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INTEGRAL EQUATIONS FOR AN THERMOELASTIC ANISOTROPIC BIMATERIAL WITH HIGH TEMPERATURE-CONDUCTING COHERENT INTERFACE

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Summary. *This paper studies the problem of an thermoelastic anisotropic bimaterial with highly conducting interface containing thin inclusions. Using the modified boundary element approach, the extended Stroh formalism and complex variable calculus the Somigliana type integral formulae and corresponding boundary integral equations for the anisotropic bimaterial with mechanically perfect and thermally imperfect interface of base materials containing internal inhomogeneties are obtained. Derived integral equations are introduced into the modified boundary method, which along with the models of thin thermoelastic inclusions allow solving various problems for thermoelastic anisotropic medium composed with two half-planes with different thermo-mechanical properties. The influence of the high temperature-conducting coherent interface on the field intensity factors at the tips of thin inhomogeneties is studied.*

Key words: *compressible elastic layer, layer of viscous compressible liquid, initial stresses, harmonic waves.*

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Problem setting. Composite materials as result of their adjustable thermo-physical and mechanic properties during design and production phases are becoming more and more popular in production of modern innovative engineering constructions. The production of the latter faces the combination of two different in mechanic and thermal anisotropic thermo-elastic materials. Such combination results in creation of thin transitive layer that perturbs the fields of stresses and temperatures in the whole piecewise homogenous environment and especially alters the physic-mechanic fields around powerful concentrators like thin heterogeneities, particularly cracks. In case of minor or considerable influence of this layer they use the conditions of ideal or non-ideal thermal and mechanic contact between the constituent parts of bi-material matrix for mathematical modeling of the problem.

Analysis of the known research results. Generally, a lot of issues were devoted to studies of anisotropic bi-materials [1-11]. Item [7] develops the boundary-element method for the problems of thermo-elasticity of anisotropic bi-materials. The project [9] resulted in obtaining of Green's two-dimension function for anisotropic bi-materials with non-ideal weak thermal contact and non-ideal deft mechanic contact. There were obtained the solution for cracks in dissimilar anisotropic media [6] and clear close-end appearance of nucleuses of integral equations for thermo-elastic anisotropic bi-material with ideal contact of constituent parts [8]. The problem of thermo-elasticity of anisotropic bi-material under weak thermo-conductivity and ideal mechanic contact between constituents was solved in [11].

Research objectives. The research project deals with comprehensive research of thermo-elastic anisotropic bi-material with internal thin inhomogeneities (particularly cracks) under high thermo-conductivity and ideal mechanic contact on the boundary of semi-planes (coherent interface of high thermo-conductivity). To solve these problems there were used results of [11], Stroh's extended formalism [12] and complex variable calculus [13].

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Task setting. Let's take a look in immobile rectangular frame of axis $Ox_1x_2x_3$ at balance equation, thermal balance and constituent correlations of flat (in plane Ox_1x_2) deformation of linearly thermo-elastic anisotropic body and flat immobile thermo-conductivity [4, 12, 14]:

$$\sigma_{ij,j} = 0\,,\ \ h_{i,i} = 0\quad (i, j = 1, 2, 3)\,; \tag{1}$$

$$\sigma_{ij} = C_{ijkm}\varepsilon_{km} - \beta_{ij}\theta\,,\ \ h_i = -k_{ij}\theta_{,j} \tag{2}$$

Here $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$ – components of deformation tensor; σ_{ij} – components of stress tensor; h_i – components of vector for thermal stream density; u_i – relocation vector; θ – temperature alteration against initial one; C_{ijkm} – elastic constants; k_{ij} – thermo-conductivity coefficients; $\beta_{ij} = C_{ijkm}\alpha_{km}$ $\left(i, j, k, m = 1,..,3\right)$ – modules of thermal extension (coefficients of thermal stresses); α_{ij} – coefficients of thermal extension. Tensors with components C_{ijkm}, k_{ij}, α_{ij} and β_{ij} are symmetric. Comma in index indicators is differentiation due to indicated after comma coordinate, i.e. $u_{i,j} = \partial u_i / \partial x_j$.

Due to Stroh's extended formalism [4, 12] the general solution to equations (1), (2) will be as follows:

$$\theta = 2\,\mathrm{Re}\left\{g'\left(z_t\right)\right\},\ \ \vartheta = 2k_t\,\mathrm{Im}\left\{g'\left(z_t\right)\right\},\ \ h_1 = -\vartheta_{,2}\,,\ \ h_2 = \vartheta_{,1}\,,\ \ k_t = \sqrt{k_{11}k_{22} - k_{12}^{\ 2}}\ ;$$

$$\mathbf{u} = 2\,\mathrm{Re}[\mathbf{A}\mathbf{f}(z_*) + \mathbf{c}g(z_t)],\ \ \boldsymbol{\varphi} = 2\,\mathrm{Re}[\mathbf{B}\mathbf{f}(z_*) + \mathbf{d}g(z_t)],\ \ \sigma_{i1} = -\varphi_{i,2}\,,\ \ \sigma_{i2} = \varphi_{i,1}\,;$$

$$z_t = x_1 + p_t x_2\,;\ \ z_\alpha = x_1 + p_\alpha x_2\,;\ \ \mathbf{f}\left(z_*\right) = \left[F_1\left(z_1\right), F_2\left(z_2\right), F_3\left(z_3\right)\right]^{\mathrm{T}}, \tag{3}$$

where ϑ – function of thermal stream; $F_\alpha\left(z_\alpha\right)$ – certain analytical functions describing Stroh's vector of complex potentials $\mathbf{f}\left(z_*\right)$; $g(z_t)$ – temperature potential; complex constant p_t is a root (with added imaginative part) of characterizing equation of thermo conductivity $k_{22}p_t^2 + 2k_{12}p_t + k_{11} = 0$. Matrixes $\mathbf{A}$ and $\mathbf{B}$, vectors $\mathbf{c}$ and $\mathbf{d}$, constants p_α $\left(\alpha = 1, 2, 3\right)$ are determined from the problem associated with characteristic values of Stroh's formalism [4].

Stroh's potentials $\mathbf{f}\left(z_*\right)$ and the vector of transposition and stresses function are linked with correlations [1]:

$$\mathbf{f}\left(z_*\right) = \mathbf{B}^{\mathrm{T}}\mathbf{u} + \mathbf{A}^{\mathrm{T}}\boldsymbol{\varphi} - \mathbf{B}^{\mathrm{T}}\mathbf{u}^t - \mathbf{A}^{\mathrm{T}}\boldsymbol{\varphi}^t,\ \ \mathbf{u}^t = 2\,\mathrm{Re}\left\{\mathbf{c}g\left(z_t\right)\right\},\ \ \boldsymbol{\varphi}^t = 2\,\mathrm{Re}\left\{\mathbf{d}g\left(z_t\right)\right\}, \tag{4}$$

And due to (3) the function $g'\left(z_t\right)$, temperature and thermal stream function are as follows:

$$g'\left(z_t\right) = \frac{1}{2}\left(\theta + i\frac{\vartheta}{k_t}\right). \tag{5}$$

Construction of integral inputs of complex potentials for bi-materials. Let us take insight on plane deformation of medium, which consists of two thermo-elastic anisotropic semi-spaces that are correspondingly located in semi-planes S_1 $(x_2 > 0)$ i S_2 $(x_2 < 0)$ (Figure 1). On the right line $x_2 = 0$ (plane Ox_2x_3), which is a line (plane) of contact between these two bodies there are fulfilled the conditions of non-ideal thermal interaction (high thermo-conductivity interface)

$$\vartheta^{(1)}(x_1, x_2)\Big|_{x_2=0} = \vartheta^{(2)}(x_1, x_2)\Big|_{x_2=0} + \eta_0\left(\theta_{,1}^{(2)}(x_1, x_2)\right)\Big|_{x_2=0}; \tag{6}$$

$$\theta^{(1)}(x_1, x_2)\Big|_{x_2=0} = \theta^{(2)}(x_1, x_2)\Big|_{x_2=0} \tag{7}$$

and ideal mechanical contact

$$\boldsymbol{\varphi}^{(1)}(x_1, x_2)\Big|_{x_2=0} = \boldsymbol{\varphi}^{(2)}(x_1, x_2)\Big|_{x_2=0},\ \mathbf{u}^{(1)}(x_1, x_2)\Big|_{x_2=0} = \mathbf{u}^{(2)}(x_1, x_2)\Big|_{x_2=0}. \tag{8}$$

Here indexes 1 and 2 are used to identify the values of the fields that act in semi-planes S_1 and S_2 respectfully. The equations (6) and (7) stipulate the boundary conditions that are relevant to mathematic model of thin layer where the temperature has the same value at antipodal points on layer surfaces, and the difference between normal constituents of thermal stream is proportional to the derivative of temperature function. In case the parameter η_0, describing the non-ideal character of thermal interaction, equals zero, then the interaction is ideal, and if η_0 equals perpetuity, then there is no transposition of thermal energy from the first semi-plane on the second one.

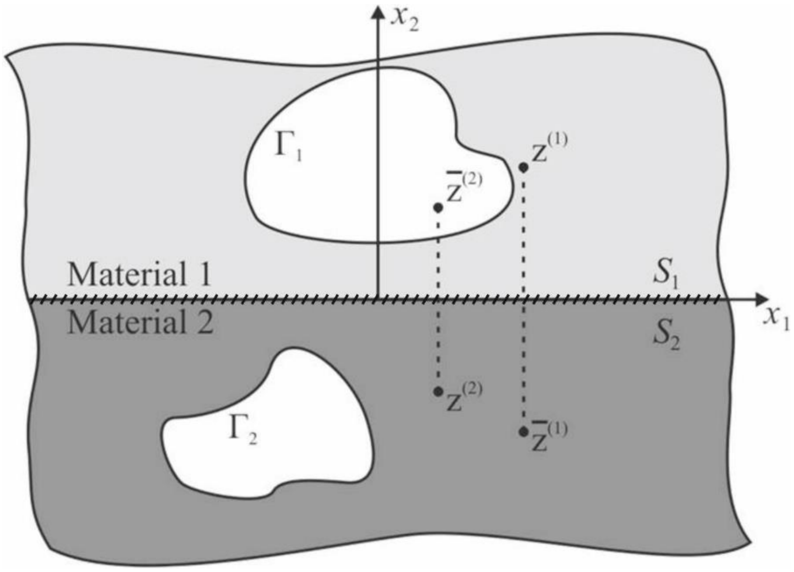

Figure 1. Thermoelastic anisotropic bi-material medium

Every semi-plane contains a system of smooth close-end contours (holes) $\Gamma_1 = \bigcup_i \Gamma_i^{(1)}$ and $\Gamma_2 = \bigcup_i \Gamma_i^{(2)}$ respectfully.

