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STATISTICAL PREDICTION OF THE RELIABILITY OF COMPOSITE MATERIALS WITH DISPERSIVE INCLUSIONS

© R. Baitsar, R. Kvit, A. Malyar

Запропоновано алгоритм розрахунку надійності (ймовірності руйнування) композитних матеріалів зі стохастично розподіленими дисперсними включеннями за умов складного напруженого стану. Розглянуто детерміністичний критерій руйнування композита типу закону кулонівського тертя зі зчепленням. Отримано функцію розподілу руйнівного навантаження композита, яка є основою для запису статистичних характеристик міцності і надійності матеріалів. Розраховані і побудовані діаграми залежності від прикладеного навантаження ймовірності руйнування плоского композитного зразка з різними структурною неоднорідністю матеріалу та кількістю включень

Ключові слова: *композитний матеріал, імовірність руйнування, функція розподілу, руйнівне навантаження, дисперсні включення*

1. Introduction

Construction of failure mathematical models of structural elements from composite materials is the subject of intensive and versatile research. The composite reliability depends on various probabilistic factors. For the descriptive parameters of a composite material structure, randomness is characteristic, certain laws of probabilistic distribution. Therefore, the problem of the composite materials strength and reliability calculating using stochastic modeling is an actual problem. A joint consideration of defect and structure randomness allows for a more accurate assessment of their strength and reliability. For modern statistical approaches to the failure problem of composite materials is characterized by the tendency to fuller use the results of deterministic theories of the defects influence on the strength and conditions of defects such as cracks propagation.

2. Literature review

The complex application of composite materials failure deterministic mechanics and probabilistic statistical methods is considered in a number of authors' works. In particular, the paper [1] presents a state-of-the-art review of ultimate strength prediction and reliability analysis for composite material structures with emphasis on laminated composite structures. In [2] a numerical simulation and analytical probabilistic methods for the reliability evaluation of composite structures are considered. The author [3] proposed a mechanical multi-scale model describing relationship between the crack-opening and composite bridging stress in brittle matrix composites with heterogeneous reinforcement. The work [4] concerned with a statistical distributions of the critical fracture toughness values with due consideration given to the scale size effect. Experimental investigations of the composite glass fiber materials tensile strength and the statistical analysis of the results obtained on the basis of the two-parameter Weibull distribution have been carried out in [5]. According to the experimental results, a probability analysis was conducted on the degradation of tensile strength [6].

3. The aim and objectives of research

The aim of research is calculation and analysis of the reliability (probability of failure) under certain loading conditions of composite materials specimens with different numbers of randomly distributed elliptic dispersive inclusions that do not interact with each other.

To achieve this aim, the following objectives need to be solved:

– determine the failure criterion of a composite material with elliptic dispersive inclusions in the conditions of a complex stress state;

– choose the distribution laws of statistically independent geometric parameters of inclusions;

– obtain the failure loading distribution function for a composite material element with one inclusion;

– calculate and construct the diagrams of dependence on the applied loading of failure probability of a flat composite sample with different structural material heterogeneity and different number of inclusions.

4. Probability of failure calculation method for the composite material with randomly distributed dispersive inclusions

Let's consider a flat macro-element of a composite material that is an elastic homogeneous matrix in which are evenly distributed N elliptic inclusions from another elastic material that do not interact with each other. The macro-element is under the conditions of uniformly distributed forces P and $Q=\eta P$ which can be considered as the main stresses for a flat stress state (Fig. 1). The elastic properties of the matrix and inclusions are given (the properties of all inclusions are the same), that is, the material is two-component. Let's believe that the inclusions are soft and have the form of flattened ellipses ($\delta = 2b/(a+b) \approx 2b/a,\ b \ll a$, a and b are the semi axes of ellipse). The influence of such inclusions on the strength is quite significant, because high local stress concentrations arise near their edges [7]. The geometric parameters of the inclusions (δ and orientation angle α) are statistically independent random variables.

