MATHEMATICAL MODELING. MATHEMATICS

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ. МАТЕМАТИКА

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MATHEMATICAL MODELING OF LED RADIATION IN THE SYSTEM OF MEDICAL DIAGNOSTICS

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Summary. *The use in the medical diagnosis light irritation with intensity lower than standard level leads to decreasing invasiveness and increasing resolution of bioobject response. This improves the quality of diagnosis the state of bioobjects by the results of such active electrophysiological study. The intensity of light irritation recently determined from properties diagram of radiation and place of location of its source. For transfer of energy through a free space, layers of biological media on to the object of irradiation choose the standard levels of intensity. That helps in order to increasing of likelyhood the diagnostic decision about the state of bioobject by result analysis of its response on to irritation. In low intensity irritation, the biomedia influences on the transfer of energy and emerges requirement to control the intensity of irradiation on the surface of the target object. The propagation of light beams in biological media are investigated in ophthalmology, astronomy, biophysics only in order to evaluate the image quality on the surface of the irradiation facility but not intensity of irradiation. In the paper is given the results of studies of the method for control of irradiation of the surface of the target object, which is located in bio media. Based on the wave theory of light, the choice of the mathematical models for the radiation of LED is discussed, as well as expressions for module of Poynting vectors for dipole model of the chip emission and the LED radiation. Expression of the factor transform of light intensity of the LED radiation into intensity of irradiation the surface of the target object is defined. Results of research had been applied for effective irritation the retina eye by light with intensity lower than standard level.*

Key words: *LED, radiation, low light intensity, biomedia, irradiation, mathematical model.*

Problem setting. The use in the medical diagnosis light irritation with intensity lower than standard level leads to decreasing invasiveness and increasing resolution of bioobject response. This improves the quality of diagnosis the state of bioobjects by the results of such active electrophysiological study [1, 2]. The intensity of light irritation recently determined from properties diagram of radiation and place of location of its source. For transfer of energy through a free space, layers of biological media on to the object of irradiation choose the standard levels of intensity. That helps in order to increasing of likelyhood the diagnostic decision about the state of bioobject by result analysis of its response on to irritation [3]. In low intensity irradiation (LII), the biomedia influences on the transfer of energy and emerges requirement to control the intensity of irradiation on the surface of the target object. The propagation of light beams in biological media are investigated in ophthalmology, astronomy, biophysics only in order to evaluate the image quality on the surface of the irradiation facility but not intensity of irradiation [4, 5]. LII control of the surface of the bioobject (not-imaging [6])

is used in the quantum physics [7]. Developing of the method for defining of LII by the light of the surface of the target object, which is located in bio media, has important meaning for bioengineering. The choice of the mathematical models for the radiation of the light, as well as expressions for module of vectors light energy flow, of factor transform of intensity of the radiation into the intensity of irradiation surface of the target object are the problems of developing this method.

Application LED for LII a bioobject through biomedia. Very perspective for using in kind of LII sources are the LEDs [8]. Explorations of the light luminous flux of the LED belong to physics: (a) of the spontaneous light emission of semiconductor structures; (b) developing of structures for propagation of light waves; (c) the combination of (a) and (b). These approaches in detail and in general applicable in analysis of electromagnetic phenomena, are based on wave theory have been used to calculate the distribution efficiency of spontaneous emission in a periodically layered structures [9]. This general model been specified to the electromagnetic analysis by the numerical simulation dipole emission inside periodically layer structures the grating, resonant cavity of LED which providing a radiating characteristic of the flux. The radiometric approach for modeling the intensity spatial distribution of encapsulated LEDs is based on to dipole model of the LED chip [10]. There are given analytical relationships between pattern of radiation and parameters of LED chip, encapsulant, and reflectors.

There are two ways for reaching LII of bioobject: of (a-c) methods or by control the LII characteristics of the LED that had been manufactured. The second way for biomedical engineering and health care industry is priority. For designing of adaptive optimal control of LII need the mathematical model of LED the LII source. The relative spatial distribution of light energy in space (i.e., diagram of LED) always known [11]. In other words, LED diagram can replace wave function in kind of the model. For getting adequation of the properties of such model to estimation of LII values need researches. These researches are directed to determining of influence of the layered medium, i.e., of diffraction, interference, refraction etc. In other words, level of energy LII that is incident through the medium on to the surface of bioobject is not researched. There are different theoretical approaches, physical models of realistic rendering by waves or particles how much light getting from one place to another, and scattering (how surfaces interact with light). The particular case is light entering the eye. It passes through the cornea, anterior chamber that is filled with aqueous, the iris, and the lens with of thousands of concentric layers [4]. The light wave front entering the eye is not flat. The point-image on the retina as the end object will suffer from wider diffraction energy spread, aberrations [4, 12]. The reflecting properties of such a system have been investigated geometrically, and the results compared with the measured reflectivity [13]. Classical account has been given by Prosser, who solved Maxwell's equations using the diffracting obstacle as a boundary condition and obtained plots of the Poynting vector past the obstacle, which exhibited undulations that could be interpreted as flow lines of energy [14].