To derive the integral formulae for Stroh's complex potentials one has to utilize Cauchy's integral formula [13], which describes the connection between arbitrary analytical function $\phi(\tau)$ on the boundary ∂S of the area S and within the mentioned above area:

$$\frac{1}{2\pi i}\int_{\partial S}\frac{\phi(\tau)d\tau}{\tau - z} = \begin{cases}\phi(z) & \forall z \in S, \\ 0 & \forall z \notin S,\end{cases} \tag{9}$$

where $\tau, z \in \mathbb{C}$ are complex variables that determine the location of source and field points correspondingly. They also state in (9) that, if the area (9) S is perpetual, then function $\phi(\tau)$ has to disappear at $z \to \infty$.

Thermo-conductivity. As the problem of thermo-conductivity is linear, then its solution can be represented as a superposition of homogeneous solution $g_{1\infty}\left(z_t^{(1)}\right)$ and $g_{2\infty}\left(z_t^{(2)}\right)$, which must satisfy the marginal conditions (6) as well as perturbed solution that is caused with available contours Γ_1 and Γ_2 due to certain marginal conditions.

Let us mark Cauchy's integrals for complex function of temperatures $g_i'\left(z_t^{(i)}\right)$ as

$$q_t^{(i)}\left(z_t^{(j)}\right) = \int_{\Gamma_i}\frac{g_i'\left(\tau_t^{(i)}\right)d\tau_t^{(i)}}{\tau_t^{(i)} - z_t^{(j)}},\ \ \overline{q}_t^{(i)}\left(z_t^{(j)}\right) = \int_{\Gamma_i}\frac{\overline{g_i'\left(\tau_t^{(i)}\right)}d\overline{\tau}_t^{(i)}}{\overline{\tau}_t^{(i)} - z_t^{(j)}}, \tag{10}$$

and extrinsic integrals due to unrestricted integration are as $-\infty < x_1 < +\infty$.

$$m_t\left(z_t^{(j)}\right) = \int_{-\infty}^{+\infty}\frac{\vartheta^{(2)}\left(x_1\right)dx_1}{x_1 - z_t^{(j)}},\ \ p_t\left(z_t^{(j)}\right) = \int_{-\infty}^{+\infty}\frac{\theta\left(x_1\right)dx_1}{x_1 - z_t^{(j)}}. \tag{11}$$

Integration with the portions of extrinsic integrals will give

$$\int_{-\infty}^{+\infty}\frac{\theta_{,1}\left(x_1\right)}{x_1 - z_t^{(j)}}dx_1 = \left.\frac{\theta\left(x_1\right)}{x_1 - z_t^{(j)}}\right|_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty}\frac{\theta\left(x_1\right)}{\left(x_1 - z_t^{(j)}\right)^2}dx_1 = p_t'\left(z_t^{(j)}\right). \tag{12}$$

Let us accept that function $\theta\left(x_1\right)$ approaches zero on perpetuity. Then the first summand in (12) equals null and the second one is derivative of the function $p_t'\left(z_t^{(j)}\right)$. Hence

$$\int_{-\infty}^{+\infty}\frac{\vartheta^{(1)}\left(x_1\right)}{x_1 - z_t^{(j)}}dx_1 = \int_{-\infty}^{+\infty}\frac{\vartheta^{(2)}\left(x_1\right)}{x_1 - z_t^{(j)}}dx_1 + \eta_0\int_{-\infty}^{+\infty}\frac{\theta_{,1}^{(2)}\left(x_1\right)}{x_1 - z_t^{(j)}}dx_1 = m_t\left(z_t^{(j)}\right) + \eta_0 p_t'\left(z_t^{(j)}\right). \tag{13}$$

Having used the correlation (5), (9) – (11), (13) and taking into account the marginal conditions on the dissimilar anisotropic media of bi-material, Cauchy's integral formula for the functions $g_1'\left(z_t^{(1)}\right)$ та $g_2'\left(z_t^{(2)}\right)$ will be as follows:

$$g_1'\left(z_t^{(1)}\right) = g_{1\infty}'\left(z_t^{(1)}\right) + \frac{1}{2\pi i}q_t^{(1)}\left(z_t^{(1)}\right) + \frac{1}{4\pi i}p_t\left(z_t^{(1)}\right) + \frac{1}{4\pi k_t^{(1)}}m_t\left(z_t^{(1)}\right) + \frac{\eta_0}{4\pi k_t^{(1)}}p_t'\left(z_t^{(1)}\right)\ \ \forall\,\mathrm{Im}\left(z_t^{(1)}\right) > 0,$$

$$g_2'\left(z_t^{(2)}\right) = g_{2\infty}'\left(z_t^{(2)}\right) + \frac{1}{2\pi i}q_t^{(2)}\left(z_t^{(2)}\right) - \frac{1}{4\pi i}p_t\left(z_t^{(2)}\right) - \frac{1}{4\pi k_t^{(2)}}m_t\left(z_t^{(2)}\right)\ \ \forall\,\mathrm{Im}\left(z_t^{(2)}\right) < 0, \tag{14}$$

and for determination of extrinsic integrals (11) via Cauchy's integrals (10) we will obtain the first order systems of linear differential equations

$$\forall\,\mathrm{Im}\left(z_t^{(1)}\right)>0:\quad \begin{aligned} & q_t^{(2)}\left(z_t^{(1)}\right)-\frac{1}{2}p_t\left(z_t^{(1)}\right)-\frac{i}{2k_t^{(2)}}m_t\left(z_t^{(1)}\right)=0,\\ & \overline{q}_t^{(1)}\left(z_t^{(1)}\right)+\frac{1}{2}p_t\left(z_t^{(1)}\right)-\frac{i}{2k_t^{(1)}}m_t\left(z_t^{(1)}\right)-\frac{i\eta_0}{2k_t^{(1)}}p_t'\left(z_t^{(1)}\right)=0; \end{aligned} \tag{15}$$

$$\forall\,\mathrm{Im}\left(z_t^{(2)}\right)<0:\quad \begin{aligned} & \overline{q}_t^{(2)}\left(z_t^{(2)}\right)-\frac{1}{2}p_t\left(z_t^{(2)}\right)+\frac{i}{2k_t^{(2)}}m_t\left(z_t^{(2)}\right)=0,\\ & q_t^{(1)}\left(z_t^{(2)}\right)+\frac{1}{2}p_t\left(z_t^{(2)}\right)+\frac{i}{2k_t^{(1)}}m_t\left(z_t^{(2)}\right)+\frac{i\eta_0}{2k_t^{(1)}}p_t'\left(z_t^{(2)}\right)=0. \end{aligned} \tag{16}$$

Accounting on the condition $m_t\left(z_t^{(j)}\right)\to 0$, at $z_t^{(j)}\to\infty$ the solution for the systems (15), (16) will be as follows

$$m_t\left(z_t^{(1)}\right)=-2ik_t^{(2)}q_t^{(2)}\left(z_t^{(1)}\right)+ik_t^{(2)}\left[\frac{2k_t^{(1)}}{i\eta_0}\overline{e}_t^{(1)}\left(z_t^{(1)}\right)-\frac{2k_t^{(2)}}{i\eta_0}e_t^{(2)}\left(z_t^{(1)}\right)\right]; \tag{17}$$

$$m_t\left(z_t^{(2)}\right)=2ik_t^{(2)}\overline{q}_t^{(2)}\left(z_t^{(2)}\right)-ik_t^{(2)}\left[\frac{2k_t^{(2)}}{i\eta_0}\overline{e}_t^{(2)}\left(z_t^{(2)}\right)-\frac{2k_t^{(1)}}{i\eta_0}e_t^{(1)}\left(z_t^{(2)}\right)\right]; \tag{18}$$

$$p_t\left(z_t^{(1)}\right)=\frac{2k_t^{(1)}}{i\eta_0}\overline{e}_t^{(1)}\left(z_t^{(1)}\right)-\frac{2k_t^{(2)}}{i\eta_0}e_t^{(2)}\left(z_t^{(1)}\right); \tag{19}$$

$$p_t\left(z_t^{(2)}\right)=\frac{2k_t^{(2)}}{i\eta_0}\overline{e}_t^{(2)}\left(z_t^{(2)}\right)-\frac{2k_t^{(1)}}{i\eta_0}e_t^{(1)}\left(z_t^{(2)}\right) \tag{20}$$

Functions $e_t^{(i)}\left(z_t^{(j)}\right)$ and $\overline{e}_t^{(i)}\left(z_t^{(j)}\right)$ are characterized with the following correlations:

$$e_t^{(i)}\left(z_t^{(j)}\right)=\int_{\Gamma_i} g_i'\left(\tau_t^{(i)}\right)\mathcal{K}\left(B^{(j)}\left(\tau_t^{(i)}-z_t^{(j)}\right)\right)d\tau_t^{(i)}\,; \tag{21}$$

$$\overline{e}_t^{(i)}\left(z_t^{(j)}\right)=\int_{\Gamma_i} \overline{g_i'\left(\tau_t^{(i)}\right)}\mathcal{K}\left(B^{(j)}\left(\overline{\tau}_t^{(i)}-z_t^{(j)}\right)\right)d\overline{\tau}_t^{(i)}, \tag{22}$$

where

$$B^{(1)}=\frac{i}{\eta_0}\left(k_t^{(1)}+k_t^{(2)}\right),\quad B^{(2)}=-\frac{i}{\eta_0}\left(k_t^{(1)}+k_t^{(2)}\right), \tag{23}$$

$\mathcal{K}(z)=e^zE_1(z)$, a $E_1(z)$ – integral ratio function

$$E_1(z)=\int_z^\infty \frac{e^{-t}}{t}dt\,. \tag{24}$$

Having put the obtained solution into (14), we will gain the integral approximation for complex functions $g_1'\left(z_t^{(1)}\right)$ and $g_2'\left(z_t^{(2)}\right)$, which do not contain the integrals along perpetual dissimilar anisotropic media

$$\forall \operatorname{Im}\left(z_t^{(1)}\right) > 0:$$

$$g_1'\left(z_t^{(1)}\right) = g_{1\infty}'\left(z_t^{(1)}\right) + \frac{1}{2\pi i}\left[q_t^{(1)}\left(z_t^{(1)}\right) + \bar{q}_t^{(1)}\left(z_t^{(1)}\right) - \frac{2k_t^{(2)}}{i\eta_0} e_t^{(2)}\left(z_t^{(1)}\right) + \frac{2k_t^{(1)}}{i\eta_0}\bar{e}_t^{(1)}\left(z_t^{(1)}\right)\right]; \quad (25)$$

$$\forall \operatorname{Im}\left(z_t^{(2)}\right) < 0:$$

$$g_2'\left(z_t^{(2)}\right) = g_{2\infty}'\left(z_t^{(2)}\right) + \frac{1}{2\pi i}\left[q_t^{(2)}\left(z_t^{(2)}\right) + \bar{q}_t^{(2)}\left(z_t^{(2)}\right) + \frac{2k_t^{(1)}}{i\eta_0} e_t^{(1)}\left(z_t^{(2)}\right) - \frac{2k_t^{(2)}}{i\eta_0}\bar{e}_t^{(2)}\left(z_t^{(2)}\right)\right]. \quad (26)$$

Having utilized (5), one can build the integral approximation for functions $g_1'\left(z_t^{(1)}\right)$ and $g_2'\left(z_t^{(2)}\right)$ via ultimate values of temperature θ and normal component of thermal stream vector $h_n = h_i n_i$ (n_i – components of outer normal unit vector to curve Γ_i).

