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DETERMINATION OF THE CHARACTERISTIC FUNCTION OF DISCRETE-TIME CONDITIONAL LINEAR RANDOM PROCESS AND ITS APPLICATION

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Summary. *The discrete-time conditional linear random process is defined, and its properties in the context of application for mathematical modelling of information signals in energy and medicine are analyzed. The relation to the continuous-time counterpart is considered on the basis of time sampling and aggregation. One-dimensional and multidimensional characteristic functions of discrete-time conditional linear random process are obtained using conditional characteristic function approach. The conditions for the investigated model to be strict sense stationary are justified.*

Key words: *mathematical model, information signal, conditional linear random process, characteristic function, strict sense stationary process.*

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Statement of the problem. In the tasks of creating modern information systems and technologies, which are based on the methods of processing stochastic signals and noises, the substantiation of their mathematical models is of great importance. Linear [1–3] and conditional linear random processes (CLRP) [2–5] are important classes of models gaining considerable popularity. Such mathematical objects are used for mathematical and computer modeling of information signals, their generation mechanism makes it possible to represent them in the form of the sum of a large number of random impulses occurring at random moments of time. Such signals include, for example, electrophysiological signals of the brain, photoplethysmography signals, cardio signals, energy consumption processes (electricity, gas, water consumption), vibration signals of energy objects, radar signals, etc. [1, 2, 4, 6]. The development of the methodology of mathematical modeling of information signals using CLRP, as well as the construction of methods for their statistical analysis and forecasting in information systems for technical and medical purposes is vital scientific and applied problem.

Analysis of available investigation results. Due to the analysis of literary sources we can conclude that for the first time the concept of «conditionally linear random process» have been proposed by Percy A. Pierre [2] in the context of his researches carried out at RAND Corporation concerning the problem of mathematical modeling of radar signals and interferences. In paper [2] CLRP is defined in the form of stochastic integral of random kernel according to the Lévy process, and the possibility of using such model to study signals represented as the sum of a large number of stochastically dependent random impulses occurring at Poisson moments of time is also substantiated. In paper [6], similar processes where random pulses are represented in the form of deterministic functions with random stochastically dependent parameters have been investigated. Volatility modulated Lévy-driven Volterra processes and fields [7] can be considered as partial case of CLRP, where the kernel in the corresponding integral representation is the product of non-random function and nonnegative stochastic process. In papers [3, 5, 8] and others conditional linear random processes with continuous time are investigated by the method of characteristic functions, the conditions under which CLRP will be stationary or cyclostationary random process are substantiated, the possibilities of applying CLRP in the problems of mathematical modeling of information signals in medicine and power engineering are shown.

Discrete time CLRP, represented as stochastic sum with stationary generating white noise, was analyzed for the first time in paper [2]. In a number of theoretical works, the central limit problem concerning randomly weighted sums of random variables [9], linear processes with random coefficients [10], etc. have been investigated. In various literature sources, however, there is no analysis of the properties of multidimensional probability distributions for the general case of discrete time CLRP, which should be taken into account in applied problems of mathematical modeling and information signals processing.

The objective of the paper is to substantiate the expressions of one-dimensional and multi-dimensional characteristic functions of discrete time conditional linear random process in general case and to analyze their properties, particularly, to establish conditions under which the investigated process is stationary in strict sense.

Statement of the task. Thus, the tasks of the paper are to define the discrete-time CLRP, to obtain expressions of the conditional and unconditional characteristic functions of the process, making it possible to determine the conditions under which discrete-time CLRP will be stationary in the strict sense.

Discrete-time conditional linear random processes. First, let us give the definition of CLRP with continuous time [3, 5, 8]. That is, conditional linear random process (with continuous time) $\xi(\omega,t), \omega\in\Omega, t\in(-\infty,\infty)$ given in certain probability space $\{\Omega, \mathsf{F}, \mathbf{P}\}$ is defined as stochastic integral in the following form:

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)\,d\eta(\omega,\tau), \tag{1}$$

where $\varphi(\omega,\tau,t),\ \tau,t\in(-\infty,\infty)$ is real *random* function (kernel);
$\eta(\omega,\tau),\ \tau\in(-\infty,\infty)$, $\mathbf{P}(\eta(\omega,0)=0)=1$ is real Hilbert stochastically continuous random process with independent increments;
random functions $\varphi(\omega,\tau,t)$ і $\eta(\omega,\tau)$ are *stochastically independent*.

Discrete time real conditional linear random process $\xi_t(\omega)$, $t\in\mathbf{Z}$, $\omega\in\Omega$, is defined as random sequence of the following form:

$$\xi_t(\omega)=\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t}(\omega)\zeta_\tau(\omega), \tag{2}$$

where $\varphi_{\tau,t}(\omega)$, $t,\tau\in\mathbf{Z}$ is real *random* function (random matrix);
$\zeta_\tau(\omega)$, $\tau\in\mathbf{Z}$ is Hilbert sequence of infinitely divisible independent random variables (infinitely divisible white noise with discrete time);
random functions $\varphi_{\tau,t}(\omega)$ and $\zeta_\tau(\omega)$ are stochastically independent by definition.