The stress state in the inclusion is homogeneous, therefore, we assume that the failure of composite material starts in the inclusion (in it a crack which has a length $2a$ is formed). Cracks may occur across the inclusion, but the most dangerous are longitudinal. The composite strength is determined by the strength of its weakest element (the hypothesis of the weakest link).

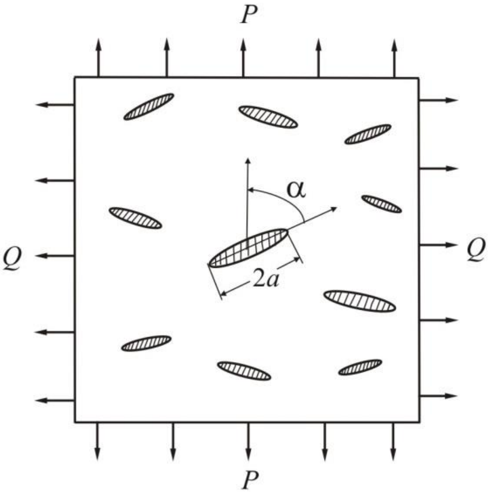

Fig. 1. Macro-element of composite material with randomly distributed elliptical inclusions

Let's consider the composite material to be macro-isotropic (all possible orientations of inclusions are equally probable). Then the distribution of the random variable α will be uniform and has a probability density distribution $f(\alpha)=2/\pi$ $(-\pi/2 \le \alpha \le \pi/2)$. The probability density distribution of a random variable δ we write the β-law [8]

$$f(\delta)=\frac{r+1}{\delta_1-\delta_0}\left(1-\frac{\delta-\delta_0}{\delta_1-\delta_0}\right)^r \quad (\delta_0 \le \delta \le \delta_1), \quad (1)$$

where δ_0 is a minimum, δ_1 is a maximum value of the parameter δ, $r \ge 0$ is a parameter of the material structural heterogeneity. With increasing r let's observe an increase in the probability of meeting small values of a random variable δ (decrease in the size of inclusions).

According to (1) the integral probability distribution function of random variable δ

$$F(\delta)=1-\left(1-\frac{\delta-\delta_0}{\delta_1-\delta_0}\right)^{r+1} \quad (\delta_0 \le \delta \le \delta_1). \quad (2)$$

The mean value of the random variable δ is determined as follows:

$$\langle\delta\rangle=\frac{\delta_0+\delta_1}{r+2}. \quad (3)$$

In accordance with the above assumption of the inclusions form, let's consider that $0 \le \delta \le 0,5$. Then from the expression (3) obtain

$$\langle\delta\rangle=\frac{1}{2(r+2)}. \quad (4)$$

The composite material heterogeneity of the structure is characterized by the joint probability distribution density of independent random variables α and δ

$$f(\alpha,\delta)=f(\alpha)f(\delta)=\frac{4(r+1)}{\pi}(1-2\,\delta)^r. \quad (5)$$

Graphs of the joint probability distribution density (5) for some parameter values r are shown in Fig. 2.

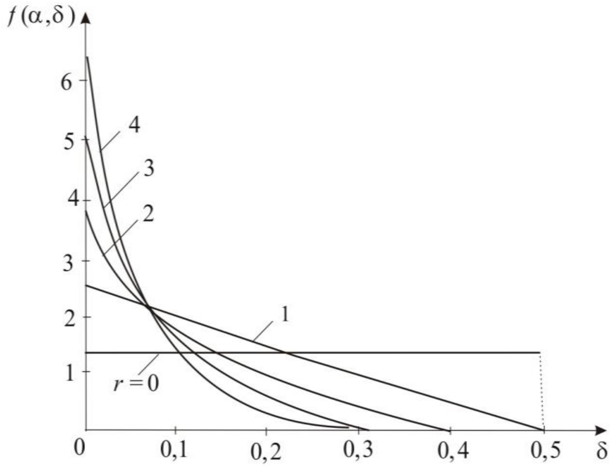

Fig. 2. Joint probability distribution density $f(\alpha,\delta)$ for different parameter values r

At $r=0$ distribution (5) becomes a uniform one. At $r=1$ have a linearly decreasing distribution. It is seen from the constructed curves that with the increasing of the parameter r, the probability of a meeting of random variables δ, which are close to zero, increases.

