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Dispersion properties of artificial topological insulators based on an infinite double-periodic array of elliptical quartz elements

Subject and Purpose. Special features of all-dielectric electromagnetic analogues of topological insulators in the microwave range are considered, aiming at studying the influence of geometrical and constitutive parameters of topological insulator elements on the dispersion properties of topological insulators based on a two-dimensional double-periodic array of dielectric elements.

Methods and Methodology. The evaluation of dispersion properties and electromagnetic field spatial distribution patterns for topological insulators is performed using numerical simulation programs.

Results. The electromagnetic analogue of a topological insulator based on a double-periodic array of elliptical quartz cylinders has been considered. By numerical simulation, it has been demonstrated that the electromagnetic properties of the structure are controllable by changing the quartz uniaxial anisotropy direction without any changes in other parameters.

A combined topological insulator made up of two adjoining ones differing in shapes of their unit cells has been considered with the numerical demonstration that frequencies of surface states are controllable by choosing the quartz uniaxial anisotropy direction. It has been shown that it is at the interface of two different in shape unit cells that the electromagnetic field concentration at a surface state frequency takes place.

Conclusion. A possibility has been demonstrated of controlling microwave electromagnetic properties of topological insulators by changing their geometric parameters and permittivity of the constituents. From a practical point of view, topological insulators can be used as components of microwave transmission lines and devices featuring very small propagation loss. Fig. 5. Ref.: 17 items.

Key words: topological insulator, uniaxial anisotropy, microwave range, photonic crystal.

Structures called topological insulators stand out in solid state physics. Their main distinguishing feature is that a topological insulator is insulating inside the bulk like a normal insulator, while on its surface it shows conductivity like a metal. Since their advent, the structures of the type have been of great interest among researchers and engineers [1, 2]. Also, with advances in telecommunication technologies, physicists got a new opportunity to study the nature of unusual conductivity for spin [3] and electromagnetic waves, too [4–6]. The significance of “topological” properties is highlighted in lots of theoretical and experimental publi-

cations in the field of topological photonics, displaying that with topological insulators a low-loss electromagnetic energy transmission is possible in both the optical [7, 8] and microwave ranges [9–14]. Topological insulators of the type act as spin and electromagnetic analogues of “classical” electronic topological insulators, demonstrating similar properties for spin and electromagnetic waves.

Topological insulators can be constructed of either metal [9, 10] or dielectric elements [11–13]. Notice that dielectric ones have a significant advantage over other materials. They can be used at the optical frequencies due to their low loss. Topo-

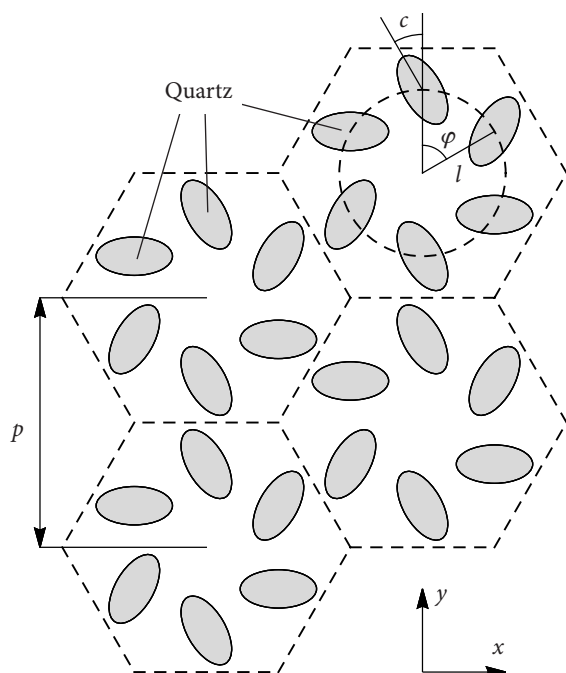


Fig. 1. The unit cell of a topological insulator based on a double-periodic array of elliptical quartz elements in the neighborhood of other unit cells of the structure

logical properties can also arise in structures based on magnetized ferrites [14]. However, these topological insulators require a fairly strong magnetic field. Another thing is with topological insulators that are formed of ordinary dielectrics and do not require an external magnetic field application. If made of ferrite, these topological insulators can work without external magnetic fields. Yet a weak magnetic field provides extra tuning options, enabling magnetically tunable topological insulators.

