UDC 624

APPLICATION OF STOCHASTIC FINITE ELEMENT METHOD TO PREDICT THE REMAINING SAFE LIFE OF CONCRETE SEWER PIPES

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1- Introduction

Underground sewer pipes are important and vital infrastructures that play a crucial role in the economy, prosperity, quality of life and especially the health of a country.These essential structures are designed to resist and operate safely under various external loads and environmental conditions.However the degradation of sewer pipes over their service life in combination with the effect of overlaying soil and surface traffic loads can sometimes causefailures in sewer pipes. Every year there are thousands of collapse reports in sewer pipe networks throughout the world which cause enormous disruption of daily life, massive costs, and widespread pollution. It is known that for cementitious sewer pipes, corrosion is the main cause of degradation. The corrosion can cause reduction in structural strength of the pipeline, leading to pipe collapse. In addition, severe localised corrosion can cause pitting,also resulting in the failure of sewer pipes.Therefore considering the effect of corrosion in the analysis and design of cementitious sewer pipes is essential for developing advanced model(s) to predict the likelihood of collapses of sewer systems.

In the field of reliability of underground pipelines a number of research studies are dedicated to the probabilistic analysis and predicting the remaining safe life of these structures. Ahammed and Melchers (1994, 1995) derived formulas to estimate the maximum circumferential and longitudinal stresses in underground pressurised pipes subject to external loading and corrosion. They used the obtained formulasto perform reliability analysis in order to evaluate the probability of failure of sewer pipes versus their service life. Li et al.,2009 used the Monte Carlo simulation technique to calculate the remaining safe life of metallic underground gas pipelines. They performed a sensitivity analysis to identify the key corrosion parameters in terms of pipeline failure. These studies and many similar works in literature have used empirical equations to estimate the stresses in the pressurised pipes subject to corrosion and external soil and traffic loads. However, there is a lack of research work in the area of reliability of cementitious sewer pipes subject to corrosion and external loads. In addition alternative approaches based on theoretical and numerical methods (rather than empirical) that provide a better estimation of the stresses in the pipes due to external loading and corrosion are required.

In order to provide an accurate prediction of remaining safe life of the sewer pipes, all the parameters that affect and control the process of deterioration and failure of pipes,the interaction of different mechanisms of failure, and their effect on remaining safe life of sewer pipes should be considered.Due to the large degree of uncertainty relating to the factors that are involved in the operation of underground sewer systems- in particular corrosion -it is more rational to model the failure of sewer pipes as a stochastic process.To fulfil this, acomprehensive model has been developed and is reported in this paper that takes into consideration all parameterscontributing towards the failure of cementitious sewer pipes using the stochastic finite element method. The developed stochastic finite element method can determine the prospect of failure of sewer pipesthroughout their intended service life. Particular attention is paid to simulate the corrosion using the stochastic finite element model and to investigate its interaction with other mechanisms of failure and their effect on the remaining safe life of sewer pipes. Moreover using the stochastic finite element method a parametric study is carried out to evaluate the significance and effect of each contributing parameters in the remaining safe life of the cementitious sewer pipes. The results provided by the proposed stochastic finite element method can help asset managers and owners to make risk-informed and cost-minimised decisions with respect to when, where, what and how interventions are required to ensure the safety and integrity of existing pipelines during their whole life of service.

2- Stochastic Finite Element Method

The stochastic finite element method (SFEM) is a powerful numerical tool in computational stochastic mechanics. SFEM can be classified as an extension of the classic deterministic finite element approach to the stochastic framework i.e. to the solution of static and dynamic problems with stochastic mechanical, geometric and/or loading properties (Stefanou, 2009).The general formulation of a SFEM can be written in the following form (Pellissetti and Schuëller, 2009):

$$\mathbf{K}(\theta)\mathbf{U}(\theta) = \mathbf{F}(\theta) \qquad (1)$$

where $\mathbf{K}$ is the global stiffness matrix,$\mathbf{U}$ and $\mathbf{F}$ represent the nodal displacement and force vectors and θ represents the randomness of the parameters. Theabove equationis the stochastic representation of the static finite element problems andthe uncertain response of structure (i.e. $\mathbf{U}(\theta)$) and other quantities of interest such as stresses $\boldsymbol{\sigma}(\theta)$, and strains $\boldsymbol{\varepsilon}(\theta)$ can be obtained by solving Equation 1.

