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COMPARISON AND ACCURACY OF THE METHODS FOR DETERMINING THE AREA OF PARCELS

The method of work is to perform a comparative analysis of the methods of determining the area of land plots. Research methods were performed: calculation of the area of land plots on the physical surface of the ellipsoid; calculation of the area of the horizontal projection of the land plot; Calculation of the area of the physical surface of land plots, as well as through practical implementation, the stated theoretical studies were confirmed. The performed calculations indicate that the results of calculating the physical areas of land plots using different coordinate systems are practically the same. However, the difference between the results obtained by dividing the land plot into triangles and the results of increasing the area of physical surfaces by the coordinates of the top reaches a value of approximately 1%. The reason for these discrepancies arises from the fact that all the vertices of the polygon lie on the same area, and when dividing this polygon (land plot) into triangles, the different slope of the land surface in different places of the land plot is taken into account. Calculation of the area of real estate objects must be performed simultaneously with an assessment of the accuracy of the results obtained. Accuracy assessment is characterized by the reliability of area information and also ensures making correct decisions about changes to the original data when re-determining the area. The work proves that during the procedure of buying and selling land plots at their market value, the purpose of land acquisition is: in one case, the horizontal dimensions of land plots are important; in the second - the dimensions of the area of physical surfaces. Thus, in agriculture, the horizontal area determines the number of upright plants, and the size of the area of the physical surface of the land affects the amount of work related to its cultivation, and therefore in land management, cadastre, and real estate evaluation, it is appropriate to use the area of the horizontal projection and the physical surface of the land plot.

***Keywords**: area of the land plot; physical surface of the ellipsoid; horizontal projection of the land plot; real estate object; cadastre; land management; real estate evaluation.*
Fig.: 3. Table: 8. References: 22.

Introduction. One of the main parameters entered into the state land cadastre of real estate is the area of parcels. The value of this attribute is decisive in the monetary evaluation of objects, thus they are used to solve fiscal problems, record land resources and real estate objects and their number, distribution among owners and other participants in market relations. They also serve as the basis for analytical processing in order to prepare the necessary data for management decisions. It is the implementation of the procedure of sale and purchase of real estate that requires accurate area sizes, as it actually determines the market value, and the deviation of these values is likely to lead to a false price and even escalate into litigation. The area value of parcels depends on their location on the physical surface, i.e the earth's surface taking into account its terrain; on the horizontal projection of the parcel; projection on the surface of the earth ellipsoid and the geodetic parcel (the image of the parcel in the adopted projection, usually in the transverse Mercator map projection [1]).

Therefore, depending on tasks set, one of the above options is used. Nowadays, as documents on the rights to parcels include the area of the geodetic site obtained when measuring by means of cartographic-topographic materials, the subsequent calculations take it as a basis. Moreover, in choosing the local coordinate system, the average terrain level is considered as the relativity surface, due to which the values of the geodetic and horizontal areas are practically the same. The total value of monetary proceeds from the parcel sale is determined in this case as the sum of the areas of these horizontal areas with a possible coefficient of their value.

Analysis of literature. Yu. H. Batrakov, V. D. Baranovskyy, V. N. Hanshyn, V. I. Hladkykh, Yu. P. Hubar, Yu. A. Karpinskyy, A. A. Lyashchenko, A. V. Maslov, M. M. Fys and many others dedicated their works to the development of methods for determining and estimating the accuracy of land areas.

Requirements for the accuracy of determining the area of parcels are related to the cost of natural resources. Some standards for accuracy are given in [2; 3]. In particular, the accuracy of calculating the area of parcels in the process of drafting a project for their allocation for use or for the issuance of a deed of ownership is characterized by relative error $m_P/P = 1/1000$. For large areas, such as agricultural land or forestry, this error can amount to 1/500. Therefore, depending on the dimensions and configuration of the parcel and on the requirements for accuracy, a particular way to determine its area will be used.

In 1995, the FIG enacted the statement, where the cadastre is defined as a site-based land information system that contains up-to-date information on rights and restrictions on land rights. This system includes a geometric description of land parcels and other information about these parcels, including information about their value. It can be used for fiscal purposes (assessment and taxation), legal purposes (transfer of rights under agreements), spatial planning purposes and to promote sustainable development and environmental protection. This resolution, in particular, recommends the use of a systematic approach to land registration as the most effective one compared to the sporadic approach. Thus, FIG has set a standard that many countries worldwide have begun to follow, mainly due to the development of technology. The cadastre concept developed by FIG has proven its potential in different parts of the world [4].

Cooperation between FIG and UN organizations increased markedly in the second half of the 1990s. This is evidenced by the adoption of the Bogor Declaration by international experts in the field of cadastre in 1996 in Bogor (Indonesia). For the first time, the Bogor Declaration officially recognized cadastral systems to be a critical element of infrastructure that supports solutions to sustainable development and natural resource management.

At the Bathurst Conference (Bathurst, Australia, 1999), the key issues related to strengthening land policy, institutions, and cadastral infrastructure were considered. The Declaration on Land Administration for Sustainable Development adopted at this conference created a strong link between efficient land administration and sustainable development and offered a number of recommendations on land use development and infrastructure [4; 5].

Since the cadastral systems of different countries differed functionally in content, architecture, technical performance, and economic efficiency, there was a need to develop a concept for further development of these systems in terms of versatility and technological improvement.

