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A.P. Egorov, V.B. Zvorykin, A.I. Mikhalyov, M.Y. Kuzmenko

**MATHEMATICAL MODEL OF PULSE AUTOMATIC  
CONTROL SYSTEM FOR TEMPERATURE CONTROL  
IN AN ELECTRIC HEATING FURNACE**

**Annotation.** *Functional and structural schemes of a pulsed automatic control system (ACS) have been developed for the temperature control in an electric heating furnace.*

*Mathematical models of PDD-regulator, reversing starter, actuating mechanism and autotransformer are obtained. Chain of links "PWM - converter - reversing starter - actuating mechanism" was linearized. The transfer function of the heating furnace and the temperature sensor was identified.*

*A mathematical model of a pulsed automatic control system for temperature control in a furnace was compiled*

**Keywords:** *electrical furnace, temperature control, impulse regulator, identification, mathematical model*

The use of electrical energy to heat billets and products facilitates the regulation of the thermal regime, allows maintaining the temperature within specified limits and ensures uniform heating of the products [1]. If necessary, in electric furnaces, local heating of individual sections can be carried out. Electric heating furnaces, in comparison, for example, with flame furnaces, are much easier to seal, which allows to reduce heat losses with waste gases, and this, in turn, provides a higher efficiency of the heat unit. All this makes actual the research of automatic control systems by the thermal mode of heating of products for various purposes and the development of thermal automation in general.

The articles [2, 3] describe objects of thermal automation control. The expediency of using a PID controller for objects with an actuator heater (refrigerator) is shown.

Consider a heating furnace in which temperature control is performed by means of a single-turn actuating mechanism. In this case, the use of a regulator with an integral part is not permissible. In the direct control channel of the system there are two integrating links: the regulator and the actuating mechanism. As a consequence, the regulatory system becomes structurally unstable.

Therefore, in such systems use the proportional differential (PD) or proportional-differential second-order (SDA) control law:

$$U_p = K_p \left[ 1 + T_d p + T_{dd} p^2 \right] \varepsilon, \quad (1)$$

where  $\varepsilon$  - the difference between the set and current value of the controlled parameter;  $K_p$  - coefficient of proportionality;  $T_d$ ,  $T_{dd}$  - the time constants of differentiation.

### Description of the installation - control systems for the temperature regime of an electric heating furnace

The functional diagram of the pulsed ACS furnace temperature is shown in fig. 1, the diagram of the functional structure is shown in fig. 2.