This paper are presented the result of modeling small-extended source on a chip with an extra optics. A mathematical model of irradiation the surface of bioobject when wave is spreading through layers of medium. Electrical dipoles are used for mathematical modeling both as LED source, so and its radiation. Modules of vector Poynting for the before and after extra optics are expressed, as well and the factor this transform.

Mathematical model of the LED radiation. Following substantially by schema that was given in [15] for theory of optics, contact of semiconductors in LED is considered as symmetrical system dipoles. The point of electromagnetic waves at P0 is emitting quasi-monochromatic light of frequency ω0. At P_0 we choose a set of Cartesian axes (x_1, x_2, x_3) with the x_3 direction along the principal ray (Figure 1). We assume that the inclination θ to the axis x_3 of the rays, which pass through LED output (i.e., lens), is not more than 15°.
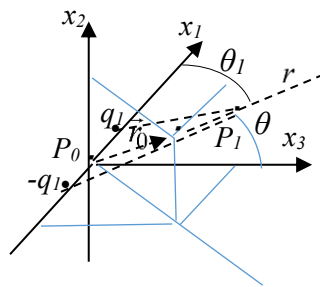
Figure 1. System schema of source point of LED radiation

The source is regarded as a dipole *Q(t)* of moment which varies both, in magnitude and direction with time, whose orientation is also a function of time. This dipole is equivalent to three linear dipoles $\pm q_j$, *j*= 1, 2, 3 (monochromatic Hertzian oscillators) with their moment vectors oriented along three mutually perpendicular directions. The components of *Q(t)* in the three directions were written in the form of Fourier integrals

$$Q_j(t) = \text{Re}\left[\sqrt{\frac{2}{\pi}}\int_0^\infty |q_j(\omega)| e^{i\delta_j(\omega)} e^{-i\omega t} d\omega\right], \tag{1}$$

where $|q_j(\omega)|$ – the amplitude, $\delta_j(\omega)$ – the phase of $q_j(\omega)$, that are rapidly and irregularly varying functions over the frequency range; $q_j(-\omega) = q_j^*(\omega)$, the asterisk denoted complex conjunction. The field of a single oscillator is weak in the neighborhood of its axis, and as the angles which the diameters are subtend at *P₀* are small, the components *Q₁(t)* and *Q₂(t)* of *Q(t)* will substantially contribute to the field. We shall therefore take as our typical oscillator one which has its axis in the *x₁x₂* – plane. The moment of this typical dipole *Q(t)* is $\text{Re}\{q(\omega)\vec{\rho}_0(\omega)e^{-i\omega t}\}$, $\vec{\rho}_0(\omega)$ being a unit vector in the direction of its axis. Such a dipole will produce at a point *P₁* in vacuum, whose distance from P₀ is large compared to the wavelength λ=2πc/ω, a field [15]:

$$\vec{E}_\omega = \text{Re}\left\{\frac{\omega^2}{c^2 r}|q(\omega)|\vec{r}_0 \times (\vec{\rho}_0(\omega) \times \vec{r}_0)e^{i[\delta(\omega)-\omega(t-r/c)]}\right\},$$

$$\vec{H}_\omega = \text{Re}\left\{\frac{\omega^2}{c^2 r}|q(\omega)|\vec{r}_0 \times \vec{\rho}_0(\omega)e^{i[\delta(\omega)-\omega(t-r/c)]}\right\}, \tag{2}$$

where $\vec{r}_0$ denotes the unit radial vector.