The development of land relations, especially land commodification, has increased the role of a cadastre in the theory of land management. In 1980, the US National Research Council published a study entitled "The Need for a Multi-Purpose Cadastre," which described a cadastral system that would integrate cartographic and land management functions based on the use of a geodetic basis. The need to create such a system marked the beginning of a new era in the development of land administration. Now all over the world, the main focus is on the technology of creating multipurpose inventories rather than justifying the need to create them. A multipurpose cadastre is a register containing many attributes of land [6].

Modern theory of land administration defines the cadastre (cadastral systems) as a central tool of spatial infrastructure of the state and emphasizes its central role in the implementation of the paradigm of land management. These systems are multipurpose, as they include land identification, registration of rights and restrictions on land rights, support in real estate valuation and taxation, and administration of current and future land use [4; 7; 8].

Multipurpose cadastral systems support four basic functions: land tenure, land value, land use, and land development. They can also be considered as land administration systems or land-information systems (LIS). LIS is a general concept for all land-related information databases that have one thing in common: the data of these systems relate to a fixed point on, in, or below the earth's surface. LIS is a concept that seeks to integrate into a single system all types of land-related data (cadastral, infrastructural, environmental, socio-economic). Within the framework of such LIS, which requires a lot of work on coordination, standardization, organization, etc., the land cadastre plays the role of a link to which many other systems add or connect their data [4; 5; 9; 10].

The uniqueness of the cadastral potential is that 80% of the information has a spatial component and, therefore, can be associated with the land by means of a unique identifier. Thus, the cadastral layer, consisting of information on land and their affiliation, becomes the core of SDI, which includes layers of data on public infrastructure, hydrology, vegetation, topography, remote sensing data, and dozens of other data [4; 11].

International experience shows that the best option for the transition of land relations to the "land information system" stage is to create a land information system based on a cadastre. In turn, when creating a cadastral system following the defined FIG concept of cadastre, we should focus on addressing several issues. They are the creation of fiscal or legal cadastre; integrated or separate system; creation of a register of rights or a register of documents; choose the concept of common or fixed boundaries; centralized or decentralized system; self-sustaining or budget system; systematic or sporadic approaches to legalizing land rights.

We should note that a cadastre tends to integrate with the real estate registration system and transform into the cadastral accounting system and, therefore, both the cadastral accounting system and created on its basis LIS must comply with the principles of state registration of rights such as mandatory registration; consent; identification of the subject and object of law; publicity; guarantees of rights. The structural elements of LIS must also comply with the principle of legal independence and be a kind of legal register, i.e., a database (information layer) that an evidential character. The court considers the information of such databases (information layers) as evidence; it cannot be annulled other than in the manner prescribed by law [12].

Global drivers will influence the characteristics of future cadastral systems, which include instrumental accuracy, object-oriented design, 3D/4D visualization, real-time information, global connections, and organic characteristics.

Research objective. Considering the area of a parcel, the following options are distinguished: the area of the land surface based on the topography, the area of the horizontal projection of the parcel, the area of the projection of the parcel on the surface of the Earth ellipsoid and the area of the parcel image on the map projection plane (the transverse Mercator map projection). The choice of the option depends on the task. Working with maps and plans, one receives the area of the parcel image in the map projection, which is called the geodetic area. If the axial meridian passes by the parcel and the average level of the territory is considered the surface of relativity, the values of the geodetic area and the horizontal projection of the parcel almost do not differ. In other cases, it is important to know the area of the entire physical surface of the parcel. The calculation of real estate areas should be done in parallel with the evaluation of the accuracy of the results obtained. The estimated accuracy is characterized by the reliability of the area information and also serves to make correct decisions about changes in the original data when the areas are redetermined.

The total area of large territories can be determined as a simple sum of horizontal areas. However, it is more practical to determine them using surveillance aircraft (airplanes, drones, drones, etc.). The areas obtained by both methods cannot be identical, because they describe the parcels differently. Even considering topography as the average level of the terrain only partially takes into account its features. In this regard, there arises a need to study the error in determining the areas by different methods [13], since the variation in their values contributes to estimating the areas differently and thus provides a different income, which, in turn, creates a conflict of interest that may give rise to litigation.

The purpose of the work is to perform a comparative analysis of the methods of determining the area of land plots in order to choose the most optimal method for targeted practical use in various branches of the economy.

Method and/or Theory

Let us outline the methodology for determining the areas of the above options.

I. Calculation of parcel areas on the physical surface of the ellipsoid

The main methods of determining the area P_E on the surface of the ellipsoid include calculation using either a known horizontal projection or a known value of the geodetic area. The horizontal projection of the area P_h is calculated through the results of direct geodetic measurements - distances, angles, and coordinate increments reduced to the horizon plane. Let us investigate the transition from the horizontal projection of the area to its projection on the ellipsoid surface. The transition from horizontal distances of the lines *d* to their projections *s* on the surface of the ellipsoid is carried out using the following ratio:

$$\frac{d}{s} = \frac{R + H_m}{R}, \tag{1}$$

where H_m is the average line height above the ellipsoid; R is the average radius of curvature of the Earth ellipsoid.

Ratio (1) can be expressed as follows:

$$d = s + \frac{H_m}{R} \cdot s. \tag{2}$$

The Left and right parts of equation (2) are squared, with irrelevant second-order values discarded, which follows:

$$d^2 \approx s^2 + 2 \cdot \frac{H_m}{R} \cdot s^2. \tag{3}$$

Since the areas are related as squares of the line lengths, the following can be inferred:

$$P_h \approx \left(1 + 2 \cdot \frac{H_m}{R}\right) \cdot P_E. \tag{4}$$

Consequently, with the horizontal projection of the area being known, the transition to the area on the ellipsoid can be performed by the formula:

$$P_Y = P_A - 2 \cdot \frac{H_m}{R} \cdot P_Y \approx P_A - 2 \cdot \frac{H_m}{R} \cdot P_A. \tag{5}$$

Thus,

$$P_E = P_h + \delta P_H, \tag{6}$$

where

$$\delta P_H = -2 \cdot \frac{H_m}{R} \cdot P_A \tag{7}$$

is elevation correction of the parcel area to pass from the area of the horizontal projection to the area on the ellipsoid.