On the basis of (5) and (25), (26) we will get integral approximations for the temperature and thermal stream in arbitrary point ξ of bi-material

$$\begin{aligned}
\theta(\xi) &= \int_\Gamma \left[\Theta^{\mathrm{HCI*}}(\mathbf{x},\xi) h_n(\mathbf{x}) - H^{\mathrm{HCI*}}(\mathbf{x},\xi)\theta(\mathbf{x})\right] ds(\mathbf{x}) + \theta^\infty(\xi);\\
h_i(\xi) &= \int_\Gamma \Theta_i^{\mathrm{HCI**}}(\mathbf{x},\xi) h_n(\mathbf{x})\, d\Gamma(\mathbf{x}) - \int_\Gamma H_i^{\mathrm{HCI**}}(\mathbf{x},\xi)\theta(\mathbf{x})\, ds(\mathbf{x}) + h_i^\infty(\xi),
\end{aligned} \quad (27)$$

where functions $\theta^\infty(\xi)$ and $h_i^\infty(\xi)$ are eigensolutions for homogeneous medium (bi-material with the same properties), and the expression for nucleuses $\Theta^{\mathrm{HCI*}}(\mathbf{x},\xi)$ will be as follows:

$$\mathbf{x} \in S_1 \wedge \xi \in S_1: \quad \Theta^{\mathrm{HCI*}}(\mathbf{x},\xi) = \frac{1}{2\pi k_t^{(1)}} \operatorname{Re}\left\{\ln W_t^{(1,1)} + K \ln \bar{W}_t^{(1,1)} + (K+1)\mathcal{K}\left(B^{(1)}\bar{W}_t^{(1,1)}\right)\right\},$$

$$\mathbf{x} \in S_2 \wedge \xi \in S_1: \quad \Theta^{\mathrm{HCI*}}(\mathbf{x},\xi) = \frac{1+K}{2\pi k_t^{(1)}} \operatorname{Re}\left\{\ln W_t^{(2,1)} + \mathcal{K}\left(B^{(1)} W_t^{(2,1)}\right)\right\},$$

$$\mathbf{x} \in S_1 \wedge \xi \in S_2: \quad \Theta^{\mathrm{HCI*}}(\mathbf{x},\xi) = \frac{1-K}{2\pi k_t^{(2)}} \operatorname{Re}\left\{\ln W_t^{(1,2)} + \mathcal{K}\left(B^{(2)} W_t^{(1,2)}\right)\right\},$$

$$\mathbf{x} \in S_2 \wedge \xi \in S_2: \quad \Theta^{\mathrm{HCI*}}(\mathbf{x},\xi) = \frac{1}{2\pi k_t^{(2)}} \operatorname{Re}\left\{\ln W_t^{(2,2)} - K \ln \bar{W}_t^{(2,2)} + (1-K)\mathcal{K}\left(B^{(2)}\bar{W}_t^{(2,2)}\right)\right\} \quad (28)$$

Here

$$W_t^{(i,j)} = Z_t^{(i)}(\mathbf{x}) - Z_t^{(j)}(\xi),\ \bar{W}_t^{(i,j)} = \bar{Z}_t^{(i)}(\mathbf{x}) - Z_t^{(j)}(\xi),$$
$$n_t^{(i)} = n_2(\mathbf{x}) - p_t^{(i)} n_1(\mathbf{x}),\quad \delta_t^{(j)} = \delta_{i2} - p_t^{(j)}\delta_{i1},\quad \mathcal{L}^{(i)}(z) = \mathcal{K}\left(B^{(i)} z\right) + \ln z.$$

The rest of nucleuses are determined from the correlations [15]

$$H^{\mathrm{HCI*}}(\mathbf{x},\xi) = -k_{ij} n_i(\mathbf{x}) \Theta_{,j}^{\mathrm{HCI*}}(\mathbf{x},\xi),\ \Theta_i^{\mathrm{HCI**}} = k_{ij}\Theta_{,j}^{\mathrm{HCI*}}(\mathbf{x},\xi),\ H_i^{\mathrm{HCI**}} = k_{ij} H_{,j}^{\mathrm{HCI*}}(\mathbf{x},\xi). \quad (29)$$

Due to correlations (3) and (4) in order to develop the integral formulae of transpositions and stresses one has to calculate the primary functions $m_t(z)$ and $p_t(z)$:

$$M_t(z) = \int m_t(z)\,dz = -\int_{-\infty}^{+\infty} \ln(x_1 - z)\,\vartheta(x_1)\,dx_1; \quad (30)$$

$$P_t(z) = \int p_t(z)\,dz = -\int_{-\infty}^{+\infty} \ln(x_1 - z)\,\theta(x_1)\,dx_1. \quad (31)$$

From the expressions (24), (25) we obtain

$\forall \operatorname{Im}\left(z_t^{(1)}\right) > 0$:

$$\begin{aligned} P_t\left(z_t^{(1)}\right) &= (1-K)\,Q_t^{(2)}\left(z_t^{(1)}\right) - (1+K)\,\bar{Q}_t^{(1)}\left(z_t^{(1)}\right) + (K-1)\,e_t^{(2)}\left(z_t^{(1)}\right) + (K+1)\,\bar{e}_t^{(1)}\left(z_t^{(1)}\right), \\ M_t\left(z_t^{(1)}\right) &= -2ik_t^{(1)}(1-K)\left[\bar{Q}_t^{(1)}\left(z_t^{(1)}\right) + Q_t^{(2)}\left(z_t^{(1)}\right) - \bar{e}_t^{(1)}\left(z_t^{(1)}\right)\right] - 2ik_t^{(2)}(1-K)\,e_t^{(2)}\left(z_t^{(1)}\right); \end{aligned} \quad (32)$$

$\forall \operatorname{Im}\left(z_t^{(2)}\right) < 0$:

$$\begin{aligned} P_t\left(z_t^{(2)}\right) &= (1-K)\,\bar{Q}_t^{(2)}\left(z_t^{(2)}\right) - (1+K)\,Q_t^{(1)}\left(z_t^{(2)}\right) + (1+K)\left(e_t^{(1)}\left(z_t^{(2)}\right) + \bar{e}_t^{(2)}\left(z_t^{(2)}\right)\right), \\ M_t\left(z_t^{(2)}\right) &= 2ik_t^{(2)}(1+K)\left[\bar{Q}_t^{(2)}\left(z_t^{(2)}\right) + Q_t^{(1)}\left(z_t^{(2)}\right) - e_t^{(1)}\left(z_t^{(2)}\right)\right] + 2ik_t^{(2)}(1-K)\,\bar{e}_t^{(2)}\left(z_t^{(2)}\right), \end{aligned} \quad (33)$$

$Q_t^{(i)}\left(z_t^{(j)}\right)$, $\bar{Q}_t^{(i)}\left(z_t^{(j)}\right)$, K – [11].

Thermo-elasticity of bi-material. Using (9) we will write Cauchy's integral formula for vectors $\mathbf{f}^{(1)}\left(z_*^{(1)}\right)$ and $\mathbf{f}^{(2)}\left(z_*^{(2)}\right)$ of Stroh's complex potentials that are analytical within the areas S_1 and S_2 respectfully. As Cauchy's integral formula have been determined for analytical functions heading to zero on perpetuity, then complete solution for the problem of bi-material thermo-elasticity can be interpreted due to Cauchy's formula as a sum of perturbed solution and represented with the functions $\mathbf{f}_\infty^{(1)}\left(z_*^{(1)}\right)$ and $\mathbf{f}_\infty^{(2)}\left(z_*^{(2)}\right)$ homogeneous solution, which satisfies the marginal conditions (8). According to it we get

$$\mathbf{f}^{(1)}\left(z_*^{(1)}\right) = \mathbf{f}_\infty^{(1)}\left(z_*^{(1)}\right) + \frac{1}{2\pi i}\left[\int_{\Gamma_1}\left\langle \frac{d\tau_*^{(1)}}{\tau_*^{(1)} - z_*^{(1)}}\right\rangle \mathbf{f}^{(1)}\left(\tau_*^{(1)}\right) + \int_{-\infty}^{+\infty}\left\langle \frac{dx_1}{x_1 - z_*^{(1)}}\right\rangle \mathbf{f}^{(1)}(x_1)\right] \quad \left(\operatorname{Im} z_*^{(1)} > 0\right); \quad (34)$$

$$\int_{\Gamma_1}\left\langle \frac{d\tau_*^{(1)}}{\tau_*^{(1)} - \bar{z}_\beta^{(1)}}\right\rangle \mathbf{f}^{(1)}\left(\tau_*^{(1)}\right) + \int_{-\infty}^{+\infty} \frac{dx_1}{x_1 - \bar{z}_\beta^{(1)}}\,\mathbf{f}^{(1)}(x_1) = 0 \quad \left(\operatorname{Im} z_\beta^{(1)} > 0\right); \quad (35)$$

$$\int_{\Gamma_2}\left\langle \frac{d\tau_*^{(2)}}{\tau_*^{(2)} - z_\beta^{(1)}}\right\rangle \mathbf{f}^{(2)}\left(\tau_*^{(2)}\right) - \int_{-\infty}^{+\infty} \frac{dx_1}{x_1 - z_\beta^{(1)}}\,\mathbf{f}^{(2)}(x_1) \quad \left(\operatorname{Im} z_\beta^{(1)} > 0\right) = 0; \quad (36)$$

$$\mathbf{f}^{(2)}\left(z_*^{(2)}\right) = \mathbf{f}_\infty^{(2)}\left(z_*^{(2)}\right) + \frac{1}{2\pi i}\left[\int_{\Gamma_2}\left\langle \frac{d\tau_*^{(2)}}{\tau_*^{(2)} - z_*^{(2)}}\right\rangle \mathbf{f}^{(2)}\left(\tau_*^{(2)}\right) - \int_{-\infty}^{+\infty}\left\langle \frac{dx_1}{x_1 - z_*^{(2)}}\right\rangle \mathbf{f}^{(2)}(x_1)\right] \quad \left(\operatorname{Im} z_*^{(2)} < 0\right); \quad (37)$$

$$\int_{\Gamma_1} \left\langle \frac{d\tau_*^{(1)}}{\tau_*^{(1)} - z_\beta^{(2)}} \right\rangle \mathbf{f}^{(1)}\left(\tau_*^{(1)}\right) + \int_{-\infty}^{+\infty} \frac{dx_1}{x_1 - z_\beta^{(2)}} \mathbf{f}^{(1)}\left(x_1\right) = 0 \quad \left(\operatorname{Im} z_\beta^{(2)} < 0\right); \tag{38}$$

$$\int_{\Gamma_2} \left\langle \frac{d\tau_*^{(2)}}{\tau_*^{(2)} - \overline{z}_\beta^{(2)}} \right\rangle \mathbf{f}^{(2)}\left(\tau_*^{(2)}\right) - \int_{-\infty}^{+\infty} \frac{dx_1}{x_1 - \overline{z}_\beta^{(2)}} \mathbf{f}^{(2)}\left(x_1\right) \quad \left(\operatorname{Im} z_\beta^{(2)} < 0\right) = 0, \tag{39}$$

Where $\left\langle F\left(z_*\right)\right\rangle = \operatorname{diag}\left[F_1\left(z_1\right), F_2\left(z_2\right), F_3\left(z_3\right)\right]$, $z_\beta^{(i)} = x_1 + p_\beta^{(i)} x_2$ $\left(\beta = 1,2,3\right)$.

