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L.G. Akhmetshina, M.G. Berdnyk

MATHEMATICAL MODELING OF TEMPERATURE FIELDS OF THE ROTATING CYLINDER

Annotation. In paper we consider a solid cylinder as a simplified model of the roll. The cylinder is under the influence of heat flow, which reflects the effect of the heated metal sheet on the roll. A three-dimensional mathematical model of temperature fields in a solid cylinder, which rotates at a constant angular velocity about the axis OZ length L, as a boundary value problem of mathematical physics is presented. As a result, found the temperature field of a solid cylinder as convergent orthogonal series of Bessel functions and of Fourie.

Key words: temperature field, a rolling roll, boundary value problem, boundary condition, the complex Fourier series, integral Hankel transform, Laplace, Fourie, Predvoditeleva criterion.

Introduction. It is historically established that Ukraine, having highly developed net of metallurgical works, produces a significant part of the world steel. In the process of rolling, the steel shits heated up to 12000C are transported with the help of the forming rollers. Consequently, the rollers are heated and, reaching critical temperatures, can be deformed causing spoilage in production [1-3]. Therefore, correct geometry of the rolled products and reduce of expenditures spent for the forming rollers directly depend on proper control of the heated roller shape and its thermal growth along the whole length of the body of roller [3].

Intensification of production especially of the new product assortment more accurate tolerance range both by flatness and crosscut gage interference requires from the millmen to focus on the thermal state of the rollers [4 - 6]. Thus, it is necessary to analyze the roller temperature and provide the needed cooling for the roller [5]. In connection with this, study of thermal processes occurred in result of the roller cooling presents great theoretical and practical interest.

In this paper, a cylinder is presented as a simplified model of the forming roller being under the impact of the heat flow, which is a sequence of interaction between the roller and the red-hot steel shit.

The overviewed literature has shown that today understanding of the heat-exchange process occurring in rotating cylinders is not comprehensive enough [7, 8]. The [9] states that numerical methods used for studying non-stationary nonaxisymmetric problems of the heat exchange in rotating cylinders are not economically justified for calculations when rotation velocity is high [9].

According to the [9], conditions for the calculation accuracy are specified by similar characteristics for the finite element method and finite difference method which both are applied for computing non-stationary non-axisymmetric temperature fields in rotating cylinders. These conditions can be expressed as:

$$1-2\frac{\Delta F_o}{\Delta\phi^2}\geq 0 \quad \text{and} \quad \frac{1}{\Delta\phi}-\frac{Pd}{2}\geq 0 \ ,$$

where Fo is Fourier criterion, and Pd is Predvoditelev criterion.

If Pd =105 corresponds to angular velocity of the steel cylinder rotation $\omega = 1{,}671\ \text{sec.}^{-1}$, with radius 100 mm, then variables $\Delta\phi$ and ΔF_o should be subject to the following conditions $\Delta\phi \leq 2\times 10^{-5}$ and $\Delta F_o \leq 2 \times 10^{-10}$.

In the case of uniformly cooled cylinder, and if Bi = 5 (Bi is Bio criterion), a time period needed in order the temperature can reach 90% of stationary state should be $Fo \approx 0.025$. It means that, within this period of time, at least 1.3×10^{8} operations should be fulfilled in order to obtain stationary temperature distribution.

Moreover, it should be mentioned that it would be necessary to make 3.14x 105 calculations within one cycle of computation as the inside state of the ring should be characterized by 3.14x105 points. It is obvious that this number of calculations needed for getting a numerical result is unrealistic

Therefore, we will employ integral transformations for solving boundary problems which occur during mathematic modeling of the 3D non-stationary heat-exchange processes in rotating cylinders.

The purpose of this work is build a three-dimensional mathematical model of temperature field distribution in a cylinder, which rotates

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with a constant angular velocity ω around the axis OZ, in the form of physicomathematical boundary problem for the heat conduction equation and to find a solution for the obtained boundary problem solving of which can be used for the temperature filed control.