The aim of the work is to determine the influence of geometrical and constitutive parameters of topological insulator elements on spectral and dispersion properties of a combined artificial topological insulator based on a two-dimensional and double-periodic array of dielectric elements. Numerical results demonstrating electromagnetic properties of two-dimensional (2D) dielectric topological insulators of various types are presented.

1. Dispersion properties of artificial topological insulators based on a double-periodic array of elliptical quartz elements. Firstly, in this section, the features of electromagnetic properties of all-dielectric electromagnetic analogues of topological insulators [11–13] are studied as applied to a double-periodic array of elliptical elements [16, 17] made of quartz. The choice of this topological

insulator type is reasoned by a possibility to transfer the microwave range results to the optical region, considering that metal elements in the optical range have higher losses than their dielectric analogs.

The use of a numerical simulation software package MPB [15] made it possible to compute dispersion diagrams of the above-mentioned 2D topological insulator calculations over a band of frequencies.

For a 2D topological insulator, a double-periodic structure with a hexagonal unit cell and a cell period p (Fig. 1) is taken. The unit cell consists of six dielectric z -infinite elliptical cylinders in a vacuum. Each cylinder axis and the unit cell axis are l spaced. The lines connecting the centers of two neighboring ellipses and the unit cell center make the angle $\varphi = 60^\circ$ [11, 16, 17].

Each ellipse with half-axes a and b is rotated around its center through an angle C measured between the main half-axis a of the ellipse and the line going from the unit cell center to the ellipse center (see Fig. 1).

Initially, the structure made of elliptical quartz cylinders is studied for the influence of geometric and constitutive parameters of the structure elements on the electromagnetic properties. The geometric parameters of the unit cell and the structure period p are selected subject to certain conditions. Namely, the topological insulator operating frequency is about 10 GHz, and the band gap should be wide enough to overlap with the band gap of another topological insulator whose unit cell has somewhat different parameters.

As known, to demonstrate the existence of surface electromagnetic oscillations and field concentration, a structure is often used that is made up of two adjoining topological insulators differing in shapes of their unit cells [11–13]. In this case, the necessary condition for the existence of surface oscillations is overlap of band gaps of these topological insulators. The further search for parameter values at the interface between different topological insulators shows that surface electromagnetic oscillations are possible. These oscillations depend on the type of wave polarization (right circular or left circular, RCP or LCP) and exist in this case at any frequency within the overlapped band gap. At some frequencies, these oscillations have the same wavelength in the dispersion diagram.

By numerical simulation, geometrical parameters of the unit cells were selected for two topological insulators and operating frequency area around 10 GHz. For elliptical quartz cylinders, the unit cell period is $p = 20.7$ mm. The distance from the unit cell center to the center of each ellipse is $l = p/3$. The semi-axes of the ellipses making elliptical cylinders are $a = 0.155 p$ and $b = 0.095 p$. The rotation angles of ellipses in two different unit cells were obtained given a sufficiently wide overlap of the band gaps of both topological insulators and measure $C_1 = 0^\circ$ and $C_2 = 90^\circ$ for topological properties to be achieved.

For the material of the elliptical cylinders, uniaxial-anisotropy quartz has been chosen. Let us begin with the influence of the anisotropy axis direction on the TI dispersion properties. The permittivity tensor of anisotropic quartz is

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}, \quad (1)$$

with any two components on the main diagonal being the same, the third one is different (uniaxial anisotropy).

For the elliptical α -quartz cylinders, the x -directed uniaxial anisotropy is initially taken so that $\varepsilon_{xx} = 4.847$, $\varepsilon_{yy} = 4.643$, $\varepsilon_{zz} = 4.643$, which corresponds to the real values of the permittivity tensor components.

The dispersion diagrams calculated for the two topological insulators with the ellipse rotation angles $C_1 = 0^\circ$ and $C_2 = 90^\circ$ (Fig. 2) are plotted in Fig. 3. In it, the ordinate is the normalized frequency fp/c , where c is the speed of light in a vacuum and p is the structure period. The abscissa is the wave vector. The points M, Γ , K are special points inside the Brillouin zone in the case of a 2D photonic crystal (PC) with a hexagonal lattice [11].