Let's denote by indices 1 and 2 the values that are related respectively to inclusion and matrix.

Let's accept as a deterministic failure criterion for the inclusion a condition of a Coulomb friction law with clutch type [9]

$$\tau_{xy}^1 \le K^1-\sigma_y^1\, tg\rho^1, \quad (6)$$

where σ_y^1, τ_{xy}^1 are the stress in inclusion, K^1 is a clutch coefficient, $tg\rho^1$ is a coefficient of material inclusion internal friction.

The stress in inclusion, which inclines at an angle α to the main axis (Fig. 1), is determined by the formulas [8]

$$\sigma_y^1=\frac{0,5P(\eta+1+(1-\eta)\cos 2\alpha)G_1(1+\varkappa_1)(1+\varkappa_2)}{G_1(1+\varkappa_1)(1+\varkappa_2)+2\delta G_2(\varkappa_1-1)},$$

$$\tau_{xy}^1=\frac{0,5P(1-\eta)\sin 2\alpha\; G_1(1+\varkappa_2)}{G_1(1+\varkappa_2)+2\delta G_2}, \quad (7)$$

where G_1, G_2 are the shear modules ($G_1/G_2<1$), $\varkappa_1$, $\varkappa_2$ are the elastic constants, which are expressed in terms of Poisson's coefficient ν ($\varkappa=\frac{3-\nu}{1+\nu}$ for a plane stress state, $\varkappa=3-4\nu$ for a plane deformation).

In accordance with the failure criterion (6) – (7), after conducting elementary transformations and neglecting the terms of order G_1^2/G_2^2, let's obtain the following expression for loading calculating, at which a crack with the length $2a$ is formed:

$$P=\frac{K^1L(D+\delta M)}{B(\eta+1+(1-\eta)\cos 2\alpha)+C(1-\eta)\sin 2\alpha}, \quad (8)$$

where

$$B=(1+\varkappa_1)tg\rho^1,$$

$$C=\varkappa_1-1,$$

$$D=\varkappa_1(1+\varkappa_2),$$

$$L=\frac{4}{1+\varkappa_2}, \quad M=\frac{CG_2}{G_1}.$$

The value of the parameter δ that corresponds to the given failure loading P ($P_{\min}\le P\le P_{\max}$) is determined as follows:

$$\delta=\frac{P(B(\eta+1+(1-\eta)\cos 2\alpha)+C(1-\eta)\sin 2\alpha)-K^1LD}{K^1LM}. \quad (9)$$

The dependence of the failure loading P on the angle of inclusion orientation α and the correlation $\eta=Q/P$ is shown in Fig. 3.

Its minimum value

$$P_{\min}=\frac{K^1LD}{B(\eta+1)+(1-\eta)\sqrt{B^2+C^2}} \quad (10)$$

is reached at the orientation angle

$$\alpha_*=0,5\,arctg\frac{C}{B}$$

and parameter $\delta_0=0$.

Maximum value for biaxial tension

$$P_{\max}=\frac{K^1L(D+0,5M)}{2B\eta} \quad (11)$$

is realized at the parameter $\delta_1=0,5$ and angle $\alpha=\pi/2$.