In this paper a SFEM code based on the MCS technique is developed to analyse the probability of failure of underground cementitious sewer pipes. In the MCS-based SFEM, a deterministic finite element problem (Equation 1) is solved a large number of times and the response variability is calculated using statistical relationships. The MCS method does not involve any simplification or assumption which makes it a robust and universal technique to treat complex SFEM problems. In the developed MCS code, the Latin-hypercube sampling technique (Florian, 1992) is used to generate the random numbers with arbitrary inverse cumulative probability functions. The developed code is employed incrementally over the time in order to account for the degradation of the sewer pipe and predict the probability of failure of the sewer pipes throughout their service life. In the MCS code, for every simulation, the limit state function(s) is checked using the finite element method (e.g. if the resultant stress has exceeded the yield stress) and the probability of failure is obtained using the following equation(Melchers, 1999):

$$P_f^i = \frac{N_f}{N} \tag{2}$$

where P_f^i is defined as the probability of failure of each limit state function, N_f is the number of simulations when the limit state function is violated, and N is the total number of simulations.

In a series system, where more than one limit state function exist, the failure of any of the limit state functions implies the failure of the system. If the individual failures are mutually independent,then the probability of the system can be obtained from (Melchers, 1999):

$$P_f = 1 - \prod_{i=1}^{m} (1 - P_f^i) \tag{3}$$

where P_f is the probability of failure of the system, and m is the number of limit state functions defined for the system.

Effect of Corrosion

Concrete corrosion due to sulphuric acid attack is known to be one of the main contributory factors in the degradation of concrete sewer pipes. In this study it is assumed that the corrosion of concrete sewer pipes is dependent on the age of the pipe and can be presentedusing a power lawmodel. The power law model to predict the biogenic sulphuric acid corrosion of concrete pipes can be presented in the form of the following equation:

$$\mathrm{C} = \alpha\, T^{\lambda} \tag{4}$$

where C is the corrosion of the pipe, α is a multiplying coefficient, λ is an exponential coefficient, and T is the age of the pipe.The data provided in Meyer, 1980are utilised to estimate the coefficients in Equation 4using an exponential regression.

Due to a large degree of uncertainty of corrosion process, the corrosion andconsequently the coefficients in Equation 4 are considered as stochastic parameters. At every finite element simulation the amount of corrosion is obtained using Equation 4 and the finite element input file is updated via re-meshing the pipe domain using the new coordinates after considering the corrosion. The strength of the concrete pipe is expected to be reduced as a result of the reduction of pipe wall thickness.

3- Numerical Example

In this section a numerical example is considered to evaluate the performance of the developed SFEM in predicting the probability of failure of concrete sewer pipes subject to stresses and corrosion. The finite element (FE) model of the problem consists of a concrete pipe with a circular section buried underground and surrounded by a homogenous soil.The model is subjected to self-weight and an external traffic load applied on the surface of the model. Due to the symmetry of the problem only half of the model is considered and appropriate boundary conditions are applied. The FE model is assumed to be two dimensional with plane strain geometrical condition. In order to draw conclusions that are not affected by a particular example, the problem is scaled with respect to the external diameter of the pipe and the variations of different normalised parameters are investigated. Figure 1 shows the geometry and normalised parameters of the problem.

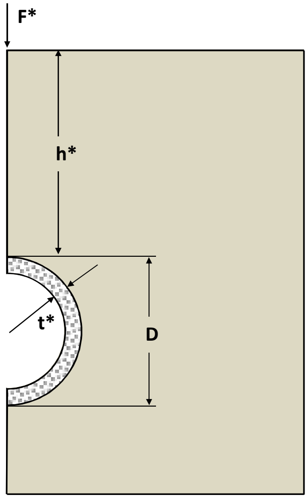


Figure 1: *Geometry and parameters used in the FE model*

In addition to the deterministic parameters such as diameter and thickness of the pipe, there are some parameters that are considered as stochastic or random parameters. Using the existing studies on the reliability analyses of underground pipelines (e.g.Ahammed and Melchers 1994, 1995) and performing a number of pilot simulations, five parameters were chosen as random variables. The normal distribution has been adopted for these random variables since only means and variances were available. These parameters, their mean and their coefficient of variation (cv) are presented in Table 1.

Results and discussion

After choosing the type of each parameter (deterministic or stochastic) and creating the FE models, the time-dependant SFEM were performed for different values of scaled parameters in order to study their effect on the probability of failure of concrete sewer pipes. Figure 2 shows the results of the SFEM on probability of failure of the example sewer pipe under various normalised traffic loads over its service life. It can be seen that as the traffic load increases the probability of failure of sewer pipes grows rapidly. In addition it can be noted that initially the probability of failure is zero or very small for all cases; however as the effect of corrosion emerges (usually after the first 20 years) the probability of failure increases rapidly.