Notice that for the topological insulator with the angle $C_1 = 0^\circ$ (Fig. 3, a), the band gap is in the frequency range $fp/c = 0.689 \dots 0.702$, and its width is 1.89%. For the topological insulator with $C_2 = 90^\circ$ (Fig. 3, b), the band gap is in the frequency range $fp/c = 0.688 \dots 0.702$, and its width is 2.03%.

Consider the action of the quartz anisotropy direction on the width and shift of the band gap. When uniaxial anisotropy of α -quartz is along the y -axis, the permittivity tensor components

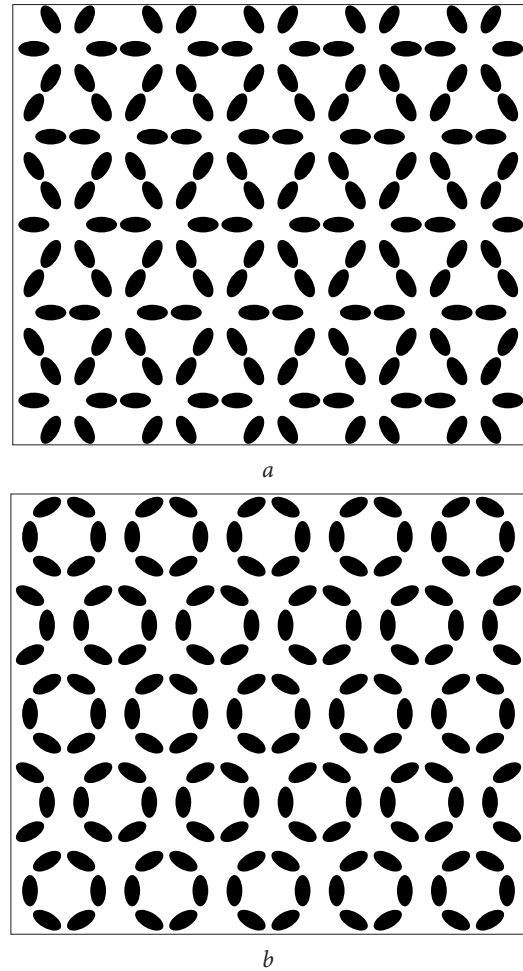


Fig. 2. Schematics of 2D topological insulators based on a double-periodic array of elliptical quartz cylinders with different angles $C_1 = 0^\circ$ (a) and $C_2 = 90^\circ$ (b) of the ellipse rotation inside the unit cells

are $\varepsilon_{xx} = 4.643$, $\varepsilon_{yy} = 4.847$, and $\varepsilon_{zz} = 4.643$. For the topological insulator with the angle $C_1 = 0^\circ$ (Fig. 3, a), the band gap is in the frequency range $fp/c = 0.688 \dots 0.702$, and it is 2.03% wide. For the topological insulator with $C_2 = 90^\circ$ (Fig. 3, b), the band gap is in the frequency range $fp/c = 0.689 \dots 0.702$, and it is 1.89% wide.

When uniaxial anisotropy of α -quartz is along the z -axis, the permittivity tensor components are $\varepsilon_{xx} = 4.643$, $\varepsilon_{yy} = 4.643$, and $\varepsilon_{zz} = 4.847$. For the topological insulator with the angle $C_1 = 0^\circ$ (Fig. 3, a), the band gap is in the frequency range $fp/c = 0.675 \dots 0.691$, and its width is 2.27%. For the topological insulator with the angle $C_2 = 90^\circ$ (Fig. 3, b), the band gap is within $fp/c = 0.676 \dots 0.691$, and its width is 2.10%.