In regions which are many wavelengths away from the sources is represented more general types of fields in the form

$$\vec{E}_0 = \vec{e}(r)e^{ik_0\ell(r)}, \quad \vec{H}_0 = \vec{h}(r)e^{ik_0\ell(r)}, \tag{3}$$

where *r* – the distance from the dipole, $\ell(r)$ – "the optical path", $\vec{e}$ and $\vec{h}$ are no longer constant vectors, but at distances sufficiently far away from the dipole (*r>>*λ₀) these vectors are, with suitable normalization of the dipole moment, independent of $k_0 = \omega/c = 2\pi/\lambda_0$, and are vector

functions of position (which may in general be complex). With (3) as trial solution, Maxwell's equations lead to a set of relations between $\vec{e}(r)$, $\vec{h}(r)$ and ℓ. For large k_0 (small λ_0) these relations demand that ℓ should satisfy certain differential equation, which is independent of the amplitude vectors $\vec{e}(r)$ and $\vec{h}(r)$:

$$\nabla \ell \times \vec{h} + \varepsilon \vec{e} = 0, \tag{4}$$

$$\nabla \ell \times \vec{e} - \mu \vec{h} = 0, \tag{5}$$

$$\vec{e} \cdot \nabla \ell = 0, \tag{6}$$

$$\vec{h} \cdot \nabla \ell = 0. \tag{7}$$

Simultaneous equations (4) and (5) is regarded as a set or six simultaneous homogeneous linear scalar equations for the Cartesian components e_{x_1}, h_{x_1} …, of $\vec{e}$ and $\vec{h}$:

$$\mu^{-1}[(\vec{e} \cdot \nabla \ell)\nabla \ell - \vec{e}(\nabla \ell)^2] + \varepsilon \vec{e} = 0, \tag{8}$$

$$\varepsilon^{-1}[(\vec{h} \cdot \nabla \ell)\nabla \ell - \vec{h}(\nabla \ell)^2] + \mu \vec{h} = 0. \tag{9}$$

These equations have non-trivial solutions only if $(\nabla \ell)^2 = n^2(x, y, z)$, where $n = (\varepsilon\mu)^{(1/2)}$ denotes the refractive index. The function ℓ is called the *eikonal*, the surfaces $\ell(r) =$ constant is called the *geometrical wave surfaces* (spheres) or the *geometrical wave-fronts* (Figure 2).

The time average of the Povnting vector $\langle \mathbf{S} \rangle = (c/8\pi)\mathrm{Re}(\vec{e} \times \vec{h}^*)$ by using (4, 5, 8) is obtained in [15]

$$\langle \mathbf{S} \rangle = \frac{c}{8\pi\mu}\{(\vec{e} \cdot \vec{e}^*)\nabla \ell - (\vec{e} \cdot \nabla \ell)\vec{e}^*\} = (2c/n^2)\langle w_e \rangle \nabla \ell. \tag{10}$$
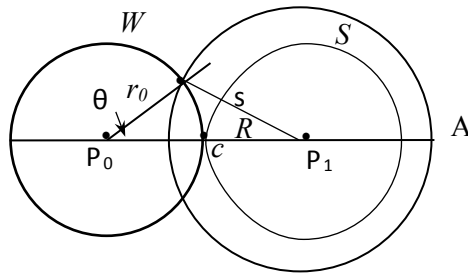

Figure 2. *W* − a wave sphere front, *S* − the reference sphere front, Q(*x'*, *y'*, *z'*) − a typical point, *s* − the distance from a typical point (*x'*, *y'*, *z'*) on the reference sphere to *P₁*, c − center of the LED lens

Let *W* (Figure 2) be as a typical geometrical wave front in the object space at a distance r_0 from P_0, which is large compared with the wavelength. The angles θ, which the rays make with the axis A of the system, are small. It follows from (2) that at any particular instant of time the vectors $\vec{E}_\omega$ and $\vec{H}_\omega$ do not vary appreciably in magnitude and direction over *W*. When choose rectangular Cartesian axes (*x, y, z*) with origin at the Gaussian image P_l of P_0, with the *z* direction along cP_l than the field at all points in the region of the LED lens aperture except those in the immediate neighborhood of the edge of the aperture can be approximately expressed in the form [15]:

$$\vec{E}_\omega(x,y,z,t) = \mathrm{Re}\left\{ \frac{\omega^2}{c^2} \vec{e}_\omega(x,y,z) e^{i\left[\delta(\omega) - \omega\left[t - \frac{1}{c} l_\omega(x,y,z) \right] \right]} \right\},$$

$$\vec{H}_\omega(x,y,z,t) = \mathrm{Re}\left\{ \frac{\omega^2}{c^2} \vec{h}_\omega(x,y,z) e^{i\left[\delta(\omega) - \omega\left[t - \frac{1}{c} l_\omega(x,y,z) \right] \right]} \right\},$$

(11)

which regarded as generalization of (2). In a homogeneous non-magnetic medium of refractive index n, these vectors satisfy the relation $\left| \vec{h}_\omega \right| = n \left| \vec{e}_\omega \right|$. Distance $P_0 P_1|_{Q \to c,\ \theta \to 0} \to \mathrm{r}_0 \triangleq \ell_\omega(x,y,z)$, where $\ell_\omega(x,y,z)$ is the optical length from the P_0 point to the point P_1 (x, y, z). A reference sphere S centered on P_1, and passes through the center c of the exit lens, so, by R denote its radius cP_1. On S just as on W the amplitude vectors f_ω and h_ω will be practically constant in magnitude and direction.

