 0 | $7 / 86$ | $1 / 86$ | $29 / 344$ |

Table 7 - Summary table of bimatrix game solutions

$$
C_{A}=\left[\begin{array}{lll}
2 & 5 & 6 \\
6 & 7 & 1 \\
6 & 3 & 6
\end{array}\right] \quad C_{B}=\left[\begin{array}{lll}
3 & 7 & 8 \\
7 & 8 & 1 \\
8 & 4 & 4
\end{array}\right]
$$

Solution for the first player $A$
$X_{A}=\left[\frac{28}{61}, \frac{4}{61}, \frac{29}{61}\right] \approx[0.46,0.07,0.47] \Rightarrow[46 \%, 7 \%, 47 \%] \quad v_{A}^{x}=\frac{108}{23} \approx 4,70$

Solution for the second player $B$

$$
X_{B}=\left[\frac{5}{23}, \frac{10}{23}, \frac{8}{23}\right] \approx[0.22,0.43,0.35] \Rightarrow[22 \%, 43 \%, 35 \%] \quad v_{B}^{x}=\frac{344}{61} \approx 5,64
$$

Matrix game


Solution of primal and dual problems

| $X_{A}^{m}=\left[\frac{15}{46}, \frac{6}{23}, \frac{19}{46}\right] \approx[0.33,0.26,0.41] \Rightarrow[33 \%, 26 \%, 41 \%]$ | $v_{A}^{x}=\frac{108}{23} \approx 4,70$ |
| :--- | :---: |
| $Y_{A}^{m}=\left[\frac{5}{23}, \frac{10}{23}, \frac{8}{23}\right] \approx[0.22,0.43,0.35] \Rightarrow[22 \%, 43 \%, 35 \%]$ | $v_{A}^{y}=\frac{108}{23} \approx 4,70$ |

$$
\text { Matrix game } C_{B}=\left[\begin{array}{lll}
3 & 7 & 8 \\
7 & 8 & 1 \\
8 & 4 & 4
\end{array}\right] \text { (matrix of the second player) }
$$

Solution of primal and dual problems

$$
\begin{array}{c|c}
\hline X_{B}^{m}=\left[\frac{28}{61}, \frac{4}{61}, \frac{29}{61}\right] \approx[0.46,0.07,0.47] \Rightarrow[46 \%, 7 \%, 47 \%] & v_{B}^{x}=\frac{344}{61} \approx 5,64 \\
\hline \hline Y_{B}^{m}=\left[\frac{25}{61}, \frac{19}{61}, \frac{17}{61}\right] \approx[0.41,0.31,0.28] \Rightarrow[41 \%, 31 \%, 28 \%] & v_{B}^{y}=\frac{344}{61} \approx 5,64
\end{array}
$$

From the last simplex table, we obtain:

$$
\begin{gathered}
y_{1}^{{ }^{* B}}=\frac{25}{344}, y_{2}^{*^{A}}=\frac{19}{344}, y_{3}^{* A}=\frac{17}{344} \cdot \mathrm{As} \\
y_{1}^{{ }^{* B}}+y_{2}^{{ }^{* B}}+y_{3}^{{ }^{* B}}=\frac{1}{v_{B}^{y}} \text { and } y_{i}^{{ }^{B B}}=\frac{y_{i}^{B}}{v_{B}^{y}}
\end{gathered}
$$

$i=1,2,3$ the game solution looks as follows:

$$
Y_{B}^{m}=\left[\frac{25}{61}, \frac{19}{61}, \frac{17}{61}\right] \approx[0.41,0.31,0.28] \Rightarrow[41 \%, 31 \%, 28 \%]
$$

$$
v_{B}^{y}=\frac{344}{61} \approx 5.64
$$

## 5 DISCUSSION

The summary table of solutions to the bimatrix game and its respective matrix games is given in Table 7.

Having analyzed the calculation results, we formulate several important statements:

- The optimum mixed solution of the first player $X_{A}=\left[\frac{28}{61}, \frac{4}{61}, \frac{29}{61}\right]$ in the bimatrix game coincides with mixed solution $X_{B}^{m}=\left[\frac{28}{61}, \frac{4}{61}, \frac{29}{61}\right]$ in the matrix game given by payoff matrix $C_{B}$ of the other player $B$. In other words, the optimum behavior of the first player is a result of solving the primal optimization problem for the second player, i.e. the first player will not take account of own matrix - all the attention is paid to the matrix of the other player; but at the same time, the value of win

$$
v_{A}^{x}=\frac{108}{23} \approx 4.70
$$

is a result of solving dual optimization problem

$$
v_{A}^{y}=\frac{108}{23} \approx 4.70
$$

for the matrix game given by own matrix $C_{A}$.