Thus, for all the three directions of the quartz uniaxial anisotropy axis, we have significant over-

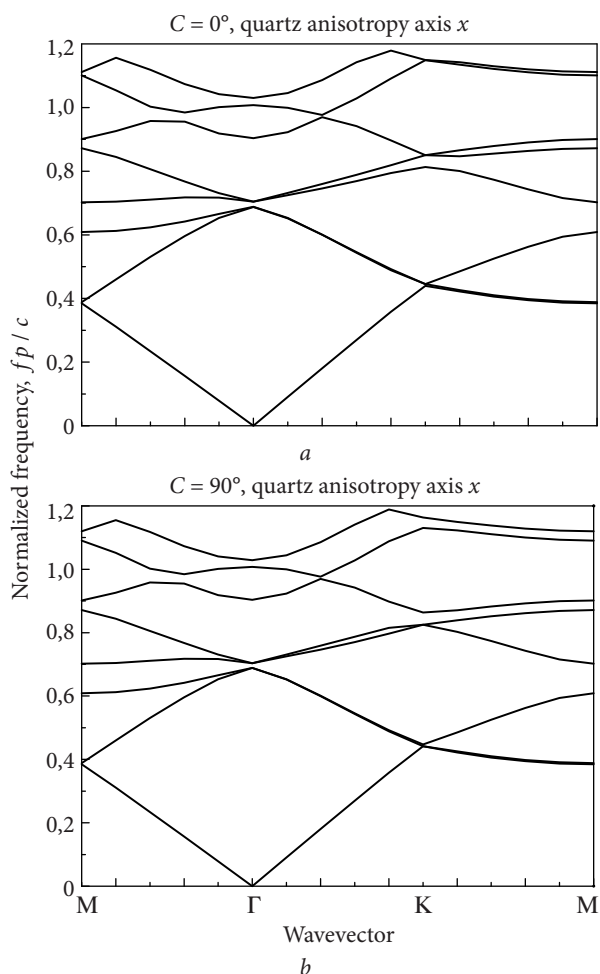


Fig. 3. Dispersion diagrams of 2D topological insulators based on a double-periodic array of elliptical quartz cylinders with uniaxial anisotropy along the x -axis and ellipse rotation angles $C_1 = 0^\circ$ (a) and $C_2 = 90^\circ$ (b)

laps of the band gaps of the topological insulators with the ellipse rotation angles $C_1 = 0^\circ$ and $C_2 = 90^\circ$. This makes it possible to design combined structures of two topological insulators, with the interface between them expected to support electromagnetic surface oscillations [11–13]. When the uniaxial anisotropy is z -directed, the band gap expands up to 10% against the other two cases (the x - and y -directed anisotropy). So, it has been just demonstrated that by choosing the uniaxial anisotropy direction of the material, the topological insulator electromagnetic properties can be controlled without any changes in other parameters of the structure.

In addition, the electromagnetic properties of the topological insulator are tunable by varying its geometric parameters. As the structure period p increases, with the relationship between elements

and period fixed, the band gap moves to the lower frequencies. Another way to fine-tune the band gap parameters is by varying distance l from the unit cell center to any ellipse center.

Thus, a possibility of controlling the electromagnetic properties of the topological insulator by changing its geometric parameters and by permittivity of its constituents has been numerically demonstrated.

2. Dispersion properties of combined artificial topological insulators based on a double-periodic array of elliptical quartz elements.

It is known that electromagnetic analogues of topological insulators can support electromagnetic surface oscillations on the surface area and behave as insulators in the bulk interior [4–6]. This effect can arise in a certain frequency range – within the band gaps of topological insulators. As soon as conditions for the maintenance of electromagnetic surface oscillations arise on the topological insulator surface, a low-loss energy transfer takes place owing to the waves travelling along the topological insulator boundary. In the previous section, these frequency ranges were determined for a certain topological insulator type. The role of the topological insulator boundary can be played by a metal wall [14] or by a topological insulator with different unit cell parameters [11–13]. This section is devoted to a combined topological insulator in which two topological insulators with different unit cells are interfaced with each other. The unit cell scheme (Fig. 1) for both topological insulators is generally the same. Yet some geometric parameters differ. The virtue is that this combined structure is all-dielectric.

An example of the 2D combined topological insulator can be provided by the two previously considered topological insulator structures with different angles $C_1 = 0^\circ$ (Fig. 2, a) and $C_2 = 90^\circ$ (Fig. 2, b) of the element rotation [11]. These angles were selected by analysis of the dispersion diagrams (Fig. 3) of these structures in view of that the band gaps of the structures should overlap in some frequency interval, and the circular polarization directions for the allowed states at the ends of the band gap at the point Γ ($k_x = k_y = 0$) in the dispersion diagram should be opposite. The so obtained combined structure of two interfaced topological insulators based on an array of elliptical quartz cylinders is schematized in Fig. 4, a.