It is essential to examine the conditions that allow the elevation correction δP_H to be neglected. Table 1 shows the calculation of the relative values of the elevation correction $\delta P_H / P$ under different values of the height H_m and $R = 6371\,\text{km}$.

Table 1 – Calculation of relative values of the elevation correction $\delta P_H / P$

H_m, m	100	300	750	1000	2000	4000	6000	8000
$\delta P_H / P$	1/31855	1/10618	1/4247	1/3186	1/1593	1/796	1/531	1/398

It is known that the correction δP_H can be neglected provided that it is 3-5 times less than the error of the area. As can be inferred from Table 1, with the relative error of the area 1/1000, the correction can be neglected for the height values up to 750m.

With the value of the area P in the transverse Mercator map projection, it is appropriate to introduce the correction [14; 15] to pass to the area on the ellipsoid surface:

$$\delta P_L = -\left(\frac{y_m}{R}\right)^2 \cdot P, \quad (8)$$

which follows

$$P_E = P - \left(\frac{y_m}{R}\right)^2 \cdot P, \quad (9)$$

where y_m is the average distance of the parcel from the axial meridian.

The conditions that require the elevation correction δP_L have been examined and the results are presented in Table 2.

With $R = 6371\text{km}$, the relative distortion of the area $\delta P_L / P$ in the transverse Mercator map projection has been calculated for different values y_m.

Table 2 – Calculation of distortion of relative area in the transverse Mercator map projection

y_m, km	25	50	100	150	200	250
$\delta P_L / P$	1/64943	1/16236	1/4059	1/1804	1/1015	1/649

According to [2], they should not exceed 1/3-1/5 of the error in the area for the projection distortions to be neglected.

Table 2 suggests that when the errors in the area are within 1/500-1/1000, the correction for the transition from the geodetic area to the area on the surface of the ellipsoid can be ignored at a distance of 100-150 km from the parcel to the axial meridian. Consequently, the ordinates y_m generally do not exceed 30 km in large cities where local coordinate systems are used, and therefore the above correction can be neglected.

There is a much more accurate formula to determine the correction for the transition from the area in the transverse Mercator map projection to the area on the surface of the ellipsoid, i.e.:

$$\delta P_L = -\left(1 - q_m^2\right) \cdot P, \quad (10)$$

where $q_m = \dfrac{1}{ch\left(\dfrac{y_m}{R_m}\right)}$; $R_m = \dfrac{b}{\left(1 - e^2 \cdot \sin^2 \dfrac{x_m}{R_E}\right)}$.

R_E is the equivalent radius (R_E=6367558,497m for the Krasovsky ellipsoid); x_m is the average abscissa of the parcel.

The area on the surface of the ellipsoid can also be calculated using the coordinates of its vertices. For example, the area of a steroidal trapezium limited by parallels with the latitudes B_1, B_2 and meridians with the longitudes L_1, L_2 (Fig. 1) can be calculated using the formula [13; 16; 17].

$$P_E = b^2 \cdot (L_2 - L_1) \cdot \int_{B_1}^{B_2} \frac{\cos B dB}{\left(1 - e^2 \sin^2 B\right)^2}, \tag{11}$$

where b is the semi-minor axis and e is the eccentricity of the earth's ellipsoid.

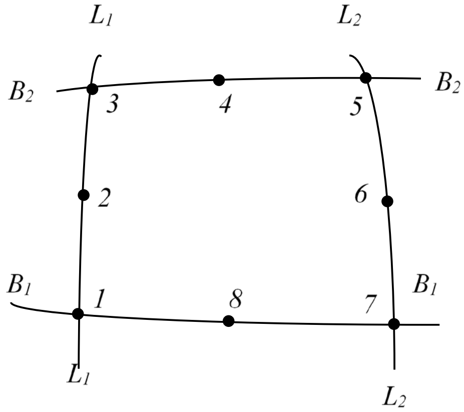

Fig. 1. Steroidal trapezoid

To perform the calculations, the integrand function (11) needs to be expanded in a series for the purpose of term-by-term integration, which follows [18; 20].

$$P_E = b^2 (L_2 - L_1) \cdot \begin{bmatrix} \sin B_2 - \sin B_1 + \frac{2}{3} e^2 \left(\sin^3 B_2 - \sin^3 B_1\right) + \frac{3}{5} e^4 \left(\sin^5 B_2 - \sin^5 B_1\right) + \\ + \frac{4}{7} e^2 \left(\sin^7 B_2 - \sin^7 B_1\right) + \ldots \end{bmatrix}. \tag{12}$$

There is also an approximate formula that allows calculating the area with an error of no more than $2 \cdot 10^{-5} \cdot P_E$ [18; 19].

$$P_E = c(L_2 - L_1) \cdot \left[\arcsin(k \cdot \sin B_2) - \arcsin(k \cdot \sin B_1)\right], \tag{13}$$

where $c = 74456{,}835$; $k = 0{,}163133$.

Formula (13) includes longitude expressed in degrees and area expressed in square kilometers.