Table 1

Random variables used for the SFEM

Symbol	Description	Mean	Coefficient of variation
f_c	Concrete maximum compressive strength	35 MPa	0.1
F^*	Traffic load	20	0.25
I_f	Load impact factor	1.5	0.15
α	Corrosion multiplying coefficient	3.5×10^{-5}	0.1
λ	Corrosion exponential coefficient	1.5	0.15

To further investigate the effect of traffic load on the service life of concrete sewer pipes subject to stresses and corrosion, the following analysis was also carried out. Let us assume that the acceptable probability of failure (P_f) is 10%, or 20% or 30% (equivalent of a remaining safe life of 90%, 80% and 70% respectively). The service life of each FE model (each model has a different normalised traffic load)can be evaluated using the results provided by the SFEM (Figure 3). It can be seen in Figure 3 that the service life of the sewer pipe is reduced significantly with a non-linear trend as the traffic load increases. For example if the traffic load is doubled (i.e. F^* increases from 10 to 20) then the service life of the sewer pipe is reduced from 60 years to zero years for the acceptable probability of failure of 10%. A similar trend can also be seen for other presumed acceptable values of probability of failure (i.e. $P_f = 20\%$ and $P_f = 30\%$).

In order to investigate the relative contribution and importance of each stochastic (random) variable in the probability of failure of concrete sewer pipes a sensitivity analysis was carried out. The results of the sensitivity analyses for two examples with normalised soil cover (h*) after 100 years of service life are presented in Figure 4a and Figure 4b. In both figures it can be seen that the exponential coefficient of corrosion is extensively contributing to the probability of failure of concrete sewer pipes. It can also be noted that this contribution enlarges as the soil cover increases. This is mainly due to the fact that, at higher values of soil cover, the dominant failure mode can become the corrosion failure as the pipe passes a certain age. A further point to discuss is the importance of the models that predict the corrosion in sewer pipes. For example at a scaled buried depth of 2.5 it can be seen that the corrosion coefficients (i.e., $\boldsymbol{\alpha}$, $\boldsymbol{\lambda}$) together contribute to nearly half (48%) of the probability of failure of sewer pipes. This shows the significant influence of corrosion in general and corrosion coefficients in particular on the prediction of probability of failure. More data on the corrosion of sewer concrete pipe, to provide a better estimation for corrosion coefficients in Equation 4 or developing alternative model(s) to predict the corrosion rate with higher accuracy, could improve the results of the SFEM. In general the results of the sensitivity analysis can be used to efficiently improve the design towards a higher reliability for sewer pipe projects or to enhance management, rehabilitation and spending on existing pipelines.

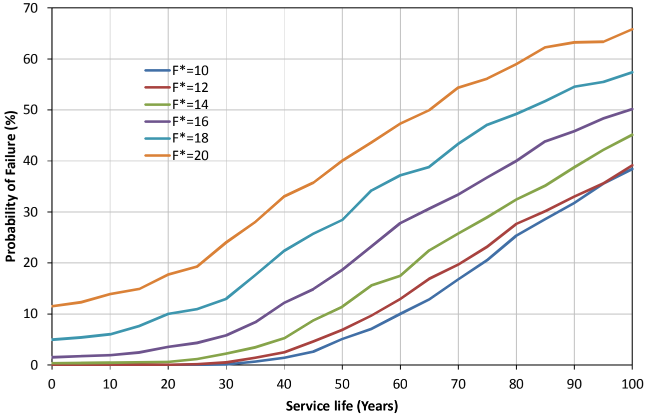


Figure 2. Probability of failure of concrete sewer pipe for different values of scaled traffic load versus service-life

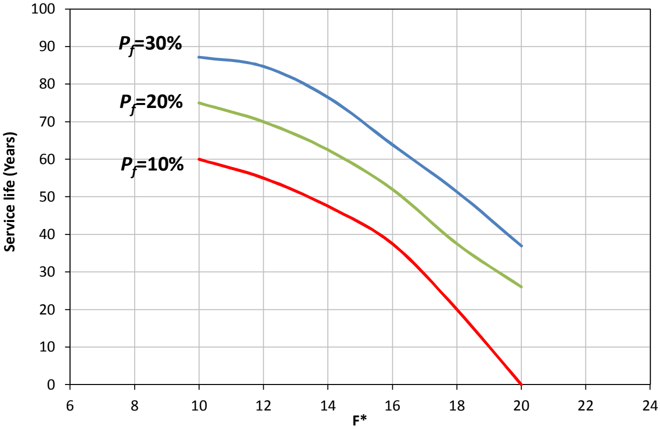
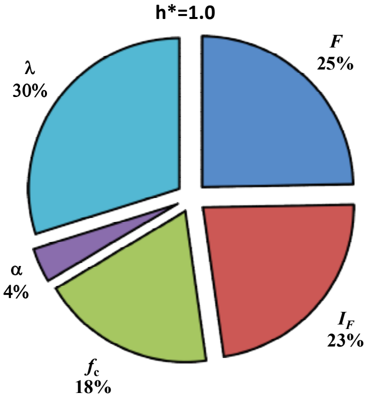