Let's consider a calculation of nonstationary non-axisymmetric temperature field in a solid cylinder in cylindrical coordinate system (r,ϕ,z). The cylinder rotates with constant angular velocity ωaround the axis OZ with finite length L, and its heat-transfer properties do not depend on temperature, and no internal sources of heat are available. At zero time point, temperature of the cylinder is constant G_0, and heat flows on surface of the cylinder depend on time $G(\phi,z,t)$.

Mathematically, a problem of defining a temperature field $T(\rho,\phi,z,t)$ in a cylinder consists of integration of differential heat-conduction equation [10] into the domain $D=\{(\rho,\phi,z,t)\ |\rho\in(0,1),\ \phi\in(0,2\pi),\ \mathrm{z}\in(0,1), F_0\in(0,\infty)\ \}$, which, with taking into consideration the accepted assumptions, can be written in the following way:

$$\frac{\partial\theta}{\partial F_0}+\omega\frac{\partial\theta}{\partial\phi}=\frac{\partial^2\theta}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\theta}{\partial\phi^2}+\chi\frac{\partial^2\theta}{\partial z^2} \qquad (1)$$

with initial condition

$$\theta(\rho,\phi,z,0)=0 \qquad (2)$$

and boundary conditions

$$\theta(1,\phi,z,F_0)=V(\phi,z,F_0), \qquad (3)$$

$$\theta(\rho,\phi,0,F_0)=0, \quad \theta(\rho,\phi,1,F_0)=0, \qquad (4)$$

where $\theta=\dfrac{T(\rho,\phi,z,t)-G_0}{T_{\max}-G_0}$ is relative temperature of the cylinder; $T_{\max}=\max\limits_{\phi,z}G(\phi,z,t)$; t is time period; $\rho=\dfrac{r}{R}$; R is external radius of the cylinder; $\chi=(R/L)^2$; $a=\dfrac{\lambda}{c\gamma}$ is thermal diffusivity coefficient; γ – is medium density; λ is heat conduction coefficient; c is specific heat capacity; $F_0 = at \cdot R^{-2}$ is Fourier criterion; $Pd = \dfrac{\omega R^2}{a}$ is Predvoditelev criterion; $V(\phi, z) \in C(D)$.

Then solution of the problem (1)-(4) $\theta(\rho, \phi, z, F_0)$ is twice continuously differentiated by ρ and ϕ, z, once - by t in eth domain D and continuous on the $\bar{D}$[10], i.e. $\theta(\rho, \phi, z, t) \in C^{2,1}(D) \cap C(\bar{D})$, and functions $V(\phi, z, F_0)$, $\theta(\rho, \phi, z, F_0)$ can be decompose into the Fourier complex series [11,12]:

$$\begin{Bmatrix} \theta(\rho, \phi, z, F_0) \\ V(\phi, z, F_0) \end{Bmatrix} = \sum_{n=-\infty}^{+\infty} \begin{Bmatrix} \theta_n(\rho, z, F_0) \\ V_n(z, F_0) \end{Bmatrix} \cdot \exp(in\phi) \tag{5}$$

where

$$\begin{Bmatrix} \theta_n(\rho, z, F_0) \\ V_n(z, F_0) \end{Bmatrix} = \frac{1}{2\pi} \int_0^{2\pi} \begin{Bmatrix} \theta(\rho, \phi, z, F_0) \\ V(\phi, z, F_0) \end{Bmatrix} \cdot \exp(-in\phi) d\phi; \tag{6}$$

$$\theta_n(\rho, z, F_0) = \theta_n^{(1)}(\rho, z, F_0) + i\theta_n^{(2)}(\rho, z, F_0), \quad \mathrm{V_n}(z) = V_n^{(1)}(z, F_0) + i\ \mathrm{V_n^{(2)}}(\mathrm{z}, F_0) \tag{7}$$

where i is imaginary unit.