Let $P_1(X,\ Y,\ Z)$ be a point in the region of the image where the intensity is to be determined. The diameters of the exit lens subtend at P_1 are small, and we apply Kirchhoff's formula on integrating the expressions (11) over that part S' of S which approximately fills the lens (in addition we also neglect the variation of the inclination factor over S'). If s is the distance from a typical point $(x',\ y',\ z')$ on the reference sphere to P_1 and since the vectors $\vec{e}_\omega(x',y',z')$ and $\vec{h}_\omega(x',y',z')$ do not vary appreciably over the surface of integration we replace them by the values $\vec{e}_\omega(0,0,-R)$ and $\vec{h}_\omega(0,0,-R)$, which they take at the center c of the LED lens. If in addition we take $n = 1$, than $\vec{e}_\omega(0,0,-R) = a(\omega)\vec{\alpha}(\omega)$, $\vec{h}_\omega(0,0,-R) = a(\omega)\vec{\beta}(\omega)$, where $\vec{\alpha}(\omega)$ and $\vec{\beta}(\omega)$ are orthogonal unit vectors in the plane perpendicular to the z direction, and

$$\vec{E}_\omega(X,Y,Z,t) = \mathrm{Re}\left\{ \frac{\omega^2}{c^2} U_\omega(X,Y,Z) a(\omega)\vec{\alpha}(\omega) e^{i[\delta(\omega) - \omega t]} \right\},$$

$$\vec{H}_\omega(X,Y,Z,t) = \mathrm{Re}\left\{ \frac{\omega^2}{c^2} U_\omega(X,Y,Z) a(\omega)\vec{\beta}(\omega) e^{i[\delta(\omega) - \omega t]} \right\},$$

(12)

$$U_\omega(X,Y,Z) = \frac{\omega}{2\pi i c} \iint\limits_{S'} \frac{e^{i\omega[\ell_\omega(x',y',z') + s]/c}}{s} dS .$$

(13)

Scalar wave function (13) is calculated from the knowledge of the eikonal function of the point P_0 radiation [15]. From (12) we deduce by calculating the Poynting vector $S_\omega = c[E_\omega \times H_\omega]/4\pi$ and taking the time average, that the intensity at the point $P_1(X,\ Y,\ Z)$ due to the single dipole (represented by $Q(t)$) at P_0 is proportional to the square of the modulus of the scalar wave function (13). To justify the use of a single scalar wave function in calculating the intensity we must carry out the time averaging not for the monochromatic component but for the total field [15]. In order to determine the intensity in the image region wrote down separate expressions for each of the Cartesian components of E and H. The contributions of each frequency component to the total field regarded as arising essentially from two dipoles at with their axes along the x_1 and x_2 directions. It follows from (1) and (12), if we also define contributions from negative frequencies that the total field in the image region. Let $\theta_1\ (\omega)$ and $\theta_2\ (\omega)$ denote the angles which the unit vectors $\alpha_1(\omega)$ and $\alpha_2(\omega)$ make with the x direction in the image space (e.g., Figure 1, θ_1 − for x_1). Since $\alpha_1(\omega)$ and $\beta_1(\omega)$ and $\alpha_2(\omega)$ and $\beta_2(\omega)$ are real, mutually orthogonal vectors which lie in a plane perpendicular to the z direction. It given in [15] that the components of E and H are approximated as:

$$E_z(X,Y,Z,t) = H_z(X,Y,Z,t) = 0,$$

$$E_x(X,Y,Z,t) = H_y(X,Y,Z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} U_\omega(X,Y,Z) f(\omega) e^{-i\omega t} d\omega, \tag{14}$$

$$E_y(X,Y,Z,t) = -H_x(X,Y,Z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} U_\omega(X,Y,Z) g(\omega) e^{-i\omega t} d\omega,$$

$$f(\omega) = \frac{\omega^2}{c^2} \left[a_1(\omega)\cos\theta_1(\omega) e^{i\delta_1(\omega)} + a_2(\omega)\cos\theta_2(\omega) e^{i\delta_2(\omega)} \right],$$

$$g(\omega) = \frac{\omega^2}{c^2} \left[a_1(\omega)\sin\theta_1(\omega) e^{i\delta_1(\omega)} + a_2(\omega)\sin\theta_2(\omega) e^{i\delta_2(\omega)} \right]. \tag{15}$$