The coordinates of vertices of an arbitrary polygonal parcel can be used to calculate the projection area of the parcel on the ellipsoid. Let us present formulas for calculating the area of small land parcels using coordinates in the following coordinate systems:

- planimetric rectangular coordinates x, y and heights z;
- spatial rectangular coordinates X, Y, Z;
- **geodetic** coordinates B, L, H.

For further calculations, let $z = H$.

Consequently, the formula for calculating the area on the ellipsoid using planimetric rectangular coordinates is the following:

$$P_E = \frac{1}{2} q_m^2 \cdot \left(\sum_{i=1}^{n} \begin{vmatrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{vmatrix} \right). \tag{14}$$

The formula for calculating the area on the ellipsoid by spatial geocentric (quasi-geocentric) rectangular coordinates X, Y, Z is the following:

$$P_E = \frac{1}{2} \sqrt{ \left(\sum_{i=1}^{n} \begin{vmatrix} X_i & Y_i \\ X_{i+1} & Y_{i+1} \end{vmatrix} \right)^2 + \left(\sum_{i=1}^{n} \begin{vmatrix} X_i & Z_i \\ X_{i+1} & Z_{i+1} \end{vmatrix} \right)^2 + \left(\sum_{i=1}^{n} \begin{vmatrix} Y_i & Z_i \\ Y_{i+1} & Z_{i+1} \end{vmatrix} \right)^2 }. \tag{15}$$

The formula to calculate the area on the ellipsoid by geodetic coordinates B, L can be expressed as follows:

$$P_E = \frac{1}{2} g_m k_m \left(\sum_{i=1}^{n} \begin{vmatrix} B_i & L_i \\ B_{i+1} & L_{i+1} \end{vmatrix} \right). \tag{16}$$

where $g_m = M_m / \rho$; $k_m = N_m \cdot \cos B_m / \rho$;

$$M_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \sin^2 B_m)^{3/2}};\ N_m = \frac{a}{\sqrt{1 - e^2 \cdot \sin^2 B_m}},$$

B_m is the average latitude of the parcel; a is the semi-major axis of the terrestrial ellipsoid.

By formulas (14) - (16), the areas on the ellipsoid surface are calculated, and therefore the height z of planimetric rectangular coordinates X, Y, Z must be equal to zero, and the spatial coordinates of each point must meet the requirement $H = 0$. To simplify the calculations, it is reasonable to discard the common first digits in the vertex coordinates or subtract the coordinates of the first vertex from them and further operate on the residuals. However, when calculating the coefficients q, g, k, it is necessary to use the value of coordinates to the full.

II. Calculation of the area of horizontal projection of the parcel

Most often, the horizontal projection area of a parcel (horizontal area) is calculated according to the results of direct geodetic measurements reduced to the horizon.

To calculate the horizontal area, with the given geodetic area of a parcel calculated from the coordinates of its vertices in the transverse Mercator map projection or measurements on the plan, it is necessary to switch from the geodetic area to the area on the ellipsoid surface, and then to the horizontal area. The transition from the geodetic area in the transverse Mercator map projection to the area on the ellipsoid surface is performed by adding the correction obtained by formula (8) to the geodetic area. To transfer to the horizontal area, a correction must be added to the result obtained according to (7):

$$\delta P_H \approx 2 \frac{H_m}{R} \cdot P_E, \tag{17}$$

where H_m is the average height of the parcel.

Thus, it follows that

$$P_h = P + \delta P_L + \delta P_H. \tag{18}$$

Let us consider how the area horizontal projection is related to the area of the physical surface of the parcel. Figure 2 shows a rectangular parcel of the area P_T with an angle of inclination ν. The lengths of the sides of the parcel are equal to S_1 and S_2.

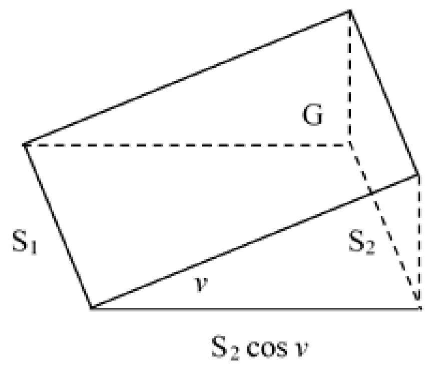

Fig. 2. Sloped parcel and its projection

The area of the tilted surface of the parcel equals $P_T = S_1 \cdot S_2$, with the area of its horizontal projection being expressed as

$$P_h = S_1 \cdot S_2 \cos\nu = P_T \cdot \cos\nu \,. \tag{19}$$

The difference in the areas is equal to:

$$\delta P_\nu = P_T - P_h = P_T - P_T \cdot \cos\nu = P_T \cdot \left(1-\cos\nu\right). \tag{20}$$

Given the equality $1-\cos\nu = 2\sin^2\frac{\nu}{2}$ and $P_T = P$, it follows that:

$$\delta P_\nu = 2P\sin^2\frac{\nu}{2}\,. \tag{21}$$

For minor slope angles, it is possible to adopt that $\sin\nu = \nu$ (ν should be given in radians), thus

$$\delta P_\nu = P \cdot \frac{\nu^2}{2} \tag{22}$$

and

$$\frac{\delta P_\nu}{P} = \frac{\nu^2}{2}\,. \tag{23}$$

Formula (20) follows that

$$P_T = P_h + \delta P_\nu = P_h + P \cdot \frac{\nu^2}{2} \tag{24}$$

and

$$P_h = P_T - \delta P_\nu = P_T - P \cdot \frac{\nu^2}{2}\,.$$

By analyzing formulas (19) - (23), it can be concluded that the horizontal projection area of a parcel is always less than its physical surface area. This difference grows with the increase of the slope angle. The relative distortion of the area $\delta P_\nu / P_T$ depends on the slope angle. This distortion is not affected by the shape of the parcel and its location on plane G.