The maximum value $P\to\infty$ for tension-compression is reached at $\delta_1=0,5$ and angle

$$\alpha_{**}=\frac{\pi}{2}-0,5\arcsin\times$$
$$\times\frac{B\left(-C(\eta+1)\right)+\sqrt{C^2(\eta+1)^2-4(B^2+C^2)\eta}}{(B^2+C^2)(1-\eta)}. \quad (12)$$

For $\alpha=0$ loading is equal

$$P_1=\frac{K^1L(D+\delta M)}{2B}. \quad (13)$$

For $\alpha=\pi/4$ we obtain a loading

$$P_2=\frac{K^1L(D+\delta M)}{B(\eta+1)+C(1-\eta)}. \quad (14)$$

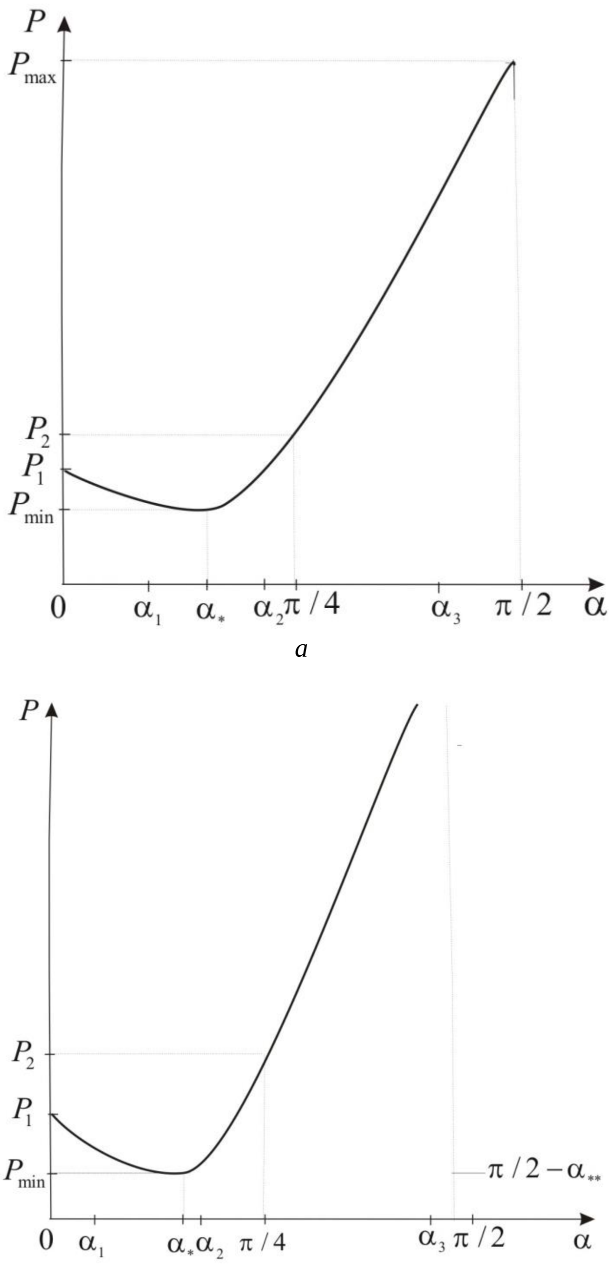

Fig. 3. Range of failure loading change: *a* – biaxial tension; *b* – tension-compression

The following orientation angles corresponds to the given failure loading P $(P_{\min} \le P \le P_{\max})$ and ratio $\eta = Q/P$:

$$\alpha_1 = 0,5\arcsin\frac{C\left(T - B(\eta+1)\right) - \sqrt{C^2\left(T - B(\eta+1)\right)^2 - (B^2+C^2)\left(T^2 + 4B^2\eta - 2TB(\eta+1)\right)}}{(B^2+C^2)(1-\eta)},$$