Using the equations (4) and (8), extrinsic integrals in the equations (34) – (39) can be written as follows

$$\int_{-\infty}^{+\infty} \frac{\mathbf{f}^{(j)} dx_1}{x_1 - z_\beta^{(i)}} = \mathbf{A}_j^{\mathrm{T}} \mathbf{m}(z_\beta^{(i)}) + \mathbf{B}_j^{\mathrm{T}} \mathbf{p}(z_\beta^{(i)}) - 2\int_{-\infty}^{+\infty} \frac{\left(\mathbf{A}_j^{\mathrm{T}} \operatorname{Re}\left[\mathbf{d}_j g_j\left(x_1\right)\right] + \mathbf{B}_j^{\mathrm{T}} \operatorname{Re}\left[\mathbf{c}_j g_j\left(x_1\right)\right]\right) dx_1}{x_1 - z_\beta^{(i)}},$$
$$\mathbf{m}\left(z_\beta^{(j)}\right) = \int_{-\infty}^{+\infty} \frac{\boldsymbol{\varphi}\left(x_1\right) dx_1}{x_1 - z_\beta^{(j)}}, \qquad \mathbf{p}\left(z_\beta^{(j)}\right) = \int_{-\infty}^{+\infty} \frac{\mathbf{u}\left(x_1\right) dx_1}{x_1 - z_\beta^{(j)}}. \tag{40}$$

In the first correlation (40) we will integrate the second summand from constituent parts and, due to (4), obtain

$$\int_{-\infty}^{+\infty} \frac{\mathbf{f}^{(1)} dx_1}{x_1 - z_\beta^{(i)}} = \mathbf{A}_1^{\mathrm{T}} \mathbf{m}(z_\beta^{(i)}) + \mathbf{B}_1^{\mathrm{T}} \mathbf{p}(z_\beta^{(i)}) + \boldsymbol{\mu}_1 M_t\left(z_\beta^{(i)}\right) - \boldsymbol{\lambda}_1 P_t\left(z_\beta^{(i)}\right) + \eta_0 \boldsymbol{\mu}_1 p_t\left(z_\beta^{(i)}\right); \tag{41}$$

$$\int_{-\infty}^{+\infty} \frac{\mathbf{f}^{(2)} dx_1}{x_1 - z_\beta^{(i)}} = \mathbf{A}_2^{\mathrm{T}} \mathbf{m}(z_\beta^{(i)}) + \mathbf{B}_2^{\mathrm{T}} \mathbf{p}(z_\beta^{(i)}) + \boldsymbol{\mu}_2 M_t\left(z_\beta^{(i)}\right) - \boldsymbol{\lambda}_2 P_t\left(z_\beta^{(i)}\right), \tag{42}$$

where such indicators as $\boldsymbol{\mu}_i$ i $\boldsymbol{\lambda}_i$ have been used for the complex constants:

$$\boldsymbol{\mu}_i = \frac{1}{k_t^{(i)}}\left(\mathbf{A}_i^{\mathrm{T}} \operatorname{Im}\left[\mathbf{d}_i\right] + \mathbf{B}_i^{\mathrm{T}} \operatorname{Im}\left[\mathbf{c}_i\right]\right), \quad \boldsymbol{\lambda}_i = \mathbf{A}_i^{\mathrm{T}} \operatorname{Re}\left[\mathbf{d}_i\right] + \mathbf{B}_i^{\mathrm{T}} \operatorname{Re}\left[\mathbf{c}_i\right]. \tag{43}$$

Cauchy's integrals in expressions (34) – (39) will be marked as

$$\mathbf{q}_j\left(z_\beta^{(i)}\right) = \int_{\Gamma_j} \left\langle \frac{d\tau_*^{(j)}}{\tau_*^{(j)} - z_\beta^{(i)}} \right\rangle \mathbf{f}^{(j)}\left(\tau_*^{(j)}\right), \ \overline{\mathbf{q}}_j\left(z_\beta^{(i)}\right) = \int_{\Gamma_j} \left\langle \frac{d\overline{\tau}_*^{(j)}}{\overline{\tau}_*^{(j)} - z_\beta^{(i)}} \right\rangle \overline{\mathbf{f}^{(j)}\left(\tau_*^{(j)}\right)}; \tag{44}$$

$$\boldsymbol{\mu}_i = \frac{1}{k_t^{(i)}}\left(\mathbf{A}_i^{\mathrm{T}} \operatorname{Im}\left[\mathbf{d}_i\right] + \mathbf{B}_i^{\mathrm{T}} \operatorname{Im}\left[\mathbf{c}_i\right]\right), \quad \boldsymbol{\lambda}_i = \mathbf{A}_i^{\mathrm{T}} \operatorname{Re}\left[\mathbf{d}_i\right] + \mathbf{B}_i^{\mathrm{T}} \operatorname{Re}\left[\mathbf{c}_i\right]. \tag{45}$$

Now, according to the developed markers (44), (45), the equations (34) – (39) can be written as follows:

$$\mathbf{f}^{(1)}\left(z_*^{(1)}\right) = \mathbf{f}_\infty^{(1)}\left(z_*^{(1)}\right) + \frac{1}{2\pi i}\left[\mathbf{q}_1\left(z_*^{(1)}\right) + \sum_{\beta=1}^{3} \mathbf{I}_\beta\left(\mathbf{A}_1^{\mathrm{T}} \mathbf{m}\left(z_\beta^{(1)}\right) + \mathbf{B}_1^{\mathrm{T}} \mathbf{p}\left(z_\beta^{(1)}\right)\right) + \right.$$
$$\left. + \left\langle M_t\left(z_*^{(1)}\right)\right\rangle \boldsymbol{\mu}_1 - \left\langle P_t\left(z_*^{(1)}\right)\right\rangle \boldsymbol{\lambda}_1 + \eta_0 \left\langle m_t\left(z_*^{(1)}\right)\right\rangle \overline{\boldsymbol{\mu}}_1 \right]; \tag{46}$$

$$\bar{\mathbf{q}}_1\left(z_\beta^{(1)}\right)+\bar{\mathbf{A}}_1^{\mathrm{T}}\mathbf{m}\left(z_\beta^{(1)}\right)+\bar{\mathbf{B}}_1^{\mathrm{T}}\mathbf{p}\left(z_\beta^{(1)}\right)+M_t\left(z_\beta^{(1)}\right)\bar{\boldsymbol{\mu}}_1-P_t\left(z_\beta^{(1)}\right)\bar{\boldsymbol{\lambda}}_1+\eta_0 p_t\left(z_\beta^{(1)}\right)\bar{\boldsymbol{\mu}}_1=0; \quad (47)$$

$$\mathbf{q}_2\left(z_\beta^{(1)}\right)-\mathbf{A}_2^{\mathrm{T}}\mathbf{m}\left(z_\beta^{(1)}\right)-\mathbf{B}_2^{\mathrm{T}}\mathbf{p}\left(z_\beta^{(1)}\right)-M_t\left(z_\beta^{(1)}\right)\boldsymbol{\mu}_2+P_t\left(z_\beta^{(1)}\right)\boldsymbol{\lambda}_2=0; \quad (48)$$

$$\begin{gathered}\mathbf{f}^{(2)}\left(z_*^{(2)}\right)=\mathbf{f}_\infty^{(2)}\left(z_*^{(2)}\right)+\frac{1}{2\pi i}\Big[\mathbf{q}_2\left(z_*^{(2)}\right)-\\ -\sum_{\beta=1}^{3}\mathbf{I}_\beta\left(\mathbf{A}_2^{\mathrm{T}}\mathbf{m}\left(z_\beta^{(2)}\right)+\mathbf{B}_2^{\mathrm{T}}\mathbf{p}\left(z_\beta^{(2)}\right)\right)-\left\langle M_t\left(z_*^{(2)}\right)\right\rangle\boldsymbol{\mu}_2+\left\langle P_t\left(z_*^{(2)}\right)\right\rangle\boldsymbol{\lambda}_2\Big];\end{gathered} \quad (49)$$

$$\mathbf{q}_1\left(z_\beta^{(2)}\right)+\mathbf{A}_1^{\mathrm{T}}\mathbf{m}\left(z_\beta^{(2)}\right)+\mathbf{B}_1^{\mathrm{T}}\mathbf{p}\left(z_\beta^{(2)}\right)+M_t\left(z_\beta^{(2)}\right)\boldsymbol{\mu}_1-P_t\left(z_\beta^{(2)}\right)\boldsymbol{\lambda}_1+\eta_0 p_t\left(z_\beta^{(2)}\right)\boldsymbol{\mu}_1=0; \quad (50)$$

$$\bar{\mathbf{q}}_2\left(z_\beta^{(2)}\right)-\bar{\mathbf{A}}_2^{\mathrm{T}}\mathbf{m}\left(z_\beta^{(2)}\right)-\bar{\mathbf{B}}_2^{\mathrm{T}}\mathbf{p}\left(z_\beta^{(2)}\right)-M_t\left(z_\beta^{(2)}\right)\bar{\boldsymbol{\mu}}_2+P_t\left(z_\beta^{(2)}\right)\bar{\boldsymbol{\lambda}}_2=0, \quad (51)$$

where $\mathbf{I}_\beta=\operatorname{diag}\left[\delta_{1\beta},\delta_{2\beta},\delta_{3\beta}\right]$.