III. Calculation of the parcel physical surface area

Depending on the purpose of the parcel, there is a need to determine the area P_T of the physical surface of the parcel, i.e. the area of the land surface taking into account the terrain that may differ from the geodetic area by 2-5%, which is important to consider in the calculations and in drawing up land planning documents (e.g., contracts of sale of parcels) [20; 21].

After calculating the geodetic area P by coordinates of polygon vertices, the physical surface area can be found by transforming the system of equations:

$$\left.\begin{aligned} \delta P_H &= -2\frac{H_m}{R}P_A \\ \delta P_L &= -\left(\frac{y_m}{R}\right)^2 P \\ P_\text{T} &= P_\text{h} + \delta P_\nu = P_\text{h} + P\frac{\nu^2}{2} \end{aligned}\right\}. \tag{25}$$

Therefore, the area of the physical surface area can be determined as follows:

$$P_\text{T} = \left[1-\left(\frac{y_m}{R}\right)^2 + \frac{2H_m}{R} + \frac{1}{2}\left(\frac{\nu''_m}{\rho''}\right)^2\right] \cdot P\,, \tag{26}$$

where v_m is the average angle of slope of the land surface on the parcel; H_m is the average height of the parcel spot; y_m is the average parcel ordinate (distance from the axial meridian); R is the Earth radius.

The formula (26) can be used for the area of the whole parcel only if the terrain is calm. If the terrain is complex, it is reasonable to divide the parcel into elementary areas with constant tilt angles, determining for each of them the geodetic area, tilt angle, height, and deviation of the axial meridian. After calculating by formula (26) the physical surface area of each site and summing them up, it is possible to find the total physical surface area of the whole parcel. It is also worth emphasizing that determining the parameters v_m, H_m, y_m is a rather complicated and time-consuming process.

With calm terrain, the area of the physical surface of a parcel can also be calculated using the coordinates of the polygon vertices. Moreover, the coordinates can be specified in any coordinate system - planimetric rectangular coordinates x, y combined with height z (27); spatial rectangular geocentric coordinates X, Y, Z (28); geodetic coordinates B, L, H, with $z = H$.

$$P_T = \frac{1}{2}\sqrt{q_m^4\left(\sum_{i=1}^{n}\begin{vmatrix} x_i & y_i \\ x_{i+1} & y_{i+1}\end{vmatrix}\right)^2 + q_m^2\left(\sum_{i=1}^{n}\begin{vmatrix} x_i & z_i \\ x_{i+1} & z_{i+1}\end{vmatrix}\right)^2 + q_m^2\left(\sum_{i=1}^{n}\begin{vmatrix} y_i & z_i \\ y_{i+1} & z_{i+1}\end{vmatrix}\right)^2} \,. \quad (27)$$

$$P_T = \frac{1}{2}\sqrt{\left(\sum_{i=1}^{n}\begin{vmatrix} X_i & Y_i \\ X_{i+1} & Y_{i+1}\end{vmatrix}\right)^2 + \left(\sum_{i=1}^{n}\begin{vmatrix} X_i & Z_i \\ X_{i+1} & Z_{i+1}\end{vmatrix}\right)^2 + \left(\sum_{i=1}^{n}\begin{vmatrix} Y_i & Z_i \\ Y_{i+1} & Z_{i+1}\end{vmatrix}\right)^2} \,. \quad (28)$$

$$P_T = \frac{1}{2}\sqrt{g_m^2 k_m^2\left(\sum_{i=1}^{n}\begin{vmatrix} B_i & L_i \\ B_{i+1} & L_{i+1}\end{vmatrix}\right)^2 + g_m^2\left(\sum_{i=1}^{n}\begin{vmatrix} B_i & H_i \\ B_{i+1} & H_{i+1}\end{vmatrix}\right)^2 + k_m^2\left(\sum_{i=1}^{n}\begin{vmatrix} L_i & H_i \\ L_{i+1} & H_{i+1}\end{vmatrix}\right)^2} \,. \quad (29)$$

Formulas (27) - (29) include the following notation [18, 21]:

$$q_m = \sqrt{1 + \frac{2z_m}{R_m}} \Bigg/ ch\left(\frac{y_m}{R_e}\right); \quad R_m = b \Bigg/ \left(1 - e^2 \sin^2 \frac{x_m}{R_e}\right); \quad g_m = (M_m + H_m)/\rho \,;$$

$$k_m = (N_n + H_m)\cos B_m/\rho \,; \quad M_m = \frac{\alpha(1-e^2)}{(1 - e^2 \sin^2 B_m)^{3/2}} \,; \quad N_m = \alpha \Big/ \sqrt{1 - e^2 \sin^2 B_m} \,,$$

where x_m, y_m, z_m, H_m, B_m are the average parcel coordinates; a, b, e^2 are the elements of the earth's ellipsoid.

To simplify the calculations by formulas (27)-(29), let us discard the first common digits or subtract from them the coordinates of the first vertex in the vertex coordinates and further deal with the residuals:

$$\begin{aligned} x_i' &= x_i - x_0 \,; & y_i' &= y_i - y_0 \,; & z_i' &= z_i - z_0 \,; \\ X_i' &= X_i - X_0 \,; & Y_i' &= Y_i - Y_0 \,; & Z_i' &= Z_i - Z_0 \,; \\ B_i' &= B_i - B_0 \,; & L_i' &= L_i - L_0 \,; & H_i' &= H_i - H_0 \,, \end{aligned} \quad (30)$$

where the coordinates with index 0 correspond to the coordinates of the first vertex.