where

$$T = \frac{K^1 L(D+\delta M)}{P},$$

$$(0 \le \alpha_1 \le \alpha_*,\ P_{\min} \le P \le P_1); \qquad (15)$$

$$\alpha_2 = 0,5\arcsin\frac{C\left(T - B(\eta+1)\right) + \sqrt{C^2\left(T - B(\eta+1)\right)^2 - (B^2+C^2)\left(T^2 + 4B^2\eta - 2TB(\eta+1)\right)}}{(B^2+C^2)(1-\eta)},$$

$$(\alpha_* \le \alpha_2 \le \pi/4,\ P_{min} \le P \le P_2); \qquad (16)$$

$$\alpha_3 = \frac{\pi}{2} - \alpha_1$$

$$\left(\frac{\pi}{4} \le \alpha_3 \le \frac{\pi}{2},\ P_2 \le P \le P_{\max}\ \text{(biaxial tension)},\right.$$

$$\left.\begin{array}{l}\dfrac{\pi}{4} \le \alpha_3 \le \dfrac{\pi}{2} - \alpha_{**}, \\ P_2 \le P < \infty\ \text{(tension-compression)}.\end{array}\right) \qquad (17)$$

The failure loading for a composite element with one inclusion is a random variable having an integral distribution function $F_1(P,\eta)$, which can be written similarly to the results of [8] as follows:

$$F_1(P,\eta) = \frac{2}{\pi}\int_{S_\alpha}\left(1 - F(\delta(P,\eta,\alpha))\right)d\alpha, \qquad (18)$$

where the parameter $\delta(P,\eta,\alpha)$ is determined from (9). The integration area S_α depends on the ratio η of the applied loading.

Taking into account the expressions of the integral distribution functions (2), (18) and the range of variation of the random variable δ, we obtain

$$F_1(P,\eta) = \frac{2}{\pi}\int_{S_\alpha}\left(1 - 2\delta(P,\eta,\alpha)\right)^{r+1}d\alpha$$

$$(0 \le \delta \le 0,5). \qquad (19)$$

Let's introduce the notation

$$F(P,\eta,\beta_1,\beta_2) = \frac{2}{\pi}\int_{\beta_1}^{\beta_2}\left(1 - 2\delta(P,\eta,\alpha)\right)^{r+1}d\alpha. \qquad (20)$$

Taking into account the notation (20) and the relationship (10)–(17), the integral distribution function (19) for a composite element with one inclusion is written as follows:

– for biaxial tension $(0 \le \eta \le 1)$

$$F_1(P,\eta) = \begin{cases} F(P,\eta,\alpha_1,\alpha_2),\ P_{\min} \le P \le P_1\ (\eta \ne 0); \\ F(P,\eta,0,\alpha_1) + F(P,\eta,\pi/4,\alpha_3),\ P_1 \le P \le P_2\ (\eta \ne 1); \\ F(P,\eta,\alpha_3,\pi/2),\ P_2 \le P < P_{\max}\ (\eta \ne 0); \end{cases} \qquad (21)$$

– for tension-compression $(-\infty \le \eta < 0)$

$$F_1(P,\eta) = \begin{cases} F(P,\eta,\alpha_1,\alpha_2),\ P_{\min} \le P \le P_1; \\ F(P,\eta,0,\alpha_1) + F(P,\eta,\pi/4,\alpha_3),\ P_1 \le P \le P_2\ (\eta \ne -1); \\ F(P,\eta,\alpha_3,\pi/2 - \alpha_{**}),\ P_2 \le P < \infty\,(\eta \ne -1). \end{cases} \qquad (22)$$

Let's note separately the case of biaxial symmetric tension $(\eta = 1,\ P = Q > 0)$. Considering normalization condition for density $f(\alpha)$, we obtain from formula (21)

$$F(P,1) = 1 - \left(1 - 2\delta(P,\eta,\alpha)\right)^{r+1},$$

$$P_{\min} \le P \le P_{\max}. \qquad (23)$$

The failure probability of a composite material macro-element containing N inclusions is determined [10] as follows:

$$P_f = 1 - \left(1 - F_1(P,\eta)\right)^N,\ P_{\min} \le P \le P_{\max}. \qquad (24)$$