In view of the fact that $\theta(\rho, \phi, z, F_0)$ is a real-valued function, let's confine ourselves by considering only $\theta_n(\rho, z, F_0)$ for n=0,1,2,..., because $\theta_n(\rho, z, F_0)$ and $\theta_{-n}(\rho, z, F_0)$ are complexly conjugate [11].

By putting values of functions from (5)-(7) into (1)-(4) we obtain the following system of differential equations:

$$\frac{\partial \theta_n^{(i)}}{\partial F_0} + \vartheta_n^{(i)} \theta_n^{(m_i)} = \frac{\partial^2 \theta_n^{(i)}}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_n^{(i)}}{\partial \rho} - \frac{n^2}{\rho^2} \theta_n^{(i)} + \chi \frac{\partial^2 \theta_n^{(i)}}{\partial z^2} \tag{8}$$

with initial condition

$$\theta_n^{(i)}(\rho, z, 0) = 0, \tag{9}$$

and boundary conditions

$$\theta_n^{(i)}(1, z, F_0) = V_n^{(i)}(z, F_0) \tag{10}$$

$$\theta_n^{(i)}(\rho, 0, F_0) = 0, \qquad \theta_n^{(i)}(\rho, 1, F_0) = 0, \tag{11}$$

where $\vartheta_n^{(1)} = -\omega n$; $\vartheta_n^2 = \omega n$; $m_1 = 2$; $m_2 = 1$; i=1,2.

Let's employ the Hankel integral transformation [13] for the system of differential equations (8):

$$\overline{f}\left(\mu_{n,k}\right)=\int_{0}^{1}\rho\ \mathrm{f}(\rho)\ J_n\left(\mu_{n,k}\right)\ d\rho\,, \qquad (12)$$

where $J_n(x)$ is Bassel function of the 1st kind of the nth order; $\mu_{n,k}$ are roots of transcendental equation $J_n\left(\mu_{n,k}\right)=0$, which can be seen in formula [14]:

$$\mu_{n,k}=\beta-\frac{m-1}{8\beta}-\frac{4(m-1)(7m-31)}{3(8\beta)^3}-\frac{32(m-1)\left(83m^2-982m+3779\right)}{15(18\beta)^5}-$$
$$-\frac{64(m-1)\left(6949m^3-15385m^2+1585743m-6277237\right)}{105(8\beta)^7},$$

where $\beta=\frac{1}{4}\pi(2n+4k-1)$, $\mathrm{m}=4\mathrm{n}^2$, and formula of inverse transformation is written in the following way:

$$f(\rho)=2\sum_{k=0}^{\infty}\frac{J_n\left(\mu_{n,k}\rho\right)}{\left[J'_n\left(\mu_{n,k}\right)\right]^2}\ \overline{f}\left(\mu_{n,k}\right). \qquad (13)$$

As a result, we receive the following system of differential equations:

$$\frac{\partial\overline{\theta}_n^{(i)}}{\partial\ F_0}+\vartheta_n^{(i)}\overline{\theta}_n^{(m_i)}=.\mu_{n,k}J'_n\left(\mu_{n,k}\right)V_n^{(i)}\left(z,F_0\right)-\mu_{n,k}^2\overline{\theta}_n^{(i)}+\chi\frac{\partial^2\overline{\theta}_n^{(i)}}{\partial z^2} \qquad (14)$$

with initial condition

$$\overline{\theta}_n^{(i)}\left(\mu_{n,k},z,0\right)=0, \qquad (15)$$

and boundary conditions

$$\overline{\theta}_n^{(i)}\left(\mu_{n,k},0,F_0\right)=0, \qquad \overline{\theta}_n^{(i)}\left(\mu_{n,k},1,F_0\right)=0, \qquad \mathrm{i}=1,2. \qquad (16)$$