Figure 3. The variation of traffic load (normalised) with the service life of sewer pipes

a)



b)

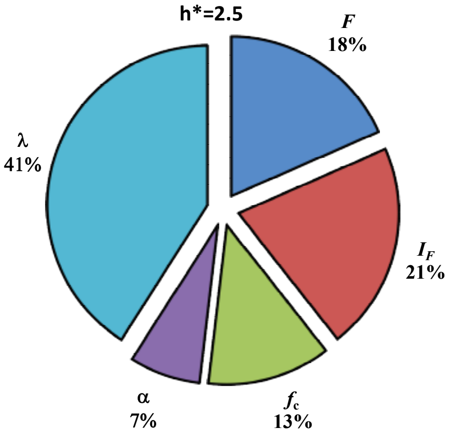


Figure 4. Relative contribution of random variables to the probability of failure of sewer pipe after 100 years of service life (a) h=1.0, (b) h*=2.5*

4- Summary and conclusions

In the present study a stochastic finite element method was utilised to predict the probability of failure of concrete sewer pipes subject to internal corrosion and stresses. Uncertainties involved in pipe material, traffic load and corrosion are considered to develop the stochastic finite element model. A nonlinear time-dependent model was chosen to predict the corrosion in concrete sewer pipes. The model parameters were chosen based on a limited set of existing data in the literature. A normalised numerical example was employed to investigate the effect of both deterministic and probabilistic parameters on the probability of failure of sewer pipes. Two mechanisms of failure (i.e. corrosion and shear failure) were adopted to define the limit state functions. The results of the numerical simulations revealed a nonlinear relationship between most of the parameters and the probability of failure of sewer pipes. In addition the results of the sensitivity analyses showed the significant contribution of the corrosion parameters.The results of the SFEM can be used to improve the performance and planning of existing sewer systems, by providing better predictions for the probability of failure of sewer pipes compared to the existing approaches. The SFEM can bring together the effect of contributing parameters in the probability of failure of the system being studied in a numerical framework with high precision. Using the SFEM it is possible to study the effect of each parameter on the failure of the system and their interaction with each other. The SFEM also provides a time-dependant reliability analysis for predicting the remaining safe life of sewers, which provides a means to better manage the existing sewers and plan resources during their whole life of service. Further improvement in the predictions provided by SFEM can be achieved by collecting additional data on the corrosion rate of concrete sewer pipes.

Acknowledgements

This research was funded by a grant from the UK Engineering and Physical Sciences Research Council (EPSRC) grant number EP/1032150/1 (Assessing Current State of Buried Sewer Systems and Their Remaining Safe Life).

REFERENCES:

1- Ahammed, M, Melchers, R.E. (1994) Reliability of underground pipelines subject to corrosion, Journal of Transportation Engineering, vol. 120; 989-1002.

2- Ahammed, M, Melchers, R.E. (1997) Probabilistic analysis of underground pipelines subject to combined stresses and corrosion, Engineering Structures, vol. 19; 988-994.

3- Florian, A., (1992) An efficient sampling scheme: Updated Latin Hypercube Sampling, Probabilistic Engineering Mechanics, vol. 7, 123-130.

4- Li, S., Yu, S., Zeng, H., Li, J., Liang, R. (2009) Predicting corrosion remaining life of underground pipelines with a mechanically-based probabilistic model, Journal of Petroleum Science and Engineering, vol. 65, 162–166.

5- Melchers, R.E. (1999) Structural Reliability Analysis and Prediction, 2nd edition.

6- Meyer, W.J. (1980) Case study of prediction of sulphide generation and corrosion in sewers, Water Pollution Control Federation, vol. 52, 2666-2674.

7- Pellissetti, M.F., Schuëller, G.I. (2009) Scalable uncertainty and reliability analysis by integration of advanced Monte Carlo simulation and generic finite element solvers, Computers & Structures, vol. 87, 930–947.

8- Stefanou, G., (2009) The stochastic finite element method: Past, present and future, Computer methods in applied mechanics and engineering, vol. 198, 1031-1051.