In calculating the coefficients q, g, k, it is crucial to use the complete coordinate values.

Basically, the areas of the parcel are determined by coordinate differences using modern satellite geodetic instruments or with the help of electronic total stations. Let us present the formulas for calculating the physical areas of spatial triangles in the three main coordinate systems.

- in the planimetric rectangular coordinate system:

$$P_T = \frac{1}{2}\sqrt{q_m^4\begin{vmatrix}\Delta x_1 & \Delta y_1\\ \Delta x_2 & \Delta y_2\end{vmatrix}^2 + q_m^2\begin{vmatrix}\Delta x_1 & \Delta z_1\\ \Delta x_2 & \Delta z_2\end{vmatrix}^2 + q_m^2\begin{vmatrix}\Delta y_1 & \Delta z_1\\ \Delta y_2 & \Delta z_2\end{vmatrix}^2}; \quad (31)$$

- in the spatial rectangular geocentric coordinate system:

$$P_T = \frac{1}{2}\sqrt{\begin{vmatrix}\Delta X_1 & \Delta Y_1\\ \Delta X_2 & \Delta Y_2\end{vmatrix}^2 + \begin{vmatrix}\Delta X_1 & \Delta Z_1\\ \Delta X_2 & \Delta Z_2\end{vmatrix}^2 + \begin{vmatrix}\Delta Y_1 & \Delta Z_1\\ \Delta Y_2 & \Delta Z_2\end{vmatrix}^2}; \quad (32)$$

- in the geodetic coordinate system:

$$P_T = \frac{1}{2}\sqrt{g_m^2k_m^2\left(\begin{vmatrix}\Delta B_1 & \Delta L_1\\ \Delta B_2 & \Delta L_2\end{vmatrix}\right)^2 + g_m^2\begin{vmatrix}\Delta B_1 & \Delta H_1\\ \Delta B_2 & \Delta H_2\end{vmatrix}^2 + k_m^2\begin{vmatrix}\Delta L_1 & \Delta H_1\\ \Delta L_2 & \Delta H_2\end{vmatrix}^2}. \quad (33)$$

In formulas (31) - (33), the indices 1 and 2 mean the coordinate differences of vertices 2-1 and 3-2, respectively. Particularly, when $\Delta z_1 = \Delta z_2 = 0$ and $q_m = 1$, formula (31) assumes the following form:

$$P = \frac{1}{2}\begin{vmatrix}\Delta x_1 & \Delta y_1\\ \Delta x_2 & \Delta y_2\end{vmatrix}. \quad (34)$$

With $\Delta H_1 = \Delta H_2 = 0$, formula (33) assumes the following form:

$$P = \frac{m_m^2 \cdot g_m \cdot k_m}{2}\begin{vmatrix}\Delta B_1 & \Delta L_1\\ \Delta B_2 & \Delta L_2\end{vmatrix}, \quad (35)$$

where m_m is the scale of the TM projection obtained by the formula $m_m = ch \cdot \frac{y_m}{R}$ [21].

The coordinates of the polygon vertices can be obtained both by satellite and traditional methods. In the latter case, it is possible to lay a polygonometric course on the boundary marks, or the coordinates can be obtained by polar cross-bearing, spatial cross-bearing with the measuring of side lengths, zenith distances and horizontal angles [22].

Results of investigations.

I. Let us examine the proposed formulas (method I) by calculating the area of the same spheroidal trapezoid (Fig. 1). The coordinates of eight points located on the boundaries of this trapezoid are presented in three systems: planimetric, spatial, and geodetic coordinates (Table 3).

Table 3 – Coordinates of points on the boundaries of trapezoid

No	Coordinates					
	planimetric rectangular, m		geocentric, m		geodetic	
1	2	3	4	5	6	7
1	x	6656825.024	X	2782664.300	B	$60^0\ 00^/\ 00^{//}$
	y	139480.191	Y	1574355.712	L	$29^0\ 30^/\ 00^{//}$
		-	Z	5500573.593		-
2	x	6659145.050	X	2780914.563	B	$60^0\ 01^/\ 15^{//}$
	y	139392.458	Y	1573365.758	L	$29^0\ 30^/\ 00^{//}$
		-	Z	5501733.794		-
3	x	6661465.083	X	2779164.453	B	$60^0\ 02^/\ 30^{//}$
	y	139304.706	Y	1572375.594	L	$29^0\ 30^/\ 00^{//}$
		-	Z	5502893.267		-

End Table 3

1	2	3	4	5	6	7
4	x	6661531.343	X	2778306.442	B	$60^0\ 02^/\ 30^{//}$
	y	141045.451	Y	1573891.159	L	$29^0\ 31^/\ 52.5^{//}$
		-	Z	5502893.267		-
5	x	6661598.426	X	2777447.604	B	$60^0\ 02^/\ 30^{//}$
	y	142786.174	Y	1575406.256	L	$29^0\ 33^/\ 45^{//}$
		-	Z	5502893.267		-
6	x	6659278.449	X	2779196.633	B	$60^0\ 01^/\ 15^{//}$
	y	142876.120	Y	1576398.329	L	$29^0\ 33^/\ 45^{//}$
		-	Z	5501733.794		-
7	x	6656958.479	X	2780945.289	B	$60^0\ 00^/\ 00^{//}$
	y	142966.047	Y	1577390.190	L	$29^0\ 33^/\ 45^{//}$
		-	Z	5500573.693		-
8	x	6656891.339	X	2781805.209	B	$60^0\ 00^/\ 00^{//}$
	y	141223.130	Y	1575873.186	L	$29^0\ 31^/\ 52.5^{//}$
		-	Z	5500573.593		-

The area of the trapezoid calculated by formula (11) is considered true $P_{\text{іст}} = 16180000{,}9\text{m}^2$. The corrections calculated by formulas (8) and (10) have the following values $\delta P_{L1} = -7952{,}6\text{m}^2$, $\delta P_{L2} = -7897{,}0\text{m}^2$.