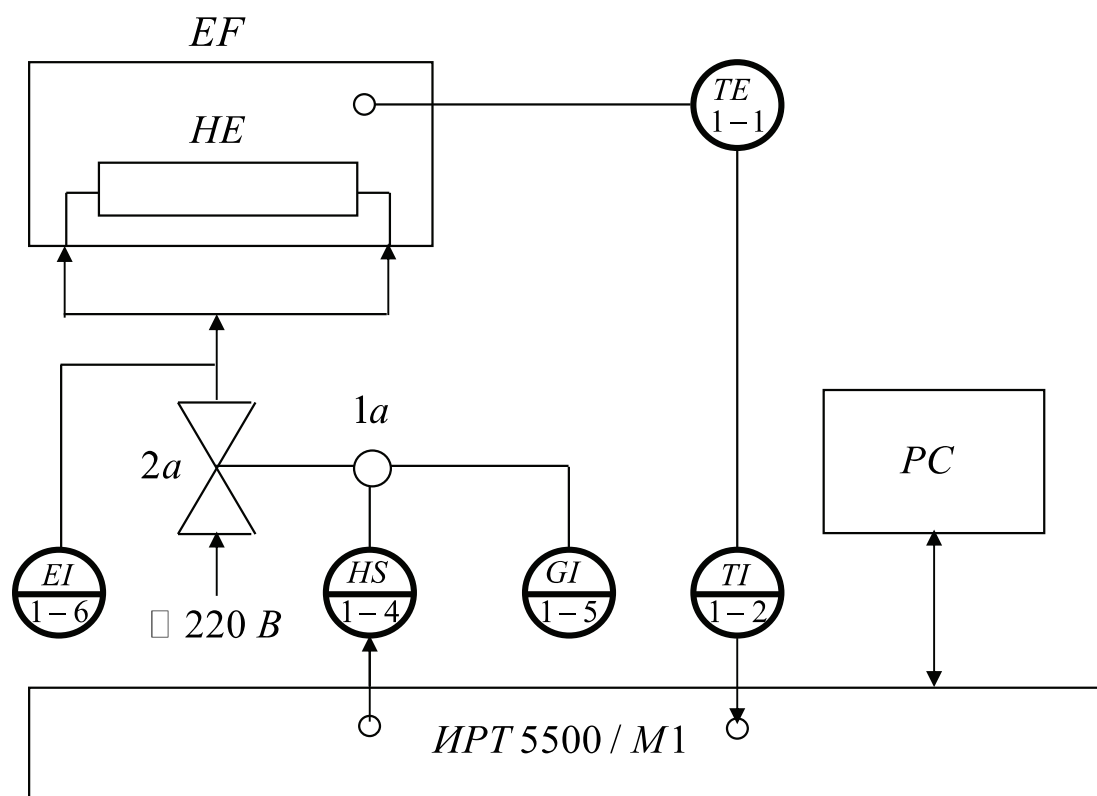


Fig. 1 – Functional diagram of the installation of a pulsed ACS furnace temperature

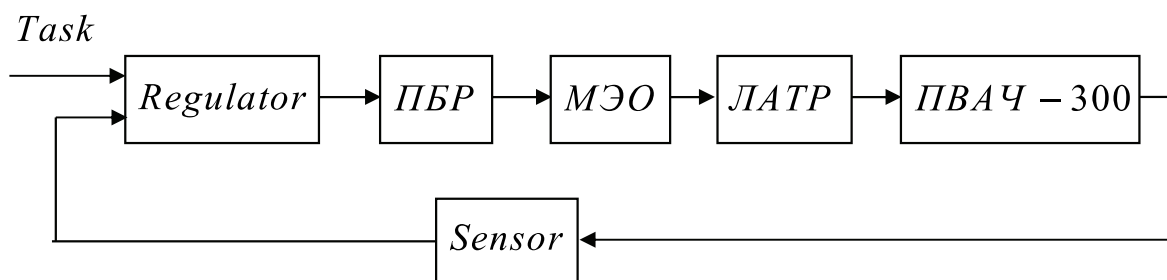


Fig. 2 – Diagram of the functional structure of a pulsed ACS temperature in an electric furnace

The installation is a system of automatic temperature control in the working space of an electric furnace with the pulse PDT regulator ИРТ 5500/М1 (pulse width modulated signal is at the regulator output) and includes:

- temperature sensor ТПУ0304/М2,
- regulating device ИРТ5501/М1 [4],
- starter contactless reversible ПБР-2М [5],
- actuating mechanism МЭО 250/63-025 [6],
- autotransformer type ЛАТР,
- electric furnace type ПВАЧ-300,
- personal computer.

Electric furnace  $EF$  has a heating element  $HE$ , fed from a single-phase alternating current network through an autotransformer 2a of the type ЛАТР. Temperature of the working space of the furnace is controlled by the thermoelectric converter ТПУ0304 (1-1, 1-2).

The voltage applied to the furnace is controlled by a voltmeter.

The position of the output shaft of the actuator 1a is controlled by a remote position indicator 1-5 of the type ДУП-М.

Personal computer  $PC$  is intended for input of parameters of a regulator and display of transient process.

The temperature in the furnace is affected by disturbances caused by a change in the supply voltage, and also by a change in the amount of air entering the furnace.

### Mathematical model of the system

From the point of view of the mathematical description, the control system is a chain of consecutively connected linear and nonlinear links (fig. 3).