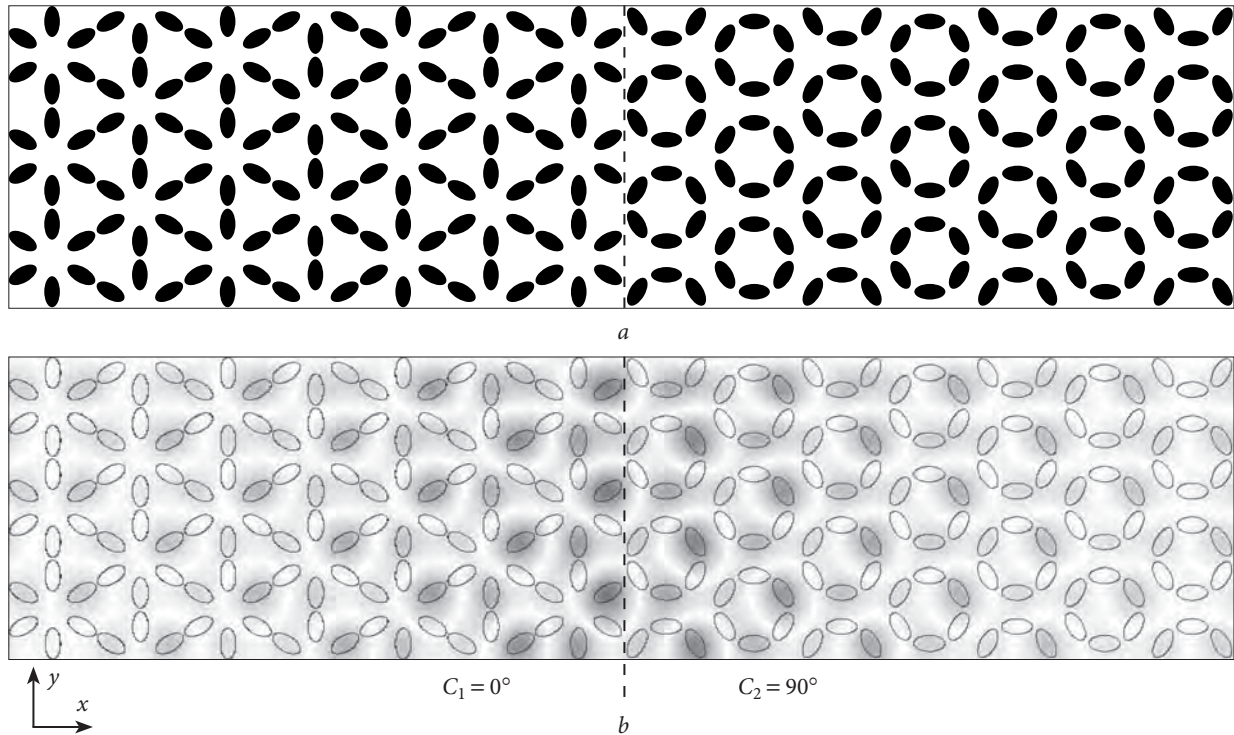


Fig. 4. The combined structure based on an array of elliptical quartz cylinders near the interface of two topological insulators with the ellipse rotation angles $C_1 = 0^\circ$ and $C_2 = 90^\circ$: (a) the 2D structure pattern and (b) the spatial distribution of the normal (z) component of the structure electric field at the normalized frequency $fp/c = 0.69029$ of the allowed state