Substituting in formula (24) expressions of failure loading distribution function (21) – (23), let's obtain a ratio for determining the failure probability of the considered composite material with different number of inclusions and different structural heterogeneity for given ratios of the applied loading:

– for biaxial tension $(0 \le \eta \le 1)$

$$P_f = \begin{cases} 1 - \left(1 - F(P,\eta,\alpha_1,\alpha_2)\right)^N,\ P_{\min} \le P \le P_1\,(\eta \ne 0); \\ 1 - \left(1 - F(P,\eta,0,\alpha_1) - F(P,\eta,\pi/4,\alpha_3)\right)^N,\ P_1 \le P \le P_2\,(\eta \ne 1); \\ 1 - \left(1 - F(P,\eta,\alpha_3,\pi/2)\right)^N,\ P_2 \le P < P_{\max}\,(\eta \ne 0); \end{cases} \qquad (25)$$

– for tension-compression $(-\infty \le \eta < 0)$

$$P_f = \begin{cases} 1-\left(1-F(P,\eta,\alpha_1,\alpha_2)\right)^N, \; P_{\min} \le P \le P_1; \\ 1-\left(1-F(P,\eta,0,\alpha_1)-F(P,\eta,\pi/4,\alpha_3)\right)^N, \; P_1 \le P \le P_2 \; (\eta \ne -1); \\ 1-\left(1-F(P,\eta,\alpha_3,\pi/2-\alpha_{**})\right)^N, \; P_2 \le P < \infty (\eta \ne -1). \end{cases} \quad (26)$$

In particular for biaxial symmetric tension

$$P_f = 1-\left(\left(1-2\delta(P,\eta,\alpha)\right)^{r+1}\right)^N,$$

$$P_{\min} \le P \le P_{\max}. \quad (27)$$

In the expressions for the loading let's introduce instead of the parameter δ its mean value $\langle\delta\rangle$ (formula (4)) and carry out the replacement of the variable $p = P/K^1$ (introduce a dimensionless loading).

Let's write the expressions to determine the probability of failure for single cases of the applied loading ratio.

Probability of failure for biaxial symmetric tension $(\eta = 1)$

$$P_f = 1-\left(\left(1-2\frac{2pB-LD}{LM}\right)^{r+1}\right)^N,$$

$$p_{\min} \le p < p_{\max}. \quad (28)$$

Probability of failure for uniaxial tension $(\eta = 0)$

$$P_f = 1- \\ -\left(1-\frac{2}{\pi}\int_0^{\alpha_1}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}d\alpha - \right. \\ \left. -\frac{2}{\pi}\int_{\pi/4}^{\alpha_3}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}d\alpha\right)^N,$$

$$p_{\min} \le p \le p_2. \quad (29)$$

Probability of failure for tensile-compression (net shear) $(\eta = -1)$

$$P_f = 1- \\ -\left(1-\frac{2}{\pi}\int_{\alpha_1}^{\alpha_2}\left(1-2\frac{2p(B\cos 2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}d\alpha\right)^N,$$

$$p_{\min} \le p < p_1 \quad (30)$$

Let's consider approbation of the obtained analytical results. In accordance with [11] and physical considerations, let's take the following values of material constants (disperse composite of gray cast iron type):

G_1/G_2=0,081,
G_2=4,4·10^{-4}MPa,
$\nu_1 = \nu_2 = 0,25$,
$tg\rho^1 = 0,6$,
$\varkappa_1 = \varkappa_2 = 2,2$ (plane stress state).

Conduct numerical research of the dispersive composite material probability of failure by formulas (28)–(30) and analyze its diagrams for the different material structural heterogeneity (parameter r) and the different number of inclusions (parameter N).