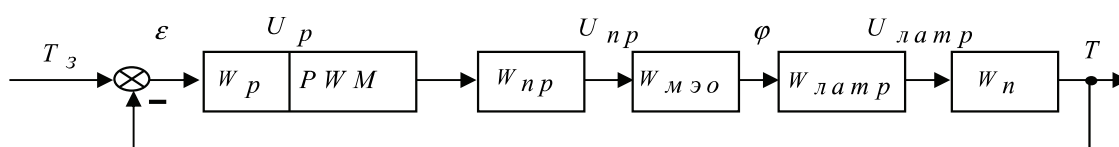


Fig. 3 – Structural diagram of the investigated automatic control system

In fig. 3 denotes:  $W_p(p)$  - transfer function of the regulator;  $W_{np}(p)$  - transfer function of the reversing starter;  $W_{mzo}(p)$  -

transfer function of the actuating mechanism;  $W_{\text{лампа}}(p)$  - transfer function of ЛАТП;  $W_n(p)$  - transfer function of the regulated object.

When modeling thermal units, the control object includes a furnace and a temperature sensor. Therefore, the transfer function  $W_n(p)$  is equal to the product of the transfer functions of the furnace and the temperature sensor.

Let's consider mathematical models of links of system.

### Temperature regulator

The output signal of the pulse regulator is a sequence of pulses whose duty cycle  $T_1, T_2, T_3, \dots$  depends on the value of the signal at the input of the regulator (fig. 4).

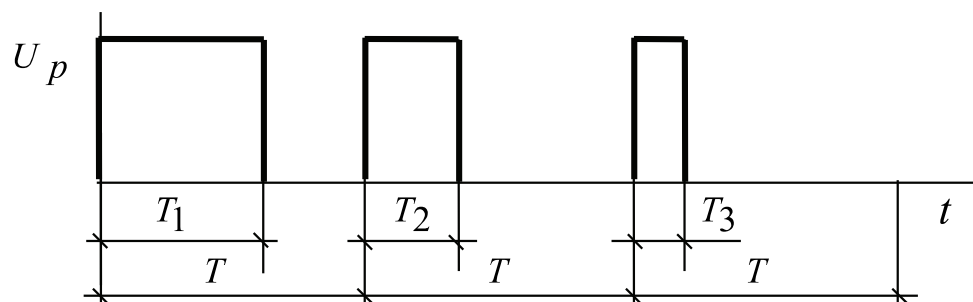


Fig. 4 – Output pulse width modulated (PWM) controller signal

Impulses follow with a period  $T$ , the duration of which is chosen in accordance with the inertia of the control object by the Kotel'nikov theorem.

### Reversing starter

The reversing starter, amplifying these pulses by voltage and power, turns on the MEO actuator motor in the "forward" or "backward" direction, depending on the PWM signal of the regulator.

The characteristic of the reversing starter has the form of a three-position relay element (fig. 5).

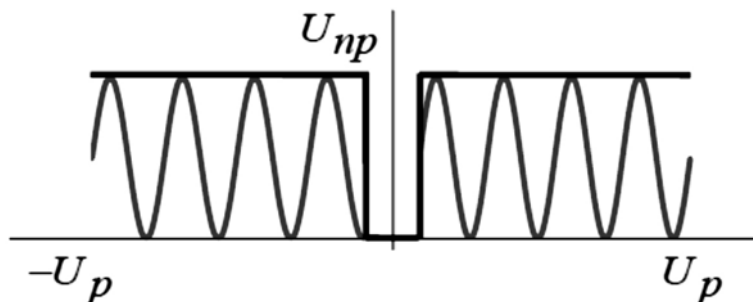


Fig. 5 – Characteristics of the reversing starter

### Actuating mechanism

Single-turn actuator MEO when its feeding voltage equal  $U_{np} = 100$  B turns the shaft at 2.18 rad for 76 s.

Set the temperature controller input task, e.g., 100°C. At the time the system is turned on, a mismatch  $\varepsilon$  at the regulator input  $W_p(p)$  will be 100%. At the output of the starter there will be voltage  $U_{np} = 100$  B and the motor of the actuator will rotate at maximum speed.

Angle of rotation of the actuator shaft:

$$\varphi = K_{MЭО} \int_0^t U_{np} dt = K_{MЭО} U_{np} t, \quad (2)$$

where  $K_{MЭО}$  - transfer coefficient of the МЭО.