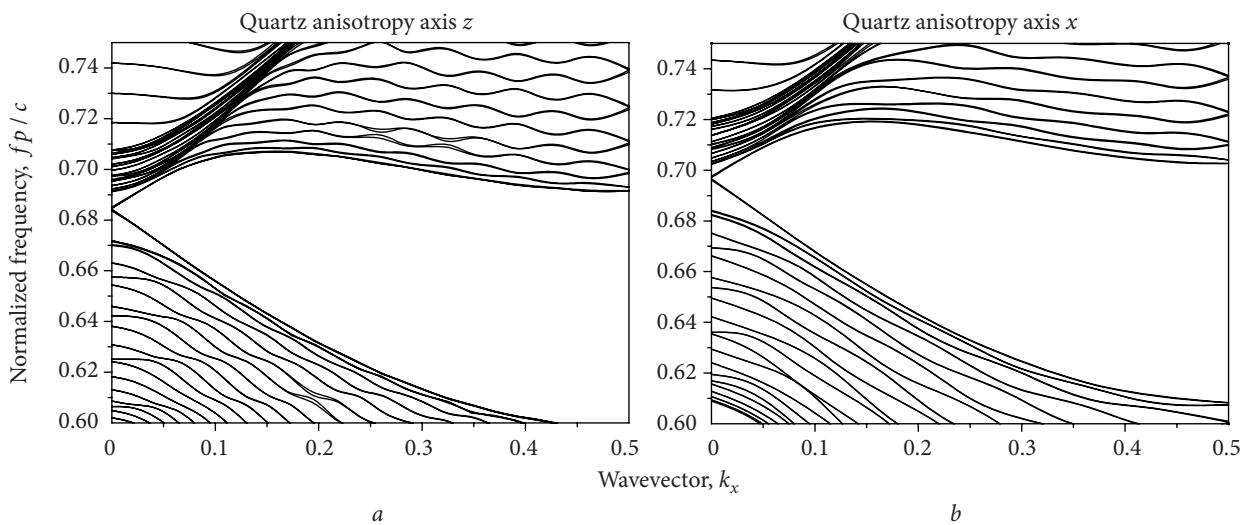


Fig. 5. Dispersion diagrams for the combined structure consisting of two adjoining topological insulators based on an array of elliptical quartz cylinders whose angles of rotation inside the unit cells are $C_1 = 0^\circ$ and $C_2 = 90^\circ$: the quartz anisotropy axis is z -directed (a) and x -directed (b).

Fig. 5 presents dispersion diagrams calculated in the Γ -K direction for the combined structure (Fig. 4, a) composed of two adjoining topological insulators with the angles $C_1 = 0^\circ$ and $C_2 = 90^\circ$ of the ellipse rotation in the unit cells. In Fig. 5, the ordinate is the normalized frequency fp/c , the abscissa is the wave vector k_x . The calculation results

for the combined structures are presented for the quartz anisotropy axis along the z -axis (Fig. 5, a) and along the x -axis (Fig. 5, b).

As seen from Fig. 5, in the band gap of the combined structure of adjoining topological insulators, there exist two degenerate surface-state modes of p -type and d -type. They are indicated by bold

lines and correspond to circular polarization waves (RCP and LCP). These surface states meet at the point Γ (at $k_x = 0$) in the dispersion diagram [11].

It has been numerically shown that by choosing the quartz uniaxial anisotropy direction, the frequencies of topological insulator surface states can be controlled. Thus, for the structure of quartz cylinders with z -directed uniaxial anisotropy, the surface-state resonant modes meet near the point $fp/c = 0.685$ (Fig. 5, *a*). For the structure with x -directed uniaxial anisotropy, they meet near the point $fp/c = 0.697$ (Fig. 5, *b*). One more way of surface-state frequency control is by varying the periods of the structures. As the periods of the structures increase with the relationship between elements and period fixed, the surface state modes move to the lower frequencies.

To observe surface oscillations in terms of electromagnetic field concentration around the interface of the two topological insulators, numerical calculations were performed for the spatial distribution of the normal (z) component of the electric field of the combined structure at the normalized frequency $fp/c = 0.69029$ (the wave vectors are $k_x = 0.02174$ and $k_y = 0.32609$) (Fig. 4, *b*). This frequency falls within the band gap of each individual topological insulator. The quartz anisotropy axis is along the z -axis. Fig. 4, *b* shows (by darkening) that certain field concentration exists at the interface between the two adjoining topological insulators and rapidly dies away from it along the x -axis.

Conclusions. The features of electromagnetic properties of electromagnetic analogues of topological insulators have been considered on the example of a two-dimensional double-periodic array of elliptical quartz cylinders.

A numerical method evaluating the spectral and dispersion properties and patterns of the topologi-

cal insulator electromagnetic field spatial distribution in the microwave range has been developed.