Fig. 4 shows a disperse composite probability of failure diagrams with different number N of inclusions at $r = 1$ for the following types of loading: biaxial symmetric tension $(\eta = 1)$, uniaxial tension $(\eta = 0)$, tensile-compression $(\eta = -1)$.

In Fig. 5 the dependence of the dispersive composite probability of failure on the number of inclusions (the dimensions of the composite) and the material structural heterogeneity for given loading ($p = 2,4$) are investigated.

The influence of loading types and material structural heterogeneity on the dispersive composite probability of failure in its fixed dimensions ($N = 50$) are analyzed in Fig. 6.

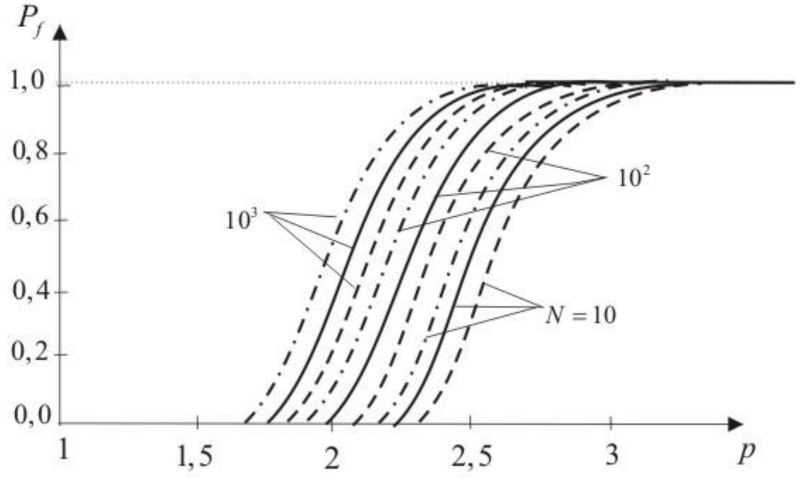

Fig. 4. Probability of failure for different types of loading (solid for $\eta=0$, dashed for $\eta=1$, dotted dashed for $\eta=-1$)

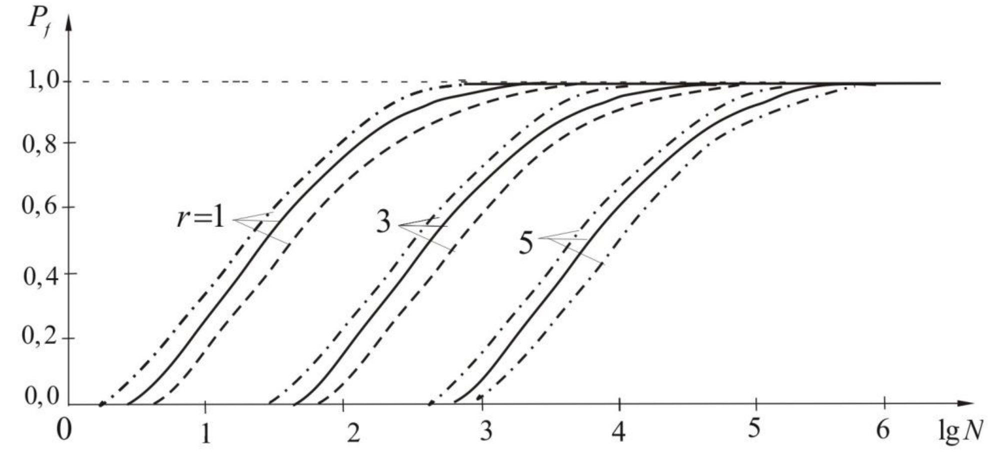

Fig. 5. Probability of failure for given loading (solid for $\eta=0$, dashed for $\eta=1$, dotted dashed for $\eta=-1$)