For boundary conditions  $t = 76$  s and  $\varphi_{max} = 2.18$  rad, we get:

$$K_{MЭО} = \frac{\varphi_{max}}{U_{np} t} = \frac{2.18}{100 \times 76} = 0.000287 \frac{rad}{Bc}. \quad (3)$$

Transfer function of the actuator MEO:

$$W_{MЭО}(p) = \frac{\varphi(p)}{U_{np}(p)} = \frac{K_{MЭО}}{p} = \frac{0.000287}{p}. \quad (4)$$

### Simulink - model of PWM-converter, reversing starter and actuating mechanism

PWM converters are used to control the average slewing speed of the actuator shaft, depending on the misalignment of the reference and feedback signals.

Simulink - model of PWM converter, reversing starter and actuator is shown in fig. 6.

The sawtooth pulse generator [7] consists of a square-wave generator with adjustable duty cycle and an integrator covered by negative feedback with a large gain. During the pause (no pulses), the integrator integrates a constant signal at its input, during a short rectangular pulse, the integrator output signal is reset.

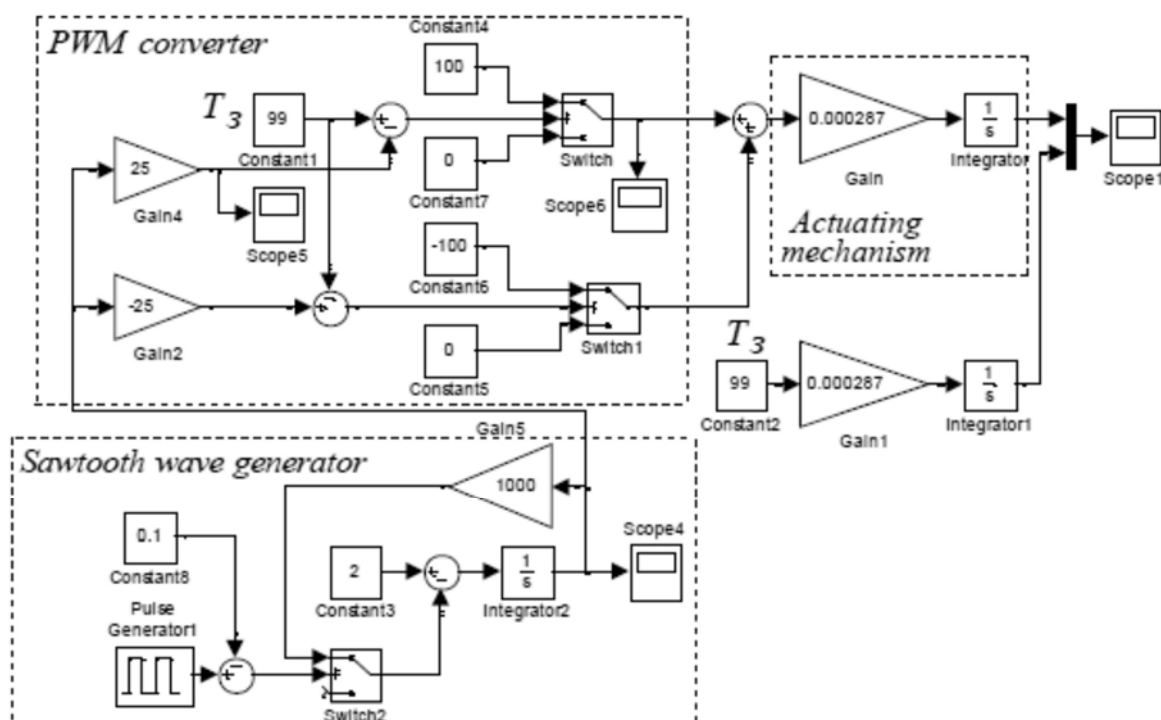


Fig. 6 – Simulink is a model of PWM converter and actuator

Disagreement  $\varepsilon$  between the reference and feedback signals at the controller input is compared with the generator output value. With a positive result of the comparison, a positive rectangular pulse is formed, and with a negative result, a negative rectangular pulse is formed. The width of the rectangular pulses depends on the time the saw tension voltage changes from the zero value to the point of equality of the error signal and the output of the generator. The amplitude of the square pulses is determined by the supply voltage  $U_{np} = 100$  B.