Table 4 presents the comparison results of areas on the ellipsoid surface, which are calculated using the geodetic areas, and the correction for them.

Table 4 – Calculation of areas on the ellipsoid by geodetic areas

Element	Correction by formula (8)		Correction by formula (10)	
	8 points	4 points	8 points	4 points
Area P_E, m²	16179953.6	16179951.6	16180009.2	16180007.1
Deviation $\Delta = P_E - P_{real}$, m²	-56.0	-58.0	-0.4	-2.5

The area was calculated twice: by the coordinates of all eight points and by the coordinates of only four points numbered 1, 3, 5, and 7 that fix the vertices of the trapezoid.

Table 5 shows the comparison results with the true value of the areas calculated by the coordinates of the same points, but in different systems.

Table 5 – Calculation of areas on the ellipsoid by geodetic areas

Element	Coordinates					
	planimetric rectangular		spatial rectangular		geodetic	
Number of points	8	4	8	4	8	4
Area P_E, m²	16180009.2	16180007.1	16180008.3	16180006.0	16180010.0	16180010.0
Deviation $\Delta = P_E - P_{real}$, m²	-0.4	-2.5	-1.3	-3.6	+0.4	+0.4

Consequently, the insignificant values of deviations of the calculated areas from the true value (Table 5) prove the quality of the formulas proposed.

II. Let us carry out the practical implementation of theoretical research (method II). Using formulas (19) - (24), calculation has been performed, with the corresponding results presented in Table 6.

Table 6 – Relative distortion of the physical area in relation to the value of the tilt angle

Slope, ν^0	1	1,3	2	3	4	5	6	7	8	9
$\delta P_\nu / P_T$	1/6567	1/3886	1/1642	1/730	1/410	1/263	1/182	1/134	1/103	1/81

In calculations (Table 6), the error for the slope has been considered to be significantly small provided it does not exceed ¼ of the error in the area. With the latter being 1/1000, the error for the angle of slope should not exceed 1/4000. As Table 6 suggests, if the angles of slope increase more than $1,3^0$, then they should be taken into account when calculating the physical area of the parcel.

III. Let us carry out the practical implementation of theoretical research (method III) and calculate the physical surface area of a pentagonal parcel. In the diagram (Fig. 3), the line a-b-c-d-e-a shows the boundary of the area divided into three triangular fragments P_1, P_2, P_3.

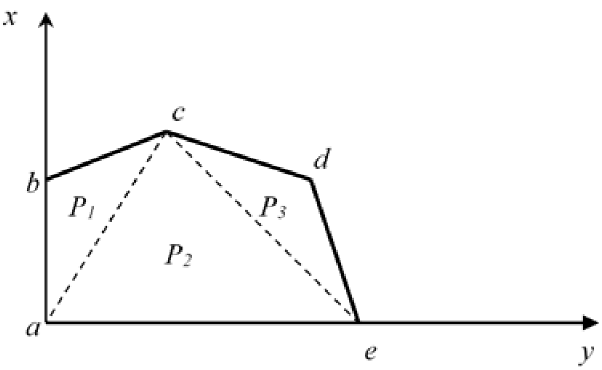

Fig. 3 Parcel diagram with a division into triangles

The coordinates of the parcel vertices a, b, c, d, e are expressed in different coordinate systems, and the areas of the parcel physical surface are calculated by formulas (27) - (29) and shown in Table 7. The calculation outcomes obtained by dividing the area into the triangles and summing their areas are presented in Table 8.

Table 7 – Calculation of physical surface areas by vertex coordinates

No	Coordinate systems					
	planimetric rectangular, m		Geocentric, m		geodetic	
1	*x*	7400000.00	*X*	2209795.74	*B*	$66^{0}\,40^{/}\,28.31^{//}$
	y	100000.00	*Y*	1238188.19	*L*	$29^{0}\,15^{/}\,45.69^{//}$
	z	1000.00	*Z*	5835156.84	*H*	1000.00
2	*x*	7400250.00	*X*	2209598.07	*B*	$66^{0}\,40^{/}\,36.38^{//}$
	y	100000.00	*Y*	1238087.82	*L*	$29^{0}\,15^{/}\,46.43^{//}$
	z	1020.00	*Z*	5835274.12	*H*	1020.00
3	*x*	7400150.00	*X*	2209612.71	*B*	$66^{0}\,40^{/}\,32.97^{//}$
	y	100150.00	*Y*	1238263.70	*L*	$29^{0}\,15^{/}\,58.34^{//}$
	z	1025.00	*Z*	5835236.99	*H*	1025.00
4	*x*	7400150.00	*X*	2209558.13	*B*	$66^{0}\,40^{/}\,32.86^{//}$
	y	100250.00	*Y*	1238347.68	*L*	$29^{0}\,16^{/}\,06.48^{//}$
	z	1000.00	*Z*	5835212.60	*H*	1000.00
5	*x*	7400000.00	*X*	2209730.27	*B*	$66^{0}\,40^{/}\,28.14^{//}$
	y	100150.00	*Y*	1238323.34	*L*	$29^{0}\,15^{/}\,57.90^{//}$
	z	1010.00	*Z*	5835163.86	*H*	1010.00
P_T	37657.55 m^2		37657.43 m^2		37657.47 m^2	