- The optimum mixed strategy of the other player

$$
X_{B}=\left[\frac{5}{23}, \frac{10}{23}, \frac{8}{23}\right]
$$

coincides with the calculation of dual optimization problem

$$
Y_{A}^{m}=\left[\frac{5}{23}, \frac{10}{23}, \frac{8}{23}\right]
$$

for the first player. Therefore, the optimum behavior of the second player is fully derived from matrix $C_{A}$ of the
first player - the values of own wins do not matter.

- The value of win

$$
v_{B}^{x}=\frac{344}{61} \approx 5.64
$$

is equal to the value of benefits of matrix game $v_{B}^{y}=\frac{344}{61} \approx 5.64$
of the dual problem for the second player.

- Solutions of the dual LO problem for the first player's matrix game and of the primal LO problem for the second player's matrix game give the full result of the bimatrix game solution. Thus, we can skip the calculation of the bimatrix game in general, but solve two separate matrix games.
- The players' behavior in the bimatrix game is sometimes subjected to strange transformations - the players' behavior strategies begin not to depend on the value of their own benefit. The players start to be only concerned about what the other player's benefit would be. We encounter the effect of behavior confrontation or antagonism. This fact can be related to the modern direction in the game theory - the behavior game theory and requires further research.

The approach proposed is taken as the basis of the recommendation system.

## CONCLUSIONS

For a project team, it's extremely important to begin implementing a project in compliance with the calendar schedule. The proposed method of analyzing the team members' proposals contributes to avoiding or solving conflict situations at the stage when they get into closer interaction. The provided calculation of the model example shows that using the proposed method enable the project manager giving a well-grounded advantage to the other team member as the expected average win of this player is bigger than of the first player . In this case, the manager has the possibility to model situations for the players (for the team) and to promptly respond to probable departures of their behavior strategies from the optimum, to establish sound relations between the team members and to select the best proposals for solving the project tasks. Further research provides for designing an information system as a project manager's tool that is based on the method elaborated.

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Received 26.02.2023.
Accepted 22.05.2023.

## УДК 004.94+519.7

## ВИКОРИСТАННЯ ПОВЕДІНКОВОГО АНТАГОНІЗМУ ТА БІМАТРИЧНОЇ ТЕОРІЇ ІГР В УПРАВЛІННІ ІТ-ПРОЕКТАМИ

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## АНОТАЦІЯ

Актуальність. У статті запропоновано методику аналізу пропозицій членів команди з метою уникнення конфліктних ситуацій на етапі формування команди.

Об'єктом дослідження є методику аналізу пропозицій учасників команди при вирішенні завдань проекту.
Мета роботи - Проаналізувати розроблену методику аналізу пропозицій учасників команди щодо уникненню або вирішенню конфліктних ситуацій на етапі спрацьовування команди

Метод. Розроблений метод базується на теорії матричних ігор. Конфлікти між окремими членами команди в основному виникають на етапі формування команди. Для керівника проекту важливо вчасно виявити конфліктну ситуацію і знайти з неї вихід, щоб задовольнити обох членів команди і без шкоди для командної роботи в цілому. Команда, створена для реалізації IT-проекту, часто стикається з ситуацією, коли двоє ії учасників мають різне бачення підходів до створення кінцевого продукту. При цьому кожен з них має великий досвід розробки подібних програмних продуктів або сервісів різними командами. Для ефективного вирішення зазначеної ситуації ми пропонуємо використовувати підходи, характерні для біматричних ігор, коли кожен із цих учасників команди розглядається як гравець. При цьому враховується той факт, що в основі біматричної теорії ігор лежить конфлікт двох гравців, інтереси яких протилежні - антагоністична гра з нульовою сумою якраз і є основою розробленого підходу.

Результати. Запропонована методика аналізу пропозицій учасників команди сприяє уникненню або вирішенню конфліктних ситуацій на етапі їх більш тісної взаємодії. Для ефективного вирішення зазначеної ситуації ми пропонуємо використовувати підходи, характерні для біматричних ігор, коли кожен із цих учасників команди розглядається як гравець. При цьому враховується той факт, що в основі біматричної теорії ігор лежить конфлікт двох гравців, інтереси яких протилежні антагоністична гра з нульовою сумою $є$ саме таким елементом, який і є основою розробленого підходу.

Висновки. Наведений розрахунок модельного прикладу показує, що використання запропонованого методу дозволяє керівнику проекту надавати обгрунтовану перевагу іншому члену команди, оскільки очікуваний середній виграш цього гравця є більшим, ніж першого гравця. У цьому випадку менеджер має можливість моделювати ситуації для гравців (для команди) і оперативно реагувати на ймовірні відхилення стратегій їх поведінки від оптимальних, налагоджувати здорові стосунки між членами команди і вибирати найкращі пропозиції щодо вирішення завдань проекту.

КЛЮЧОВІ СЛОВА: біматричні ігри, теорії матричних ігор, управління IT проектами, формування команди.


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