Let us consider the possibility of representing the chain of links "PWM converter - reversing starter - actuator" in the form of a linear link.

In fig. 7 shows the curves for changing the angle of rotation  $\varphi$  shaft of the actuator with different mismatch  $\varepsilon$  at the regulator input (1, 3 - actuating mechanism with a PWM converter, 2, 4 - actuating mechanism, with a constant reference numerically equal to the error at the regulator input).

Comparison plots of transient processes in the actuator, the illustrated linear transfer functions (Fig. 7, curves 2 and 4), and in the actuating mechanism with PWM-converter (Fig. 7, curves 1 and 3) leads to the following conclusion.

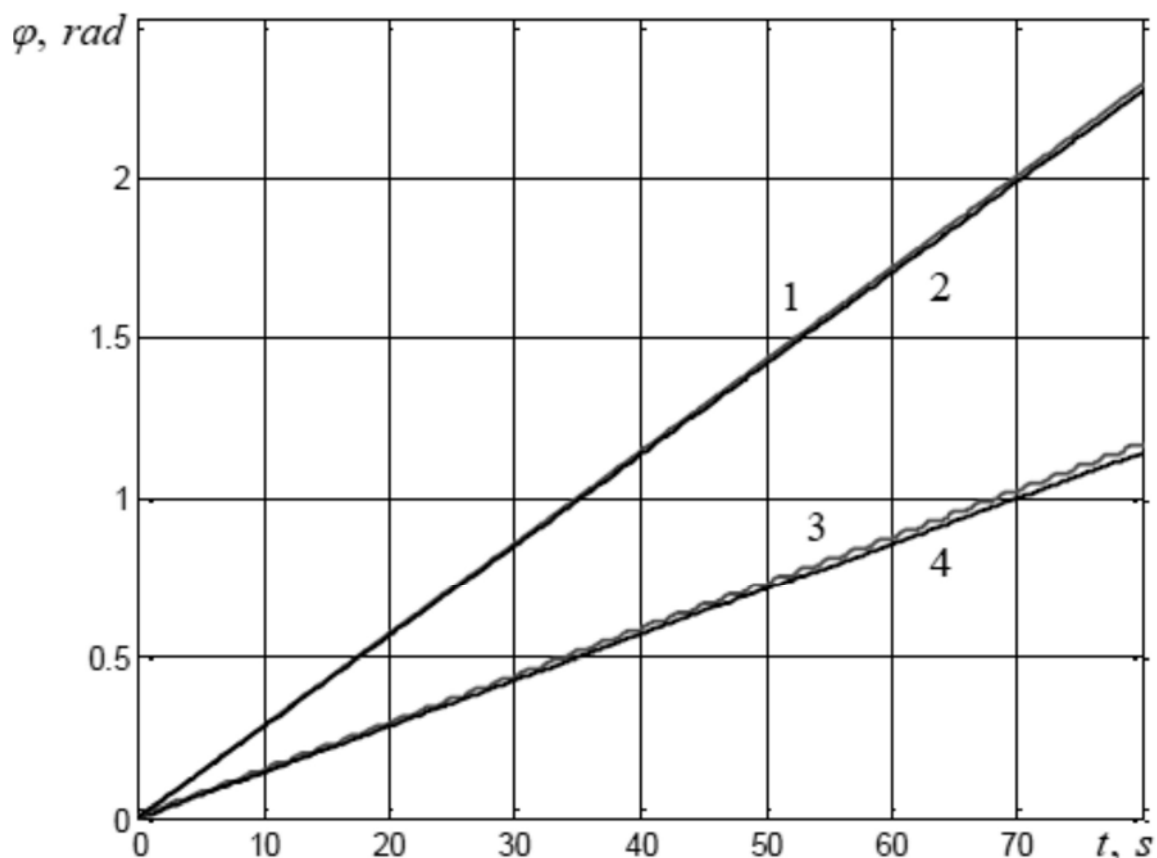


Fig. 7 – Transient processes in the actuator with 100% mismatch at the controller input (curves 1, 2) and at 50% mismatch at the regulator input (curves 3, 4)