The equations (47), (48), (50), and (51) facilitate description of extrinsic integrals (41) through the integrals in accordance with the contours Γ_j:

$$\begin{gathered}\mathbf{m}\left(z_\beta^{(1)}\right)=\left(\bar{\mathbf{A}}_1\bar{\mathbf{B}}_1^{-1}-\mathbf{A}_2\mathbf{B}_2^{-1}\right)^{-\mathrm{T}}\left(\bar{\mathbf{B}}_1^{-\mathrm{T}}\mathbf{y}_1\left(z_\beta^{(1)}\right)-\mathbf{B}_2^{-\mathrm{T}}\mathbf{y}_2\left(z_\beta^{(1)}\right)\right),\\ \mathbf{p}\left(z_\beta^{(1)}\right)=\left(\bar{\mathbf{B}}_1\bar{\mathbf{A}}_1^{-1}-\mathbf{B}_2\mathbf{A}_2^{-1}\right)^{-\mathrm{T}}\left(\bar{\mathbf{A}}_1^{-\mathrm{T}}\mathbf{y}_1\left(z_\beta^{(1)}\right)-\mathbf{A}_2^{-\mathrm{T}}\mathbf{y}_2\left(z_\beta^{(1)}\right)\right),\\ \mathbf{y}_1\left(z_\beta^{(1)}\right)=-\bar{\mathbf{q}}_1\left(z_\beta^{(1)}\right)-M_t\left(z_\beta^{(1)}\right)\bar{\boldsymbol{\mu}}_1+P_t\left(z_\beta^{(1)}\right)\bar{\boldsymbol{\lambda}}_1-\eta_0 p_t\left(z_\beta^{(1)}\right)\bar{\boldsymbol{\mu}}_1,\\ \mathbf{y}_2\left(z_\beta^{(1)}\right)=\mathbf{q}_2\left(z_\beta^{(1)}\right)-M_t\left(z_\beta^{(1)}\right)\boldsymbol{\mu}_2+P_t\left(z_\beta^{(1)}\right)\boldsymbol{\lambda}_2;\end{gathered} \quad (52)$$

$$\begin{gathered}\mathbf{m}\left(z_\beta^{(2)}\right)=\left(\bar{\mathbf{A}}_2\bar{\mathbf{B}}_2^{-1}-\mathbf{A}_1\mathbf{B}_1^{-1}\right)^{-\mathrm{T}}\left(\bar{\mathbf{B}}_2^{-\mathrm{T}}\mathbf{y}_3\left(z_\beta^{(2)}\right)-\mathbf{B}_1^{-\mathrm{T}}\mathbf{y}_4\left(z_\beta^{(2)}\right)\right),\\ \mathbf{p}\left(z_\beta^{(2)}\right)=\left(\bar{\mathbf{B}}_2\bar{\mathbf{A}}_2^{-1}-\mathbf{B}_1\mathbf{A}_1^{-1}\right)^{-\mathrm{T}}\left(\bar{\mathbf{A}}_2^{-\mathrm{T}}\mathbf{y}_3\left(z_\beta^{(2)}\right)-\mathbf{A}_1^{-\mathrm{T}}\mathbf{y}_4\left(z_\beta^{(2)}\right)\right),\\ \mathbf{y}_3\left(z_\beta^{(2)}\right)=\bar{\mathbf{q}}_2\left(z_\beta^{(2)}\right)-M_t\left(z_\beta^{(2)}\right)\bar{\boldsymbol{\mu}}_2+P_t\left(z_\beta^{(2)}\right)\bar{\boldsymbol{\lambda}}_2,\\ \mathbf{y}_4\left(z_\beta^{(2)}\right)=-\mathbf{q}_1\left(z_\beta^{(2)}\right)-M_t\left(z_\beta^{(2)}\right)\boldsymbol{\mu}_1+P_t\left(z_\beta^{(2)}\right)\boldsymbol{\lambda}_1-\eta_0 p_t\left(z_\beta^{(2)}\right)\boldsymbol{\mu}_1.\end{gathered} \quad (53)$$

Having input the obtained solution (52), (53) into the equations (46), (49), we will obtain Cauchy's integral formulae for the bi-material with non-ideal thermal interaction that do not contain any integrals along perpetual interval of integration:

$$\begin{gathered}\mathbf{f}^{(1)}\left(z_*^{(1)}\right)=\mathbf{f}_\infty^{(1)}\left(z_*^{(1)}\right)+\frac{1}{2\pi i}\Bigg[q_1\left(z_*^{(1)}\right)+\sum_{\beta=1}^{3}\mathbf{I}_\beta\left(\mathbf{G}_1^{(1)}\bar{q}_1(z_\beta^{(1)})+\mathbf{G}_2^{(1)}q_2(z_\beta^{(1)})\right)+\\ +\left\langle\bar{Q}_t^{(1)}\left(z_*^{(1)}\right)\right\rangle\boldsymbol{\delta}_1^{(1)}+\left\langle Q_t^{(2)}\left(z_*^{(1)}\right)\right\rangle\boldsymbol{\delta}_2^{(1)}+\left\langle e_t^{(2)}\left(z_*^{(1)}\right)\right\rangle\boldsymbol{\kappa}_2^{(1)}+\left\langle\bar{e}_t^{(1)}\left(z_*^{(1)}\right)\right\rangle\boldsymbol{\kappa}_1^{(1)}\Bigg];\end{gathered} \quad (54)$$

$$\begin{gathered}\mathbf{f}^{(2)}\left(z_*^{(2)}\right)=\mathbf{f}_\infty^{(2)}\left(z_*^{(2)}\right)+\frac{1}{2\pi i}\Bigg[q_2\left(z_*^{(2)}\right)-\sum_{\beta=1}^{3}\mathbf{I}_\beta\left(\mathbf{G}_1^{(2)}q_1(z_\beta^{(2)})+\mathbf{G}_2^{(2)}\bar{q}_2(z_\beta^{(2)})\right)+\\ +\left\langle Q_t^{(1)}\left(z_*^{(2)}\right)\right\rangle\boldsymbol{\delta}_1^{(2)}+\left\langle\bar{Q}_t^{(2)}\left(z_*^{(2)}\right)\right\rangle\boldsymbol{\delta}_2^{(2)}+\left\langle e_t^{(1)}\left(z_*^{(2)}\right)\right\rangle\boldsymbol{\kappa}_1^{(2)}+\left\langle\bar{e}_t^{(2)}\left(z_*^{(2)}\right)\right\rangle\boldsymbol{\kappa}_2^{(2)}\Bigg],\end{gathered} \quad (55)$$

where

$$\boldsymbol{\kappa}_i^{(j)} = \begin{cases} -\boldsymbol{\delta}_i^{(j)} - 2ik_t^{(j)}\left(\boldsymbol{\mu}_i + \mathbf{G}_i^{(j)}\bar{\boldsymbol{\mu}}_i\right) \ (i = j), \\ -\boldsymbol{\delta}_i^{(j)} + 2ik_t^{(i)}\mathbf{G}_i^{(j)}\boldsymbol{\mu}_i, \ (i \neq j) \end{cases} \quad (56)$$

and $\boldsymbol{\delta}_i^{(j)}$ are the same as in [11].

The obtained equations (54), (55) facilitate writing the correlations that link stresses and transpositions at any point of thermo-elastic bi-material with values on contour Γ_i of temperature, thermal stream, transposition and stress vectors. Eventually, having used (3), (4), (54), (55), we will get Somigliana type integral formulae for bi-material with non-ideal thermal interaction between its constituents.

$$\begin{aligned} \mathbf{u}(\xi) &= \mathbf{u}^{\infty}(\xi) + \int_{\Gamma}\left[\mathbf{U}^{\text{bm}}(\mathbf{x},\xi)\mathbf{t}(\mathbf{x}) - \mathbf{T}^{\text{bm}}(\mathbf{x},\xi)\mathbf{u}(\mathbf{x}) + \mathbf{r}^{\text{HCI}}(\mathbf{x},\xi)\theta(\mathbf{x}) + \mathbf{v}^{\text{HCI}}(\mathbf{x},\xi)h_n(\mathbf{x})\right]ds(\mathbf{x}), \\ \boldsymbol{\sigma}_j(\xi) &= \boldsymbol{\sigma}_j^{\infty}(\xi) + \int_{\Gamma}\left[\mathbf{D}_j^{\text{bm}}(\mathbf{x},\xi)\mathbf{t}(\mathbf{x}) - \mathbf{S}_j^{\text{bm}}(\mathbf{x},\xi)\mathbf{u}(\mathbf{x}) + \mathbf{q}_j^{\text{HCI}}(\mathbf{x},\xi)\theta(\mathbf{x}) + \mathbf{w}_j^{\text{HCI}}(\mathbf{x},\xi)h_n(\mathbf{x})\right]ds(\mathbf{x}), \end{aligned} \quad (57)$$

where nucleuses $\mathbf{U}^{\text{bm}}(\mathbf{x},\xi)$, $\mathbf{T}^{\text{bm}}(\mathbf{x},\xi)$, $\mathbf{D}_j^{\text{bm}}(\mathbf{x},\xi)$, $\mathbf{S}_j^{\text{bm}}(\mathbf{x},\xi)$ are the same as for bi-material with ideal contact between semi-plates [5], and others are marked with such correlations:

$\mathbf{x} \in S_1 \wedge \xi \in S_1$:

$$\begin{aligned} \mathbf{v}^{\text{HCI}}(\mathbf{x},\xi) = \frac{1}{\pi}\operatorname{Im}\Bigg\{\mathbf{A}_1\Bigg[&-\left\langle f^*\left(W_*^{(1,1)}\right)\right\rangle\boldsymbol{\mu}_1 - \sum_{\beta=1}^{3}\left\langle f^*\left(\bar{W}_\beta^{(1,1)}\right)\right\rangle\mathbf{G}_1^{(1)}\mathbf{I}_\beta\bar{\boldsymbol{\mu}}_1 + \\ &+\frac{i}{2k_t^{(1)}}\left\langle f^*\left(\bar{W}_{t*}^{(1,1)}\right)\right\rangle\boldsymbol{\delta}_1^{(1)} - \frac{i}{2k_t^{(1)}B^{(1)}}\mathcal{I}^{(1)}\left(\bar{W}_{t*}^{(1,1)}\right)\boldsymbol{\kappa}_1^{(1)}\Bigg] - \\ &-\frac{i\mathbf{c}_1}{2k_t^{(1)}}\left[f^*\left(W_t^{(1,1)}\right) + Kf^*\left(\bar{W}_t^{(1,1)}\right)\right] + \frac{\mathbf{c}_1}{B^{(2)}\eta_0}\mathcal{I}^{(1)}\left(\bar{W}_t^{(1,1)}\right)\Bigg\}; \end{aligned}$$

$\mathbf{x} \in S_2 \wedge \xi \in S_1$:

$$\begin{aligned} \mathbf{v}^{\text{HCI}}(\mathbf{x},\xi) = -\frac{1}{\pi}\operatorname{Im}\Bigg\{\mathbf{A}_1\Bigg[&\sum_{\beta=1}^{3}\left\langle f^*\left(W_\beta^{(2,1)}\right)\right\rangle\mathbf{G}_2^{(1)}\mathbf{I}_\beta\boldsymbol{\mu}_2 + \frac{i}{2k_t^{(2)}}\left\langle f^*\left(W_{t*}^{(2,1)}\right)\right\rangle\boldsymbol{\delta}_2^{(1)} + \\ &+\frac{i}{2k_t^{(2)}B^{(1)}}\mathcal{I}^{(1)}\left(W_{t*}^{(2,1)}\right)\boldsymbol{\kappa}_2^{(1)}\Bigg] + \frac{i\mathbf{c}_1(1-K)}{2k_t^{(2)}}f^*\left(W_t^{(2,1)}\right) + \frac{i(K-1)\mathbf{c}_1}{2k_t^{(2)}B^{(1)}}\mathcal{I}^{(1)}\left(W_t^{(2,1)}\right)\Bigg\}; \end{aligned}$$