Let us denote the mathematical expectation and variance of white noise as follows: $\mathbf{E}\zeta_\tau(\omega)=a_\tau<\infty$, $\mathrm{Var}\left[\zeta_\tau(\omega)\right]=\sigma_\tau^2<\infty$, $\forall\tau$.

The random sequence (2) can serve as the model of random signal with discrete time, obtained by sampling of the corresponding signal with continuous time (for example, in the problems of biomedical signal analysis) or by aggregation of the process with continuous time (for example, for the problems of energy consumption monitoring). Let us consider the relationship of models (1) and (2) for both cases, which have an important practical significance.

According to the method of constructing the stochastic integral (1) [8], we get: $\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)d\eta(\omega,\tau)=\underset{\substack{a\to-\infty\\ b\to\infty}}{l.i.m.}\int_{a}^{b}\varphi(\omega,\tau,t)d\eta(\omega,\tau)$, in addition $\int_{a}^{b}\varphi(\omega,\tau,t)d\eta(\omega,\tau)$ it can be represented as the limit in the mean-square sense of integral sums sequence of the following form: $I_n(\omega,t)=\sum_{i=1}^{n}\varphi(\omega,\tilde{\tau}_i,t)\Delta\eta(\omega,\tau_i)$, where $a=\tau_0<\tau_1<\tau_2<\ldots<\tau_n=b$, $\Delta\eta(\omega,\tau_i)=\eta(\omega,\tau_i)-\eta(\omega,\tau_{i-1}),\ \tilde{\tau}_i\in[\tau_{i-1},\tau_i),\ i=\overline{1,n}$. That is, if $\max_{i=\overline{1,n}}(\tau_i-\tau_{i-1})\to 0$ at $n\to\infty$, then $\underset{n\to\infty}{l.i.m.}\sum_{i=1}^{n}\varphi(\omega,\tilde{\tau}_i,t)\Delta\eta(\omega,\tau_i)=\int_{a}^{b}\varphi(\omega,\tau,t)d\eta(\omega,\tau)$.

Now if we choose $\tau_i=ih,\ \tilde{\tau}_i=\tau_{i-1},\ i=1,2,\ldots,\ t_k=kh,\ k\in\mathbf{Z}$ (where h is the sample time of the process) and denote $\varphi(\omega,\tau_{i-1},t_k)=\varphi_{i,k}(\omega)$, $\Delta\eta(\omega,\tau_i)=\zeta_i(\omega)$ (increment of the process with independent increments $\eta(\omega,\tau)$ on interval $[\tau_{i-1},\tau_i)$), then the sequence of integral sums can be represented as follows: $I_{n,k}(\omega)=\sum_{i=1}^{n}\varphi_{i,k}(\omega)\zeta_i(\omega)$.

Independent random variables $\zeta_i(\omega),\ i=1,2,3,\ldots$ are infinitely divisible because they are increments of the process with independent increments.

Now let $n\to\infty$ such that $(\tau_i-\tau_{i-1})=h,\ i=\overline{1,n}$, but $a\to-\infty,\ b\to\infty$. At the same time if there is the limit in the mean-square sense of the sequence defined above $I_{n,k}(\omega)$, then $\underset{\substack{n\to\infty\\ a\to-\infty\\ b\to\infty}}{l.i.m.}I_{n,k}=\xi_k=\sum_{i=-\infty}^{\infty}\varphi_{i,k}\zeta_i,\ k\in\mathbf{Z}$ is discrete time CLRP.

It is also possible to obtain discrete time conditional linear random process by aggregating the corresponding continuous-time process, i.e.: $\xi_t(\omega)=\int_{(t-1)h}^{th}\xi(\omega,s)ds,\ t\in\mathbf{Z},$ where $h>0$ is the aggregation step. For example, in the tasks of monitoring the consumption of energy resources, there can be $\xi(\omega,t)$ is electricity load, and $\xi_t(\omega)$ is electricity consumption during the time interval $[(t-1)h,\ th]$ (if $h=1$ hour, then $\xi_t(\omega)$ is hourly electricity consumption).

Characteristic functions. In order to obtain the expressions of one-dimensional and multidimensional characteristic functions of CLRP (2), we will use the properties of infinitely divisible distributions, as well as the apparatus of conditional characteristic functions [2, 8, 11, 12].