When the sampling period PWM - converter significantly smaller than the transient time in the actuating mechanism (in this case the sampling period is 40 times smaller than the time displacement of the actuating mechanism from the zero position to the fully open position), the chain units "PWM - converter - reversing starter - actuating mechanism" can be described by a linear integrator (4).

#### LATR

Transfer function of ЛАТР:

$$W_{латр}(p) = K_{латр} = \frac{U_{латр}}{\varphi_{max}} = \frac{60}{2.18} = 27.52 \frac{B}{рад}, \quad (5)$$

where  $U_{латр} = 60$  В - voltage at the terminals of the ЛАТР at the maximum angle of rotation of the actuator shaft  $\varphi_{max} = 2.18$  рад.

### The object of regulation

To obtain the transfer function of the control object, we construct a graph of temperature change in the furnace, giving a jump in the supply voltage  $U_{лампы} = 60$  В. We obtain the curve for the acceleration of the furnace, shown in fig. 8.

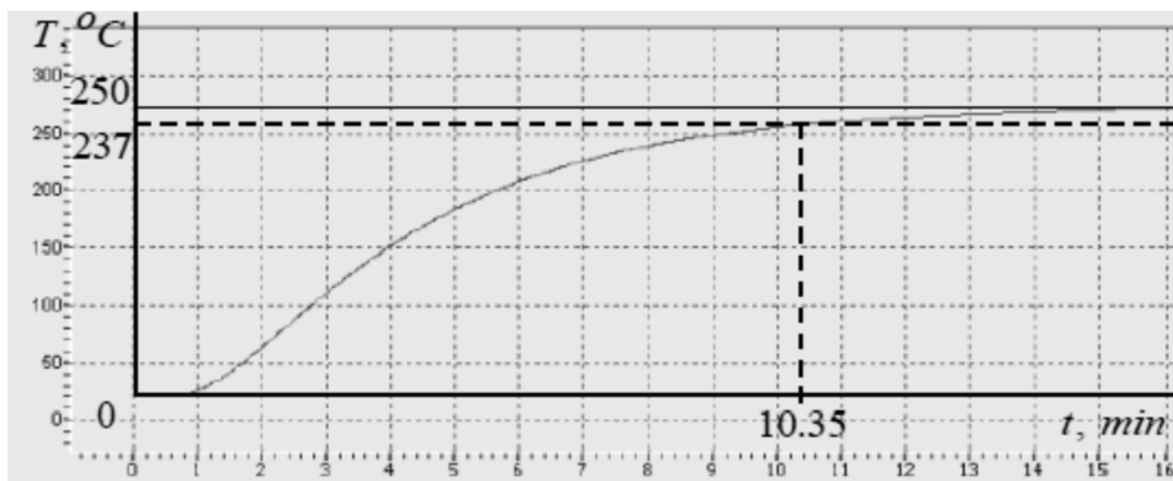


Fig. 8 – Experimental kiln acceleration curve

Since the initial temperature of the furnace corresponds to the ambient temperature  $22^{\circ}\text{C}$  shift the origin point at  $22^{\circ}\text{C}$  (abscissa shifting up) and obtain temperature scale shifted relative to the abscissa scale (fig. 8).

To determine the delay, let us consider the initial section of the acceleration curve of the furnace (fig. 9).