It follows from (14) that the magnitude of the Poynting vector S_ω can be expressed in form

$$|\mathbf{S}| = \frac{c}{4\pi} \left[E_x^2 + E_y^2 \right] = \frac{c}{4\pi} \left[H_x^2 + H_y^2 \right], \tag{16}$$

and we must time averaging of this quantity. For reasons of convergence assume that the radiation field exists only between the instants $t = -T$ and $t = T$, where $T >> 2\pi/\omega_0$; it is easy to pas to the limit $T \to \infty$. So, at a point $P_1(X, Y, Z)$ is belong to the region of the image the intensity $I(X,Y.Z)$ is to be determined as the time average of the energy U^2 which crosses a unit area [15]

$$I(X,Y,Z) = \frac{c}{4\pi T} \int_0^\infty |U_\omega(X,Y,Z)|^2 [|f(\omega)|^2 + |g(\omega)|^2] d\omega = C \int_0^\infty |U_\omega(X,Y,Z)|^2 d\omega, \tag{17}$$

$$C = \frac{c}{4\pi T} \int_0^\infty [|f(\omega)|^2 + |g(\omega)|^2] d\omega. \tag{18}$$

If $|\Delta\omega|$ is sufficiently small than $|U_\omega|$ will be practically independent of ω over the effective frequency range, so that $|U_\omega|$ may then be replaced by U_{ω_0}, and taken outside the integral

$$I(X,Y,Z) = C |U_{\omega_0}(X,Y,Z)|^2. \tag{19}$$

Conclusions.

Electrical dipoles (1) were used to mathematical model light field (2, 3) the LED chip. Expressions (10, 16) for module of Poynting vector were received for radiation of light, before and after lens of the LED, as well as for factor (18) transform of these radiations.

Scalar wave functions (13) can be used for determining amplitude vectors $\vec{e}(r)$ and $\vec{h}(r)$ LEDs that are manufactured. This allows synthesis new LEDs by the ways electrophysics, technology or reconstruction as well as designing control location and powering LED for the low intensity irradiations. Results of research had been applied for low intensity irradiation of eye retina.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ВИПРОМІНЮВАННЯ СВІТЛОДІОДА В СИСТЕМІ МЕДИЧНОЇ ДІАГНОСТИКИ

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Резюме. Застосування в медичній діагностиці світлового подразнення з інтенсивністю нижче стандартного рівня призводить до зменшення інвазивності та підвищення роздільної здатності реакції біооб'єктів. Це покращує якість діагностики стану біооб'єктів за результатами такого активного електрофізіологічного дослідження. Інтенсивність світлового подразнення останнім часом визначається з властивостей діаграми випромінювання і місця розташування його джерела. Для передавання енергії через вільний простір, шари біологічних середовищ на об'єкт опромінення вибирають стандартні рівні інтенсивності. Це допомагає для підвищення правдоподібності діагностичного рішення про стан біооб'єкта за результатом аналізу його реакції на подразнення. При низькій інтенсивності подразнення, біосередовище впливає на перенесення енергії й виникає потреба контролювати інтенсивність опромінення на поверхні об'єкта опромінення. Поширення світлових пучків у біологічних середовищах досліджені в офтальмології, астрономії, біофізиці тільки для того, щоб оцінити якість зображення на поверхні об'єкта опромінення, а не інтенсивність опромінення. У статті наведено результати досліджень методу контролю опромінення поверхні об'єкта, який розміщується в біосередовищах. На основі хвильової теорії світла обговорюється вибір математичних моделей для випромінювання світлодіода, а також вирази модуля векторів Пойнтинга для дипольної моделі випромінювання чіпа і світлодіодного випромінювання. Вираз коефіцієнта перетворення інтенсивності світла світлодіодного випромінювання в інтенсивність опромінення поверхні цільового об'єкта означується. Результати дослідження були застосовані для ефективного подразнення сітківки ока світлом з інтенсивністю нижче стандартного рівня.

Ключові слова: LED, випромінювання, низька інтенсивність світла, біосередовище, опромінення, математична модель.