It has been shown that by choosing the uniaxial anisotropy direction of the topological insulator material, the electromagnetic properties of topological insulators are tuned without any changes in other parameters of the structure. The influence of the quartz anisotropy axis direction on the width and position of the band gap of a topological insulator based on a 2D double-periodic array of elliptical quartz cylinders has been studied.

The dispersion diagrams and patterns of electromagnetic field spatial distribution have been evaluated for a combined structure consisting of two adjoining topological insulators whose unit cells differ in the angles, 0° and 90° , of ellipse rotation. It has been numerically shown that the frequencies of the surface states in the dispersion diagram of the combined structure of adjoining topological insulators can be controlled by choosing the quartz uniaxial anisotropy direction. It has been demonstrated that the field concentration occurs at the interface between the two topological insulators and rapidly decreases away from it.

As for practical applications, artificial combined topological insulators as those discussed in the paper can be part of microwave transmission lines and devices offering very low-loss wave propagation.

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REFERENCES

1. Thouless, D.J., Kohmoto, M., Nightingale, M.P., den Nijs, M., 1982. Quantized Hall Conductance in a Two-Dimensional Periodic Potential. *Phys. Rev. Lett.*, **49**(6), pp. 405–408. DOI: 10.1103/PhysRevLett.49.405.
2. Haldane, F.D.M., 1988. Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the “Parity Anomaly”. *Phys. Rev. Lett.*, **61**(18), pp. 2015–2018. DOI: 10.1103/PhysRevLett.61.2015.
3. Navabi, A., Liu, Y., Upadhyaya, P., Murata, K., Ebrahimi, F., Yu, G., Ma, Bo, Rao, Y., Yazdani, M., Montazeri, M., Pan, L., Krivorotov, I.N., Barsukov, I., Yang, Q., Amiri, P.K., Tserkovnyak, Ya., and Wang, K.L., 2019. Control of Spin-Wave Damping in YIG Using Spin Currents from Topological Insulators. *Phys. Rev. Applied.*, **11**(3), pp. 034046(1–7).
4. Khanikaev, A.B., Shvets, G., 2017. Two-dimensional topological photonics. *Nature Photon.*, **11**(12), pp. 763–773. DOI: 10.1038/s41566-017-0048-5.
5. Khanikaev, A., Mousavi, S.H., Tse, W.-K., Kargarian, M., MacDonald, A.H., Shvets, G., 2013. Photonic topological insulators. *Nature Mater.*, **12**(3), pp. 233–239. DOI: 10.1038/nmat3520.