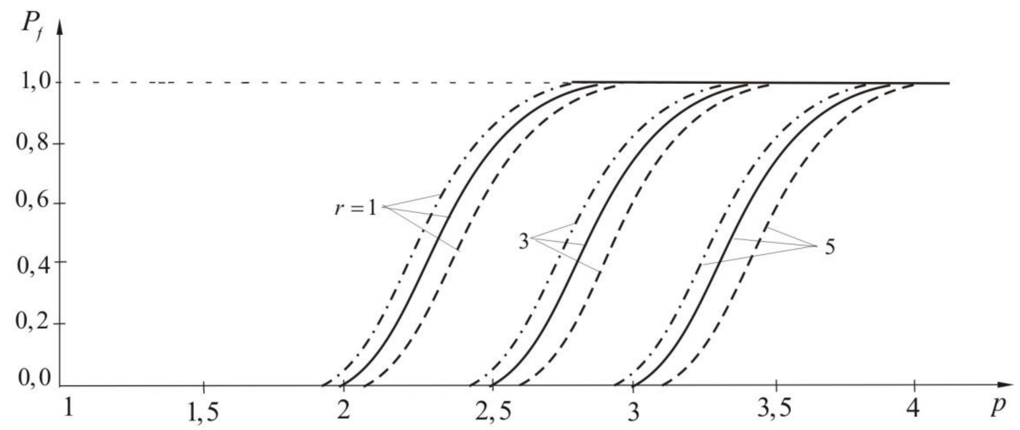

Fig. 6. Probability of failure for various material structural heterogeneity (solid for $\eta=0$, dashed for $\eta=1$, dotted dashed for $\eta=-1$)

5. Research results

Fig. 4 observes the dependence of the probability of failure on the type of stress state (from η). The probability of failure increases with the increase in the number of inclusions N for a fixed loading. At a certain loading range we observe a small probability of failure.

Fig. 5 shows the dependence of the probability of failure on the material structural heterogeneity (parameter r) and the number of inclusions for different types of applied loading. Each structural material heterogeneity and the loading level correspond to the composite dimensions, which increases the probability of failure.

In Fig. 6 the influence of a composite material structural heterogeneity on different types of loading in the case of its fixed sizes (fixed number of inclusions) is analyzed. With an increase of the parameter r at a fixed loading, the probability of failure decreases, which we observe for each type of stress state.

Similar statistical pattern are observed in [12] when calculating the reliability (probability of failure) of orthotropic composite materials with uniformly distributed defects such as cracks that have a prevailing orientation in the direction of reinforcement.

6. Conclusions

1. Written failure criterion of composite material with elliptic dispersive inclusions in a complex stress state allows to investigate the reliability of composite material, taking into account the stochastic nature of its structure.

2. Selected distribution laws of the statistically independent inclusions geometric parameters α and δ make it possible to write the failure loading distribution function $F_1(P,\eta)$ for a composite element with one inclusion.

3. The received failure loading distribution function $F_1(P,\eta)$ for a composite element with one inclusion has all the properties of a random variable integral distribution function and is the basis for obtaining a number of strength statistical characteristics.

4. The constructed diagrams of the probability of failure P_f of the composite sample allow to investigate its dependence on the material structural heterogeneity (parameter r), its dimensions (number of inclusions N), and the type of stress state (parameter η).

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Roman Baitsar, Doctor of Technical Sciences, Professor, Department of Measuring Information Technologies, Lviv Polytechnic National University, S. Bandery str., 12, Lviv, Ukraine, 79013
E-mail: baitsar@ukr.net

Roman Kvit, PhD, Associate Professor, Department of Mathematics, Lviv Polytechnic National University, S. Bandery str., 12, Lviv, Ukraine, 79013
E-mail: kvit_rom@ukr.net

Andriy Malyar, Doctor of Technical Sciences, Professor, Department of Electromechatronics and Computerized Electromechanical Systems, Lviv Polytechnic National University, S. Bandery str., 12, Lviv, Ukraine, 79013
E-mail: andrii.v.maliar@lpnu.ua