$\mathbf{x} \in S_1 \wedge \xi \in S_2$:

$$\begin{aligned} \mathbf{v}^{\text{HCI}}(\mathbf{x},\xi) = \frac{1}{\pi}\operatorname{Im}\Bigg\{\mathbf{A}_1\Bigg[&\sum_{\beta=1}^{3}\left\langle f^*\left(W_\beta^{(1,2)}\right)\right\rangle\mathbf{G}_1^{(2)}\mathbf{I}_\beta\boldsymbol{\mu}_1 - \\ &-\frac{i}{2k_t^{(1)}}\left\langle f^*\left(W_{t*}^{(1,2)}\right)\right\rangle\boldsymbol{\delta}_1^{(2)} + \frac{i}{2k_t^{(1)}B^{(2)}}\mathcal{I}^{(2)}\left(W_{t*}^{(1,2)}\right)\boldsymbol{\kappa}_1^{(2)}\Bigg] - \\ &-\frac{i\mathbf{c}_2}{2k_t^{(1)}}(1+K)f^*\left(W_t^{(1,2)}\right) - \frac{i\mathbf{c}_2(K+1)}{2k_t^{(1)}B^{(2)}}\mathcal{I}^{(2)}\left(W_t^{(1,2)}\right)\Bigg\}; \end{aligned}$$

$\mathbf{x} \in S_2 \wedge \xi \in S_2$:

$$\mathbf{v}^{\mathrm{HCI}}(\mathbf{x},\xi)=\frac{1}{\pi}\operatorname{Im}\Big\{\mathbf{A}_2\Big[-\Big\langle f^*\big(W_*^{(2,2)}\big)\Big\rangle\boldsymbol{\mu}_2+\sum_{\beta=1}^{3}\Big\langle f^*\big(\bar{W}_\beta^{(2,2)}\big)\Big\rangle\mathbf{G}_2^{(2)}\mathbf{I}_\beta\bar{\boldsymbol{\mu}}_2+$$
$$+\frac{i}{2k_t^{(2)}}\Big\langle f^*\big(\bar{W}_{t*}^{(2,2)}\big)\Big\rangle\boldsymbol{\delta}_2^{(2)}-\frac{i}{2k_t^{(2)}B^{(2)}}\mathcal{I}^{(2)}\big(\bar{W}_{t*}^{(2,2)}\big)\boldsymbol{\kappa}_2^{(2)}\Big]- \tag{58}$$
$$-\frac{i\mathbf{c}_2}{2k_t^{(2)}}\Big[f^*\big(W_t^{(2,2)}\big)-Kf^*\big(\bar{W}_t^{(2,2)}\big)\Big]-\frac{i\mathbf{c}_2(1-K)}{2k_t^{(2)}B^{(2)}}\mathcal{I}^{(2)}\big(\bar{W}_t^{(2,2)}\big)\Big\},$$

and expressions

$$r_i^{\mathrm{HCI}}(\mathbf{x},\xi)=k_{pq}v_{i,q}^{\mathrm{HCI}}(\mathbf{x},\xi)\,n_p(\mathbf{x}),$$
$$q_{ij}^{\mathrm{HCI}}(\mathbf{x},\xi)=-C_{ijmp}r_{m,p}^{\mathrm{HCI}}(\mathbf{x},\xi)+\beta_{ij}H^{\mathrm{HCI}*}(\mathbf{x},\xi),\ w_{ij}^{\mathrm{HCI}}(\mathbf{x},\xi)=-C_{ijmp}v_{m,p}^{\mathrm{HCI}}(\mathbf{x},\xi)-\beta_{ij}\Theta^{\mathrm{HCI}*}(\mathbf{x},\xi). \tag{59}$$

In variable-free expressions for nucleuses there were used the following markers:

$$\boldsymbol{\rho}_j=\mathbf{A}_j^T\operatorname{Re}[p_t^{(j)}\mathbf{d}_j]+\mathbf{B}_j^T\operatorname{Re}[p_t^{(j)}\mathbf{c}_j],\ \mathbf{n}^{(i)}=\boldsymbol{\lambda}_i n_2(\mathbf{x})-\boldsymbol{\rho}_i n_1(\mathbf{x}),\ Z_*^{(i)}(\mathbf{x})=x_1+p_*^{(i)}x_2,$$
$$W_*^{(i,j)}=Z_*^{(i)}(\mathbf{x})-Z_*^{(j)}(\xi),\ W_\beta^{(i,j)}=Z_\beta^{(i)}(\mathbf{x})-Z_*^{(j)}(\xi),\ \bar{W}_\beta^{(i,j)}=\bar{Z}_\beta^{(i)}(\mathbf{x})-Z_*^{(j)}(\xi),$$
$$W_{t*}^{(i,j)}=Z_t^{(i)}(\mathbf{x})-Z_*^{(j)}(\xi),\ \bar{W}_{t*}^{(i,j)}=\bar{Z}_t^{(i)}(\mathbf{x})-Z_*^{(j)}(\xi),\ \delta_*^{(j)}=\Big\langle\delta_{i2}-\delta_{i1}p_*^{(j)}\Big\rangle.$$

The correlations (58) and (59) indicate that in order to determine the temperature, thermal stream, transpositions and stresses at any point of bi-material it is necessary that marginal conditions on contours Γ would have been determined for all components of the functions $\theta(\mathbf{x})$, $h_n(\mathbf{x})$, $\mathbf{u}(\mathbf{x})$ and $\mathbf{t}(\mathbf{x})$ $(\forall\mathbf{x}\in\Gamma)$. However, during setting of boundary problems they input only a mutually sequenced half of these components. To determine other unknown boundary values we will apply Sokhotski-Plemelj formula [13], which links the boundary function value with its main value. Thus, according to (58), (59) and [13, 15] for smooth close-end contours Γ in thermo-elastic bi-material we obtain the following integral equations to determine previously unknown boundary functions values:

$$\frac{1}{2}\theta(\mathbf{y})=\theta^\infty(\mathbf{y})+\mathrm{RPV}\int_\Gamma\Theta^{\mathrm{HCI}*}(\mathbf{x},\mathbf{y})h_n(\mathbf{x})ds(\mathbf{x})-\mathrm{CPV}\int_\Gamma H^{\mathrm{HCI}*}(\mathbf{x},\mathbf{y})\theta(\mathbf{x})ds(\mathbf{x}),$$
$$\frac{1}{2}\mathbf{u}(\mathbf{y})=\mathbf{u}^\infty(\xi)+\mathrm{RPV}\int_\Gamma\mathbf{U}^{\mathrm{bm}}(\mathbf{x},\mathbf{y})\mathbf{t}(\mathbf{x})ds(\mathbf{x})-\mathrm{CPV}\int_\Gamma\mathbf{T}^{\mathrm{bm}}(\mathbf{x},\mathbf{y})\mathbf{u}(\mathbf{x})ds(\mathbf{x})+ \tag{60}$$
$$+\mathrm{RPV}\int_\Gamma\mathbf{r}^{\mathrm{HCI}}(\mathbf{x},\mathbf{y})\theta(\mathbf{x})ds(\mathbf{x})+\int_\Gamma\mathbf{v}^{\mathrm{HCI}}(\mathbf{x},\mathbf{y})h_n(\mathbf{x})ds(\mathbf{x}),$$

where RPV – the main value of extrinsic integral; CPV – the main value of Cauchy's integral.

Integral equations (60) become extinct when certain close-end contours Γ_j of line Γ become the boundaries of mathematical cuts Γ_{Cj} (simple open-end semicircular arcs). In this case one has to apply the theory of dual hyper-singular equations [8, 14, 15] and integrals due to corresponding contours Γ_j are transformed into the integrals due to Γ_{Cj}, where the temperature, thermal stream, stress and transposition vectors are associated with multiplication factors around the nucleuses.

According to [16] the functions of stresses and transpositions leaps around the tops of thin inhomogeneities, which are not located within dissimilar anisotropic medium, describe generalized rates of stresses intensity (RSI) that characterize the first terms of asymptotic dependence within physical-mechanical fields around the fronts of thin inhomogeneities:

$$\mathbf{k}^{(1)} = \lim_{s\to 0}\sqrt{\frac{\pi}{8s}}\mathbf{L}\cdot\Delta\mathbf{u}(s),\ \ \mathbf{k}^{(2)} = -\lim_{s\to 0}\sqrt{\frac{\pi s}{2}}\Sigma\mathbf{t}(s). \qquad (61)$$

Here $\mathbf{k}^{(1)}$, $\mathbf{k}^{(2)}$ – vectors of generalized RSI K_{ij} [14-16]; $\mathbf{L} = -2\sqrt{-1}\mathbf{B}\mathbf{B}^{\mathrm{T}}$ – Barnett – Lothe real tensor [9].

Integral equations (60) alongside with certain mathematical model of thin thermo-elastic component [14] facilitate solving 2D-problems of thermo-elasticity for unlimited or limited bi-material body with non-ideal thermal interaction between components containing thin inhomogeneities or holes with predetermined arbitrary marginal conditions on their surface.

Analysis of numerical results. To solve certain 2D-problems for bi-material with coherent interface of high thermo-conductivity and internal inhomogeinities one should apply integral equations (60), mathematical model of intercalation [14] (or any other) and modified method of boundary elements [14, 15]. There was investigated an example of anisotropic bi-material with similar mechanical and thermal properties of its components and isotropic rigid intercalation $2a$ long and $2h = 0,02a$ thick, on the angle α up to the boundary oh high thermo-conductivity in semi-plane $x_2 > 0$ (Figure. 2). The center of inclusion is located at the distance d to dissimilar anisotropic medium. In semi-plane $x_2 < 0$ at similar distance d from dissimilar anisotropic medium and at distance a to the right and to the left from omitted inclusion center to the boundary of perpendicular there are located the source and drainage of thermal energy that is the same in terms of intensity value. The bi-material components are made of anisotropic fiberglass with the following properties: $E_1 = 55$ GPa, $E_2 = 21$ GPa, $G_{12} = 9,7$ GPa, $\nu_{12} = 0,25$, $\alpha_{11} = 6,3\cdot 10^{-6}$ K^{-1}, $\alpha_{22} = 2,0\cdot 10^{-5}$ K^{-1}, $k_{11} = 3,46$ W/(m·K), $k_{22} = 0,35$ W/(m·K). The value of these material constants correspond to directions of main axes of orthotropy converging with axes of coordinates ($\alpha = 0$). In calculations for distribution of inclusions we used only 20 3-nodal boundary elements including 2 special final ones.