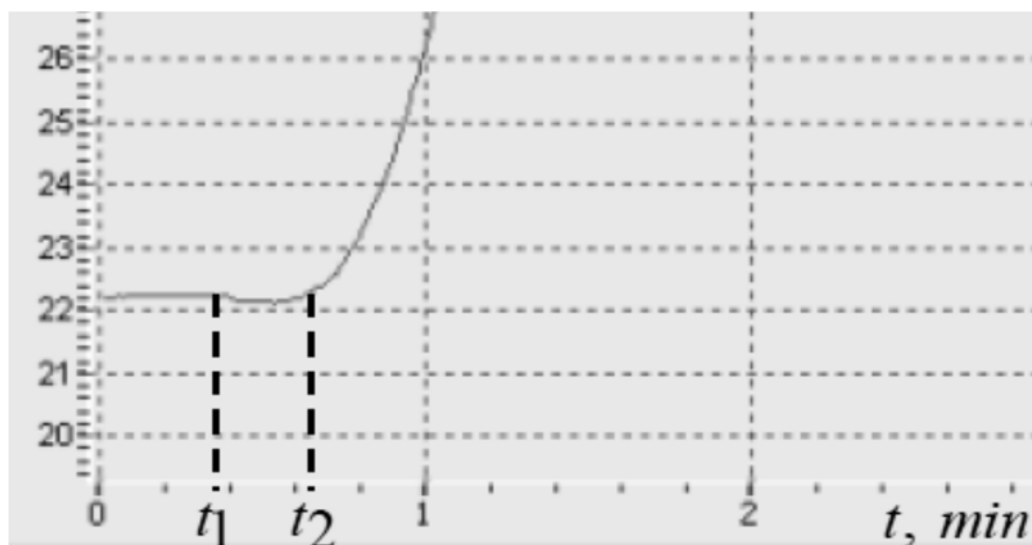


Fig. 9 – Section of delay



The delay  $\tau$  we calculate as the difference between the beginning of the temperature change in the furnace  $t_2$  and the feeding time oven supply voltage  $t_1$ :

$$\tau = t_2 - t_1 = 0.65 - 0.4 = 0.25 \text{ мин} = 15 \text{ с} . \quad (6)$$

The curve of the transient process of temperature variation in the furnace (fig. 8) is similar to the transient characteristic of a first-order aperiodic link. We obtain the coefficients of the transfer function of the control object along this curve.

A first-order differential equation describing the temperature change in the furnace with a sudden change the signal at the input:

$$T_n \frac{dT}{dt} + T = K_n U_{\text{лампы}} , \quad (7)$$

where  $T_n$  - furnace time constant, s;  $K_n$  - furnace transfer coefficient and the temperature sensor, °C/B.

The solution of this equation is the expression:

$$T = K_n U_{\text{лампы}} \left( 1 - e^{-\frac{T_n}{t}} \right) . \quad (8)$$

Equation (8) corresponds to the control time:

$$t_p = 3T_n . \quad (9)$$

As follows from the curve of the acceleration of the furnace (fig. 8), the steady-state value of the temperature is  $T_{ycm} = 250^{\circ}\text{C}$ . Time of regulation  $t_p$  find how the time to reach the temperature  $0.95T_{ycm}$  (entry of the transient into a 5% tube). To do this, draw a horizontal dotted line from the ordinate  $0.95T_{ycm} = 237^{\circ}\text{C}$  before it crossing with the curve of the acceleration of the furnace. From the intersection point, let us drop the perpendicular to the abscissa axis, obtaining a point corresponding to the control time  $t_p = 10.35 \text{ мин} = 621 \text{ с}$ .

In accordance with equation (9), we determine the furnace time constant:

$$T_n = \frac{t_p}{3} = \frac{621}{3} = 207 \text{ s.} \quad (10)$$

Transmission ratio  $K_n$  we calculate how the ratio of the steady-state value of the temperature to the output voltage of the ЛАТР.

$$K_n = \frac{T_{уст}}{U_{ЛАТР}} = \frac{250}{60} = 4.17 \frac{^{\circ}\text{C}}{\text{В}}. \quad (11)$$

The effect of delay  $\tau$  on the transient process of temperature variation in the furnace is neglected in view of its smallness.