Let's employ the Fourier integral transformation [13] for the system of differential equations (14):

$$\widehat{f}\left(\lambda_m\right)=\int_{0}^{1}f(x)\ \sin(\pi\ \cdot \mathrm{m}\ \cdot \mathrm{x})\ \ \mathrm{dx}\,,$$

where $\lambda_m=\pi\ \cdot m$; m=1,2,…, and formula of inverse transformation is written in the following way:

$$f(x)=2\sum_{m=1}^{\infty}\sin(\pi\ \cdot \mathrm{m}\ \cdot \mathrm{x})\ \cdot\widehat{f}\left(\lambda_m\right). \qquad (17)$$

As a result, we receive the following system of ordinary differential equations:

$$\frac{d\bar{\bar{\theta}}_n^{(i)}}{d\ F_0} + \vartheta_n^{(i)}\bar{\bar{\theta}}_n^{(m_i)} = d_n^{(i)}(\mu_{n,k}, \lambda_m, F_0) - \left(\mu_{n,k}^2 + \chi \cdot \lambda^2{}_m\right)\bar{\bar{\theta}}_n^{(i)} \quad (18)$$

with initial conditions

$$\bar{\bar{\theta}}_n^{(i)}\left(\mu_{n,k}, \lambda_m, 0\right) = 0, \quad (19)$$

where $d_n^{(i)}(\mu_{n,k}, \lambda_m, F_0) = \mu_{n,k} J'_n\left(\mu_{n,k}\right)\hat{V}_n^{(i)}\left(\lambda_m, F_0\right)$, i=1,2.

Let's employ the Laplace integral transformation [13] for the system of ordinary differential equations: $\tilde{f}\left(s\right) = \int\limits_0^\infty f\left(\tau\right)\ e^{-s\tau}\ \mathrm{d}\tau$.

As a result, we receive the following system of algebraic equations relatively to $\tilde{\bar{\bar{\theta}}}_n^{(i)}$:

$$s\tilde{\bar{\bar{\theta}}}_n^{(i)} + \vartheta_n^{(i)} \cdot \tilde{\bar{\bar{\theta}}}_n^{(m_i)} = \tilde{d}_n^{(i)}(\mu_{n,k}, \lambda_m, s) - \left(\mu_{n,k}^2 + \lambda^2{}_m\right)\tilde{\bar{\bar{\theta}}}_n^{(i)} \quad (20)$$

where i=1,2.

By employing formulas of the Laplace inverse transformations [13] for the expression of the functions (20), we can receive originals of the functions:

$$\bar{\bar{\theta}}_n^{(i)}(F_0) = \int\limits_0^{Fo}\left[d_n^{(i)}(\mu_{n,k}, \lambda_m, F'_0) \cdot \cos nPd(F_0 - F'_0) + \delta_i d_n^{(m_i)}(\mu_{n,k}, \lambda_m, F'_0)\sin nPd(F_0 - F'_0)\right] \cdot$$

$$\exp\left[(\mu_{n,j}^{(i)} + x\beta_m^2)\left(F'_0 - F_0\right)\right]dF'_0 \quad (21)$$

where $\delta_1 = -1$, $\delta_2 = 1$; i=1,2.

Thereby, by taking into account the inverse transformation formulas (13) and (17), we can determine a temperature field in the solid cylinder, which rotates with constant angular velocity ω around the axis OZ with the length L:

$$\theta\left(\rho, \phi, z, F_0\right) = \sum_{n=-\infty}^{+\infty}\left\{2\sum_{\mathrm{k}=1}^{\infty}\left\langle 2\sum_{m=1}^{\infty}\left[\ \bar{\bar{\theta}}_n^{(1)}\left(F_0\right) + i\bar{\bar{\theta}}_n^{(2)}\left(F_0\right)\ \right]\ \sin\left(\pi \mathrm{mz}\right)\right\rangle\ \frac{J_n\left(\mu_{n,k}\rho\right)}{\left[J'_n\left(\mu_{n,k}\right)\right]^2}\right\}\exp\left(in\phi\right) \quad (22)$$

where values $\bar{\bar{\theta}}_n^{(1)}\left(F_0\right)$, $\bar{\bar{\theta}}_n^{(2)}\left(F_0\right)$ are defined by the formulas (21).