Table 8 – Calculation of areas of physical surfaces by areas of triangles

No of triangle	Coordinate system		
	planimetric rectangular	planimetric rectangular	planimetric rectangular
1	18881.33 m^2	18881.23 m^2	18881.43 m^2
2	11332.76 m^2	11332.72 m^2	11332.54 m^2
3	7767.65 m^2	7767.68 m^2	7767.77 m^2
P_T	37980.74 m^2	37980.63 m^2	37980.74 m^2

In conclusion, the calculations made in Tables 7 and 8 prove that the results of calculating the physical areas on the surface of parcels using different coordinate systems are almost identical. However, the difference between the results obtained by dividing the parcel into triangles (Table 8) and the results given in Table 1 constitutes about 1%. The reason for these discrepancies is that the formulas (27)-(29) assume that all vertices of a polygon lie in one plane, although in dividing this polygon (the parcel) into triangles, the different slope of the land surface in different places of the parcel is considered.

Recommendations and conclusions. The fact that, in our opinion, the purpose of land purchase affects its market value: in some cases, the horizontal dimensions of parcels matter; in others, the dimensions of physical surface areas do. For example, in agriculture, the horizontal area determines the number of upright plants, and the size of the physical surface area affects the amount of work on its cultivation. Therefore, it is advisable to use the areas of horizontal projection and the physical surface of the parcel in the land management, cadastre, and real estate valuation.

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ПОРІВНЯННЯ ТА ОЦІНКА ТОЧНОСТІ МЕТОДІВ ВИЗНАЧЕННЯ ПЛОЩ ЗЕМЕЛЬНИХ ДІЛЯНОК

Актуальність дослідження полягає в тому, що загальна площа значних територій може визначатись як проста сума горизонтальних площ, проте більш практичним є їх визначення за допомогою аерознімальних засобів (літаків, безпілотних літальних апаратів, дронів тощо). Очевидно, що застосування для визначення площ різних методів не дозволить досягнути їх однакової точності, тому що ці методи по різному описують земельні ділянки. Навіть врахування *рельєфу у вигляді середнього рівня місцевості лише частково враховує її особливості. У зв'язку з цим і виникає необхідність дослідження похибки визначення площ різними методами, адже різниця їх значень дає різні оцінки значень площ дають різні суми грошових надходжень. Це в свою чергу створює конфлікт інтересів, що може породжувати собою судові суперечки, що на нашу думку формує актуальність статті. Здійснено дослідження методів обчислення площ земельних ділянок: безпосередньо на фізичній поверхні еліпсоїда; горизонтальної проекції земельної ділянки та фізичної поверхні земельних ділянок. Шляхом практичної реалізації підтверджено викладені теоретичні дослідження. Виконані розрахунки доводять, що результати обчислення фізичних площ поверхні земельних ділянок із використанням різних систем координат практично не відрізняються. Однак, розходження між результатами, отриманими шляхом розбиття земельної ділянки на трикутники і результатами обчислення площ фізичних поверхонь за координати вершин досягають величини приблизно 1%. Причина цих розходжень полягає у тому, що всі вершини багатокутника лежать в одній площині, а під час розбиття цього багатокутника (земельної ділянки) на трикутники враховується різний нахил земної поверхні у різних місцях земельної ділянки. Обчислення площ об'єктів нерухомості потрібно виконувати паралельно із оцінкою точності одержаних результатів. Оцінка точності характеризується достовірністю інформації про площі, а також слугує прийняттю вірних рішень про зміни первісних даних при повторних визначеннях площ. Розглядаючи площу ділянки, розрізняють наступні варіанти: площа земельної поверхні з урахуванням рельєфу, площа горизонтальної проекції ділянки, площа проекції ділянки на поверхні земного еліпсоїда і площа зображення ділянки на площині картографічної проекції (проекція Гауса-Крюгера). Вибір варіанту залежить від поставленого завдання. Під час роботи з картами і планами одержують площу зображення ділянки в проекції карти, яку прийнято називати геодезичною площею. Якщо при цьому осьовий меридіан проходить біля ділянки і за поверхню відносності прийнято середній рівень території, значення геодезичної площі і горизонтальної проекції площі практично не відрізняються. В інших випадках важливим є знання площі всієї фізичної поверхні земельної ділянки. Метою роботи є виконання порівняльного аналізу методів визначення площ земельних ділянок для вибору найоптимальнішого методу для цільового практичного використання у різних галузях господарства. У роботі доведено, що під час здійснення процедури купівлі-продажу земельних ділянок на її ринкову вартість впливає мета придбання земельних ділянок: в одному випадку важливі горизонтальні розміри земельних ділянок; в інших – розміри площ фізичних поверхонь. Так, у сільському господарстві горизонтальна площа визначає кількість прямовисно стоячих рослин, а величина площі фізичної поверхні землі впливає на обсяг робіт щодо її обробітку і тому у землевпорядкуванні, кадастрі та оцінці нерухомості доцільно використовувати саме площі горизонтальної проекції та фізичної поверхні земельної ділянки.*

Ключові слова: *площа земельної ділянки; фізична поверхня еліпсоїда; горизонтальна проєкція земельної ділянки; об'єкт нерухомості; кадастр; землеустрій; оцінка нерухомості.*

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