- Lu, L., Joannopoulos, J., Soljačić, M., 2014. Topological photonics. *Nature Photon.*, **8**(11), pp. 821–829. DOI: 10.1038/nphoton.2014.248.
- Rechtsman, M., Zeuner, J., Plotnik, Y., Lumer, Ya., Podolsky, D., Dreisow, F., Nolte, S., Segev, M., Szameit, A., 2013. Photonic Floquet topological insulators. *Nature*, **496**(7444), pp. 196–200. DOI: 10.1038/nature12066.
- Shalae, M.I., Walasik, W., Tsukernik, A., Xu, Y., Litchinitser, N.M., 2019. Robust topologically protected transport in photonic crystals at telecommunication wavelengths. *Nature Nanotech.*, **14**(1), pp. 31–34. DOI: 10.1038/s41565-018-0297-6.
- Lai, K., Ma, T., Bo, X., Anlage, S., Shvets, G., 2016. Experimental Realization of a Reflections-Free Compact Delay Line Based on a Photonic Topological Insulator. *Sci. Rep.*, **6**, pp. 28453(1–7). DOI: 10.1038/srep28453.
- He, M., Zhang, L., Wang, H., 2019. Two-dimensional photonic crystal with ring degeneracy and its topological protected edge states. *Sci. Rep.*, **9**, pp. 3815(1–6). DOI: 10.1038/s41598-019-40677-5.
- Huang, H., Huo, S., Chen, J., 2019. Reconfigurable Topological Phases in Two-Dimensional Dielectric Photonic Crystals. *Crystals*, **9**(4), pp. 221(1–9). DOI: 10.3390/cryst9040221.
- Yang, Y., Xu, Y.F., Xu, T., Wang, H.-X., Jiang, J.-H., Hu, X. and Hang, Z.H., 2018. Visualization of a Unidirectional Electromagnetic Waveguide Using Topological Photonic Crystals Made of Dielectric Materials. *Phys. Rev. Lett.*, **120**(21), pp. 217401(1–7). DOI: 10.1103/PhysRevLett.120.217401.
- Slobozhanyuk, A., Shchelokova, A.V., Ni, X., Hossein Mousavi, S., Smirnova, Daria A., Belov, P.A., Alù, A., Kivshar, Y.S., Khanikaev, A.B., 2019. Near-field imaging of spin-locked edge states in all-dielectric topological metasurfaces. *Appl. Phys. Lett.*, **114**(3), pp. 031103(1–6). DOI: 10.1063/1.5055601.
- Wang, Z., Chong, Y., Joannopoulos, J., Soljacic, J.M., 2009. Observation of unidirectional backscattering-immune topological electromagnetic states. *Nature*, **461**(7265), pp. 772–775. DOI: 10.1038/nature08293.
- Johnson, S.G., Joannopoulos, J.D., 2001. Block-iterative frequency-domain methods for Maxwell's equations in a planewave basis. *Opt. Express*, **8**(3), pp. 173–190. DOI: 10.1364/OE.8.000173.
- Blanco, M. de Paz, Vergniory, M.G., Bercioux, D., García-Etxarri A., Bradlyn B., 2019. Engineering fragile topology in photonic crystals: Topological quantum chemistry of light. *Phys. Rev. Research*, **1**(3), 032005(R). DOI: 10.1103/PhysRevResearch.1.032005.
- Jiang, Z., Gao, Y.-F., He, L., Sun, J.-P., Songa, H., Wang, Q., 2019. Manipulation of pseudo-spin guiding and flat bands for topological edge states. *Phys. Chem. Chem. Phys.*, **21**(21), pp. 11367–11375. DOI: 10.1039/C9CP00789J.

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ДИСПЕРСІЙНІ ВЛАСТИВОСТІ ШТУЧНИХ ТОПОЛОГІЧНИХ ІЗОЛЯТОРІВ НА ОСНОВІ НЕСКІНЧЕННОГО ДВОПЕРІОДИЧНОГО МАСИВУ ЕЛІПТИЧНИХ КВАРЦОВИХ ЕЛЕМЕНТІВ

Предмет і мета роботи. У роботі розглянуто особливості електромагнітних властивостей повністю діелектричних електромагнітних аналогів топологічних ізоляторів (ТІ) у мікрохвильовому діапазоні. Метою роботи є визначення впливу параметрів елементів (геометрії та властивостей матеріалу) на дисперсійні властивості ТІ на основі двовимірного дво-періодичного масиву діелектричних елементів.

Методи і методологія роботи. Розрахунки дисперсійних властивостей, а також картин просторового розподілу електромагнітного (ЕМ) поля для ТІ були виконані за допомогою програм чисельного моделювання.

Результати роботи. Досліджено електромагнітний ТІ, основу якого становить двоперіодичний масив еліптичних циліндрів з кварцу. За допомогою чисельного моделювання показано можливість регулювання ЕМ-властивостей ТІ шляхом зміни напрямку одновісної анізотропії кварцу без змінення інших параметрів структури. Розглянуто комбінований штучний ТІ, що складається з двох різних за формою елементарної комірки ТІ, які межують між собою. Чисельно продемонстровано можливість керування частотою поверхневих станів для комбінованого ТІ шляхом вибору напрямку одновісної анізотропії кварцу. Показано, що концентрація ЕМ-поля спостерігається саме на межі двох різних за формою елементарної комірки ТІ на частоті поверхневого стану.

Висновок. Продемонстровано можливість керувати електромагнітними властивостями ТІ мікрохвильового діапазону шляхом зміни його геометричних параметрів, а також діелектричної проникності складових його елементів. На практиці ТІ можуть знайти застосування як елементи мікрохвильових ліній передачі та приладів, які мають дуже малі втрати на розповсюдження хвиль.

Ключові слова: топологічний ізолятор, одновісна анізотропія, мікрохвильовий діапазон, фотонний кристал.