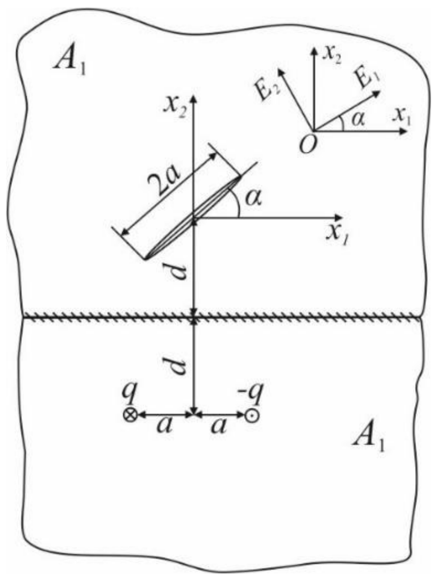

Figure 2. Bimaterial with identical properties of components

Research results. The figures below display the dependence between generalized RSI and parameter μ ($\eta_0 = k_{11} \cdot a \cdot 10^{\mu}$), which describes the rate of non-ideal thermal interaction. RSIs are limited with $K_0 = \sqrt{\pi a} \cdot E_1 \cdot \alpha_{11} / k_{11} \cdot q$. The graphs are made for different values of non-dimensional parameter $d_0 = a/d$, tilting angle of inclusion α and for fixed value of relative rigidness of inclusion $k = 10^5$.

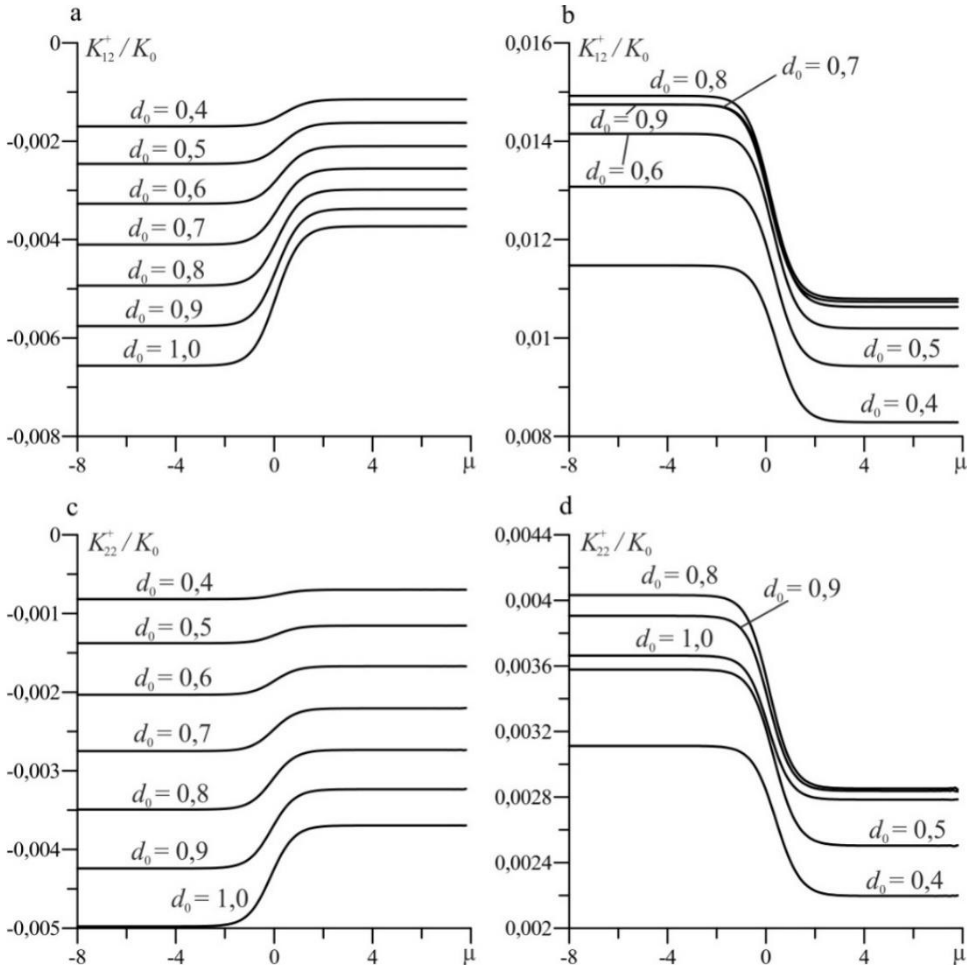

Figure 3. Stress intensity factors for the inclusion inclination angel $\alpha = 0^{\circ}$ (a, c) and $\alpha = 15^{\circ}$ (b, d)

Figure 3 – 6 shows that the closer the source and drainage of thermal energy and inclusion to bi-material dissimilar anisotropic medium, the bigger RSIs are. The growth of inclination angle α of inclusion at first increase all RSIs, and their maximal values are reached at $\alpha = 45°$. Remarkably that for all RSIs the biggest growth takes place when parameter μ of limited contact resistance of bi-material dissimilar anisotropic medium is around zero (ideal thermal interaction).

In μ it should be mentioned that at $\alpha = 30°$ RSI K_{12} changes its behavior comparing to $\alpha = 0°$ and $\alpha = 15°$, and the growth of value of limited contact resistance μ results in decrease of the RSIs at $\alpha \geq 30°$.

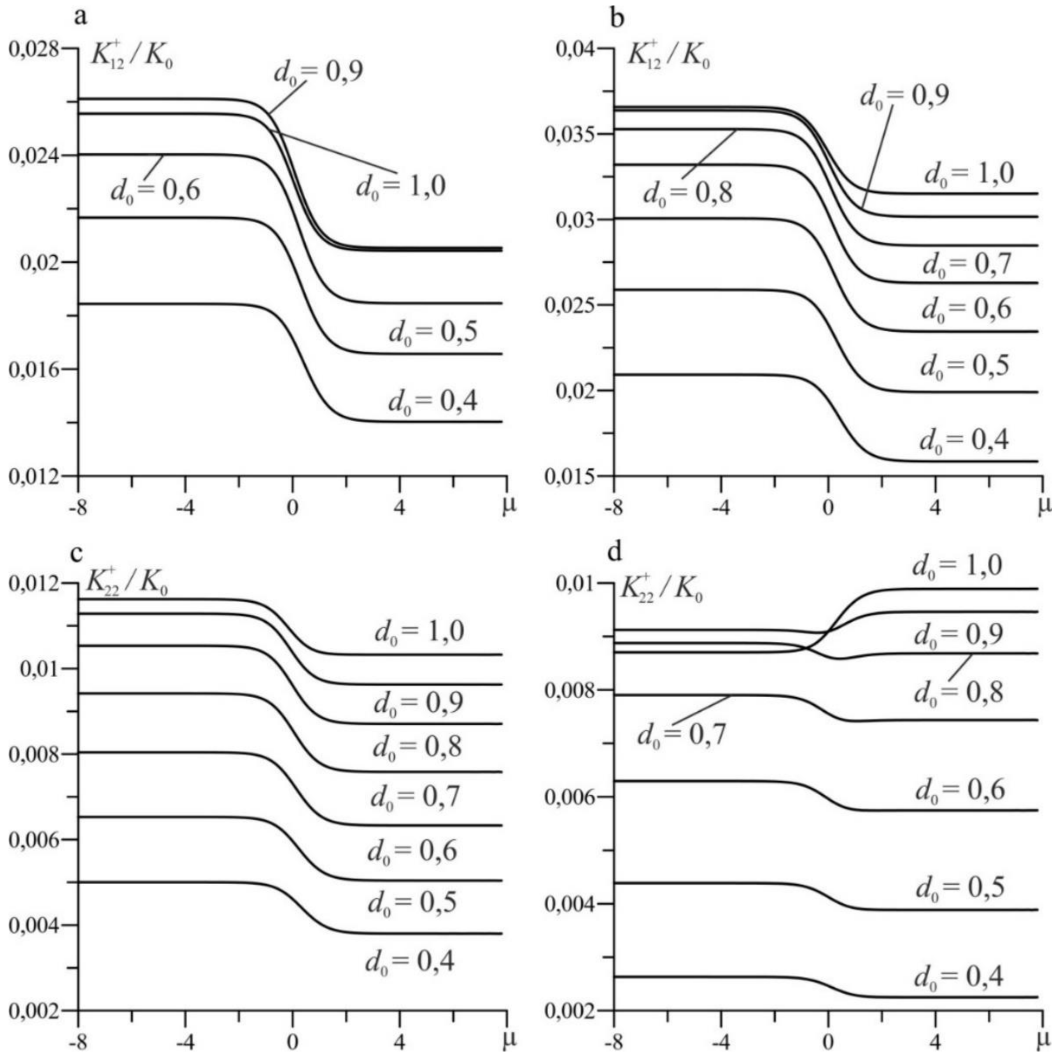

Figure 4. Stress intensity rates for the inclusion inclination angle $\alpha = 30^{\circ}$ (a, c) and $\alpha = 45^{\circ}$ (b, d)

Figure 4 shows that in case $\alpha = 45°$ the growth of distance between center of inclusion and contact boundary results in smooth alteration of behavior of intensity rate K_{22} : the growth of μ parameter is followed with growth of RSIs and d_0 parameter.

Figure 5 displays that growth of inclusion inclination angle (α) and distance from its center to dissimilar anisotropic medium (d_0) is followed with RSI maximal values at $d_0 \approx 0,8$, and later they go down.

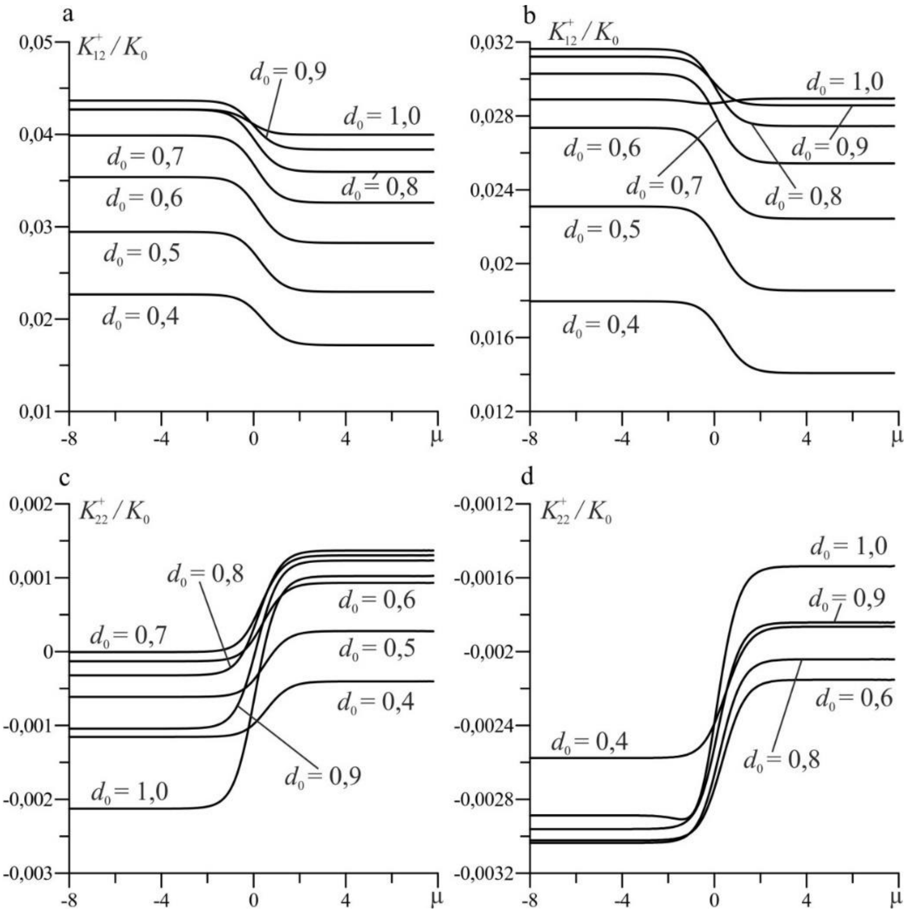

Figure 5. Stress intensity rates for the inclusion inclination angel $\alpha = 60^{\circ}$ (a, c) and $\alpha = 75^{\circ}$ (b, d)

If the inclusion is perpendicular to the interface ($\alpha = 90°$) RSI values are minimal (Figure 6), if RSI d_0 parameter grows, they also increase unlike all previous cases.