With the view of calculating the temperature fields $\theta\left(\rho, \phi, z, F_0\right)$ by formula (22), a new software was created in the object-oriented programming language C#, which was integrated into the development framework Microsoft Visual Studio 2010, and which can operate with any operational system on the basis of the version Microsoft

.NET Framework or higher. In the numerical calculation s, sums of series were replaced by the partial sums with the accuracy of 10^{-4}.

Temperature on the cylinder surface was given as:

$$\theta(1,\phi,z,t) = \Theta_1(\phi)\cdot\eta(l_1 - z) + \Theta_2(\phi)\cdot\left[\eta(1-l_2-z) - \eta(l_1-z)\right] + \Theta_3(\phi)\cdot\left[\eta(1-z) - \eta(1-l_2-z)\right],$$

$$\Theta_2(\phi) = 2\cdot\pi^{-1}\cdot\phi\cdot\eta(0,5\pi-\phi) + \eta(1,5\pi-\phi) - \eta(0,5\pi-\phi) + \cdot \left[1 + 2\cdot\pi^{-1}\cdot\eta(1,5\pi-\phi)\right]\left[\eta(2\cdot\pi-\phi) - \eta(1,5\pi-\phi)\right],$$

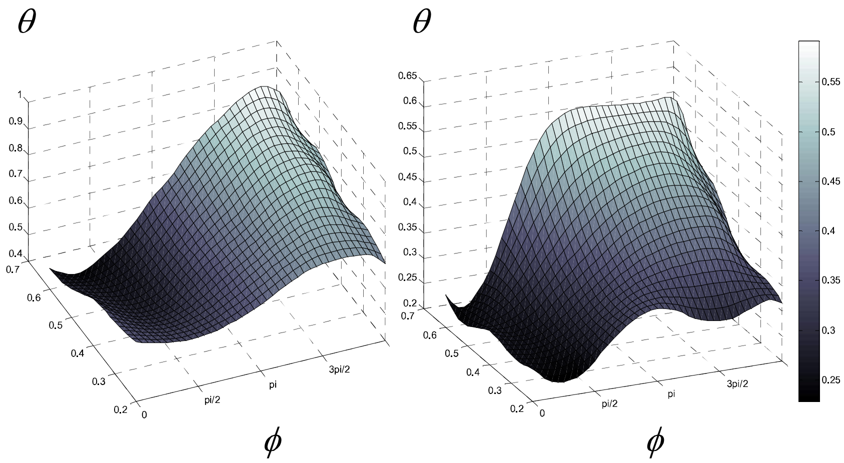
where $\Theta_1(\phi)$=$\Theta_3(\phi)$=0,03; $l_1 = l_2$=0,25; l=1;

$$\eta(z) = \begin{cases} 1 & \text{if} \quad z \ge 0 \\ 1 & \text{if} \quad z \ge 0 \end{cases}.$$

The following values are needed for calculating the cylinder material properties: $\lambda = 34.8$ W/(m.·Кo); $c = 560$ J./(kg Кo); $\gamma = 7800$ kg m.^{3}.

Experimental results. Results of the numerical experiments are shown in Fig.1 in the form of temperature distribution curves [15] at the following parameters: z=0.5, Pd=10.

Conclusions. A temperature field in the solid cylinder, which rotated with a constant angular velocity ω around the axis OZ with length L, was determined in the form of convergent orthogonal series with the help of the Bessel and Fourier functions. The obtained analytical solution for the boundary problem of the heat-exchange in the rotating cylinder can be used for modeling temperature fields which occur in many technical systems (satellites, forming rollers, turbines, etc.).



a b

Figure 1 – Distribution of temperature field in the cylinder at different value of the Fourier criteria: a) $F_0 = 0.9$; b) $F_0 = 0.1$.

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