Thus, taking into account equations (10), (11), the transfer function of the furnace and the temperature sensor takes the form:

$$W_n(p) = \frac{K_n}{T_n p + 1} = \frac{4.17}{207 p + 1}. \quad (12)$$

Comparison of the transient graph in an aperiodic link (fig. 10, curve 1) and graphics obtained as a result of the experiment (fig. 10, curve 2) shows good agreement between them (root mean square value of the error is  $4.1^{\circ}\text{C}$ )

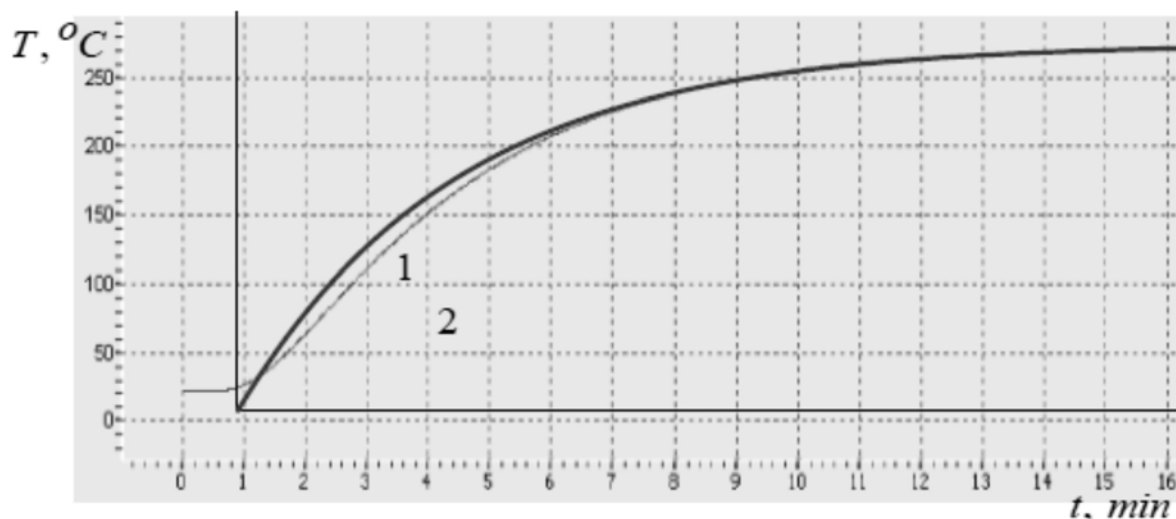


Fig. 10 - The transient process in the aperiodic link (1) and the experimental curve for accelerating the furnace (2)

Taking into account the mathematical models of the individual links of the system, as well as the structural diagram depicted in Fig. 3, a mathematical model of a pulsed automatic control system for temperature control in an oven is compiled. The corresponding structural diagram is shown in fig. 11.

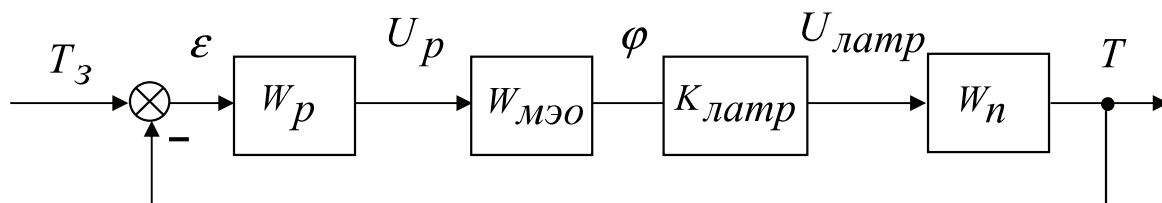


Fig. 11 – Block diagram of the linearized pulsed ACS temperature control in the furnace

### Conclusions:

- mathematical models of separate links in the control system of the furnace temperature are obtained;
- when the sampling period PWM - converter significantly smaller than the transient time in the actuator, chain units "PWM - converter - reversing starter - actuating mechanism" may be described by a linear integrator;
- transfer function of the object control - heating furnaces and sensor temperature is identified;
- based on the comparison the simulation results and the experiment proved conformity transfer function furnaces and sensor temperatures real object management.
- The mathematical model of linearized ACS pulse regulation in the furnace temperature is compiled.

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