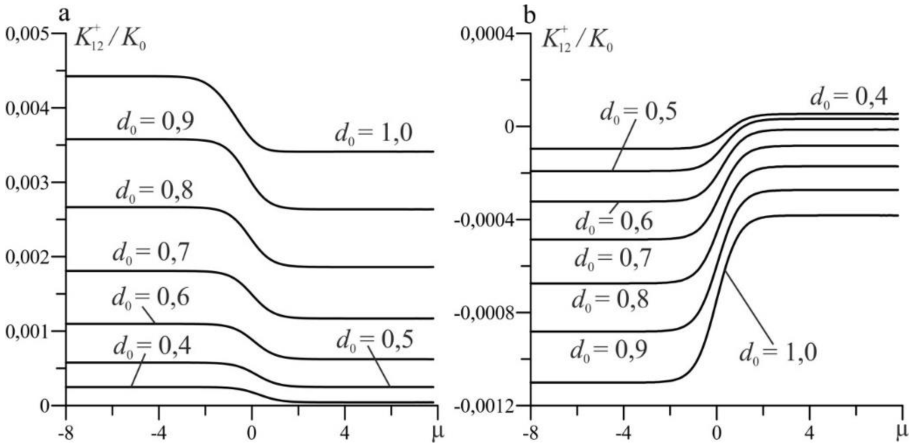

Figure 6. Stress intensity rates for the inclusion inclination angle $\alpha = 90^{\circ}$

Conclusions. The article deals with the comprehensive research of Somigliana type integral formulae and boundary integral equations for 2D-problem of thermo-elasticity of anisotropic bi-material with high thermo-conductivity of dissimilar anisotropic medium and ideal mechanical interaction between them in case of available internal holes or/and thin inhomogeinities like cracks particularly. The nucleuses of all integral equations (Green's functions that consider the peculiarities of interaction between thermo-elastic anisotropic semi-planes) were represented in explicit close-end form. The combination of obtained integral equations with dependences of mathematical models of intercalations, marginal thermal and mechanical conditions on holes boundaries as well as application of the scheme for modified boundary method enables efficient solving of practically arbitrary problems both for unlimited and limited bodies.

References

1. Kaessmair S., Javili A., Steinmann P. Thermomechanics of solids with general imperfect coherent interfaces, Archive of Applied Mechanics, vol. 84 (9-11), 2014, pp. 1409 – 1426.
2. Benvensite Y. A general interface model for a three-dimensional curved thin anisotropic interphace between two anisotropic media, J. Mech. Phys. Solids, vol. 54, 2006, pp. 708 – 734.
3. Chen T. Thermal conduction of a circular inclusion with variable interface parameter, Int. J. Solids. Struct., vol. 38, 2001, pp. 3081 – 3097.
4. Hwu C. Anisotropic elastic plates, London, Springer, 2010, 673 p.
5. Kattis M.A., Mayroyannis A. Feeble interfaces in biomaterials, Acta Mech., vol. 185, 2006, pp. 11 – 29.
6. Hwu C. Thermoelastic interface crack problems in dissimilar anisotropic media, Int. J. Solids. Struct., vol. 18, 1992, pp. 2077 – 2090.
7. Pan E., Amadei B. Boundary element analysis of fracture mechanics in anisotropic bimaterials, Engineering Analysis with Boundary Elements, vol. 23, 1999, pp. 683 – 691.
8. Pasternak Ia., Pasternak R., Sulym H. Boundary integral equations and Green's functions for 2D thermoelectroelastic biomaterial, Engineering Analysis with Boundary Elements, vol. 48, 2014, pp. 87–101.
9. Wang X., Pan E. Thermal Green's functions in plane anisotropic bimaterials with spring-type and Kapitza-type imperfect interface, Acta. Mech., vol. 209, 2010, pp. 115 – 128.
10. Qin Q. Green's function and boundary elements of multifield materials, Oxford, Elsevier, 2007, 254 p.
11. Pasternak Ia., Sulym H, Tomashivskyy M. Boundary integral equations for an anisotropic bimaterial with thermally imperfect interface and internal inhomogeneities, Acta. Mech, vol. 10, 2016, pp. 66 – 74.
12. Ting T.C. Anisotropic elasticity: theory and applications, New York, Oxford University Press, 1996, 567 p.
13. Muskhelyshvyky N.Y. Sunhuliarnie ynterhralnie uravnenyia, Moskva, Nauka, 1968, 512 p. [in Russian].
14. Pasternak Ia. Boundary integral equations and the boundary element method for fracture mechanics analysis in 2D anisotropic thermoelasticity, Engineering Analysis with Boundary Elements, vol. 36, no. 12, pp. 1931 – 1941.
15. Pasternak Ia., Pasternak R., Sulym H. A comprehensive study on the 2D boundary element method for anisotropic thermoelectroelastic solids with cracks and thin inhomogeneities, Engineering Analysis with Boundary Elements, vol. 37, no. 2, 2013, pp. 419 – 433.
16. Sulym H.T. Osnovy matematychoi teorii termopruzhoi rivnovahy deformivnykh tverdykh til z tonkymy vkliuchenniamy, Lviv, Doslidno-vydavnychyi tsentr NTSh, 2007, 716 p. [in Ukrainian].

Список використаної літератури

1. Kaessmair, S. Thermomechanics of solids with general imperfect coherent interfaces. S. Kaessmair, A. Javili, P. Steinmann, Archive of Applied Mechanics. 2014, vol. 84 (9-11), pp. 1409 – 1426.
2. Benvensite, Y. A general interface model for a three-dimensional curved thin anisotropic interphace between two anisotropic media. Y. Benvensite, J. Mech. Phys. Solids. 2006, vol. 54, pp. 708 – 734.
3. Chen, T. Thermal conduction of a circular inclusion with variable interface parameter. T. Chen , Int. J. Solids. Struct. 2001, vol. 38, pp. 3081 – 3097.
4. Hwu, C. Anisotropic elastic plates. C. Hwu, London: Springer, 2010, 673 p.
5. Kattis, M. A. Feeble interfaces in biomaterials. M.A. Kattis, G. Mayroyannis, Acta Mech. 2006, vol. 185, pp. 11 – 29.
6. Hwu, C. Thermoelastic interface crack problems in dissimilar anisotropic media. C. Hwu, Int. J. Solids. Struct. 1992, vol. 18, pp. 2077 – 2090.
7. Pan, E. Boundary element analysis of fracture mechanics in anisotropic biomaterials. E. Pan, B. Amadei, Engineering Analysis with Boundary Elements. 1999, vol. 23, pp. 683 – 691.
8. Pasternak, Ia. Boundary integral equations and Green's functions for 2D thermoelectroelastic biomaterial. Ia. Pasternak, R. Pasternak, H. Sulym, Engineering Analysis with Boundary Elements. 2014, vol. 48, pp. 87 – 101.

9. Wang, X. Thermal Green's functions in plane anisotropic bimaterials with spring-type and Kapitza-type imperfect interface. X. Wang, E. Pan, Acta. Mech. 2010, vol. 209, pp. 115 – 128.
10. Qin, Q. Green's function and boundary elements of multifield materials. Q. Qin, Oxford: Elsevier, 2007, 254 p.
11. Pasternak, Ia. Boundary integral equations for an anisotropic bimaterial with thermally imperfect interface and internal inhomogeneities. Ia. Pasternak, H. Sulym, M. Tomashivskyy, Acta. Mech. 2016, vol. 10, pp. 66 – 74.
12. Ting, T.C. Anisotropic elasticity: theory and applications. New York: Oxford University Press, 1996, 567 p.
13. Мусхелишвили, Н.И. Сингулярные интегральные уравнения [Текст] / Н.И. Мусхелишвили. – Изд. 3-е, испр. и дополн. – М.: Наука, 1968. – 512 с.
14. Pasternak, Ia. Boundary integral equations and the boundary element method for fracture mechanics analysis in 2D anisotropic thermoelasticity. Ia. Pasternak, Engineering Analysis with Boundary Elements. 2012, vol. 36, no. 12, pp. 1931 – 1941.
15. Pasternak, Ia. A comprehensive study on the 2D boundary element method for anisotropic thermoelectroelastic solids with cracks and thin inhomogeneities. Ia. Pasternak, R. Pasternak, H. Sulym, Engineering Analysis with Boundary Elements. 2013, vol. 37, no. 2, pp. 419 – 433.
16. Сулим, Г.Т. Основи математичної теорії термопружної рівноваги деформівних твердих тіл з тонкими включеннями [Текст] / Г.Т. Сулим. – Львів: Дослідно-видавничий центр НТШ, 2007. – 716 с.

УДК 539.3

ІНТЕГРАЛЬНІ РІВНЯННЯ ДЛЯ ТЕРМОПРУЖНОГО АНІЗОТРОПНОГО БІМАТЕРІАЛУ ІЗ КОГЕРЕНТНИМ ІНТЕРФЕЙСОМ ВИСОКОЇ ТЕПЛОПРОВІДНОСТІ

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Резюме. *З використанням граничноелементного методу функцій стрибка розглянуто задачу плоскої термопружності анізотропного біматеріалу з високоінтенсивним неідеальним тепловим та ідеальним механічним контактом межі поділу базових матеріалів і внутрішніми тонкими включеннями, тріщинами або отворами, на межі яких можна задавати довільні температурні й механічні крайові умови (когерентний інтерфейс високої теплопровідності). Використовуючи розширений формалізм Стро та теорію функції комплексної змінної, отримано інтегральні співвідношення типу Сомільяни та відповідні крайові інтегральні рівняння для контурів отворів і серединних ліній (поверхонь) включень з ядрами, що містять функції Гріна, які автоматично враховують ефект інтерфейсу. Поєднання цих рівнянь із математичними моделями тонких деформівних включень та належним чином модифікованого методу граничних елементів дає можливість здійснити розрахунки фізикомеханічних полів та їхньої концентрації на неоднорідностях.*

Ключові слова: *анізотропний біматеріал, термопружність, неідеальний тепловий контакт, висока теплопровідність, тонке включення, тріщина.*

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