Let us denote $\mathsf{F}_{\varphi}\subset\mathsf{F}$ as σ-subalgebra generated by the random function $\varphi_{\tau,t}(\omega)$, for which the below given conditions are fulfilled with probability 1:

$$\sum_{\tau=-\infty}^{\infty}\left|\varphi_{\tau,t}(\omega)a_{\tau}\right|<\infty \text{ and } \sum_{\tau=-\infty}^{\infty}\left|\varphi_{\tau,t}(\omega)\right|^{2}\sigma_{\tau}^{2}<\infty, \ \forall t.$$

Let us denote $I_{k,l,t}(\omega)=\sum_{\tau=k}^{l}\varphi_{\tau,t}(\omega)\zeta_{\tau}(\omega)$, $k<l\in\mathbf{Z}$. Elements $\varphi_{\tau,t}(\omega)\zeta_{\tau}(\omega)$ included in the specified sum are conditionally independent (F_{φ}-independent) infinitely divisible random variables. Therefore, the conditional (relatively to F_{φ}) characteristic function (F_{φ}-characteristic function) of the sum $I_{k,l,t}(\omega)$ of F_{φ}-independent random variables is equal to the product of their infinitely divisible F_{φ}-characteristic functions.

Thus, F_{φ}-characteristic function of the random variable $I_{k,l,t}(\omega)$, $\forall t\in\mathbf{Z}$ has the following form:

$$f_{k,l}^{\mathsf{F}_{\varphi}}(\omega,u;t)=\mathbf{E}\left(\exp\left[iuI_{k,l,t}(\omega)\right]\middle|\mathsf{F}_{\varphi}\right)=$$

$$=\exp\left[iu\sum_{\tau=k}^{l}\varphi_{\tau,t}(\omega)a_{\tau}+\sum_{\tau=k}^{l}\int_{-\infty}^{\infty}\left(e^{iux\varphi_{\tau,t}(\omega)}-1-iux\varphi_{\tau,t}(\omega)\right)\frac{d_{x}K(x;\tau)}{x^{2}}\right], u\in\mathbf{R}, i=\sqrt{-1},$$

where $K(x;\tau)$ is Poisson jump spectrum in Kolmogorov’s form of white noise $\zeta_{\tau}(\omega)$.

Taking into account the convergence properties of conditional characteristic functions [11], we can state that there is $\lim_{\substack{k\to-\infty\\ l\to\infty}} f_{k,l}^{\mathsf{F}_{\varphi}}(\omega,u;t)=f_{\xi}^{\mathsf{F}_{\varphi}}(\omega,u;t)=\mathbf{E}\left(\exp\left[iu\xi_{t}(\omega)\right]\middle|\mathsf{F}_{\varphi}\right)$ with probability 1.

Let $f_{k,l}(u;t)=\mathbf{E}\left(\exp\left[iuI_{k,l,t}(\omega)\right]\right)=\mathbf{E}\left[\mathbf{E}\left(\exp\left[iuI_{k,l,t}(\omega)\right]\middle|\mathsf{F}_{\varphi}\right)\right]=\mathbf{E}f_{k,l}^{\mathsf{F}_{\varphi}}(\omega,u;t)$ be the unconditional characteristic function of the sum $I_{k,l,t}(\omega)$. Since $\left|f_{k,l}^{\mathsf{F}_{\varphi}}(\omega,u;t)\right|\le 1$ with probability 1 [2], then we can find the one-dimensional unconditional characteristic function $f_{\xi}(u;t)=\mathbf{E}\left(\exp\left[iu\xi_{t}(\omega)\right]\right)$ of the random sequence (2) based on the following relations:

$$f_{\xi}(u;t)=\lim_{\substack{k\to-\infty\\ l\to\infty}}f_{k,l}(u;t)=\lim_{\substack{k\to-\infty\\ l\to\infty}}\mathbf{E}f_{k,l}^{\mathsf{F}_{\varphi}}(\omega,u;t)=\mathbf{E}\lim_{\substack{k\to-\infty\\ l\to\infty}}f_{k,l}^{\mathsf{F}_{\varphi}}(\omega,u;t)=\mathbf{E}f_{\xi}^{\mathsf{F}_{\varphi}}(\omega,u;t).$$

So, one-dimensional characteristic function of the discrete time conditional linear random process is as follows:

$$f_{\xi}(u;t)=\mathbf{E}\left[\mathbf{E}\left(\exp\left[iu\xi_{t}(\omega)\right]\middle|\mathsf{F}_{\varphi}\right)\right]=$$

$$=\mathbf{E}\exp\left[iu\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t}(\omega)a_{\tau}+\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi_{\tau,t}(\omega)}-1-iux\varphi_{\tau,t}(\omega)\right)\frac{d_{x}K(x;\tau)}{x^{2}}\right]. \quad (3)$$

According to this, the expression for m-dimensional characteristic function of the discrete time CLRP is as follows:

$$f_{\xi}(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=\mathbf{E}\left(\exp\left[i\sum_{k=1}^{m}u_k\xi_{t_k}(\omega)\right]\right)=\mathbf{E}\left[\mathbf{E}\left(\exp\left[i\sum_{k=1}^{m}u_k\xi_{t_k}(\omega)\right]\middle|\mathsf{F}_{\varphi}\right)\right]=$$
$$=\mathbf{E}\exp\left[i\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k}(\omega)a_{\tau}+\right. \tag{4}$$
$$\left.+\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)\right)\frac{d_xK(x;\tau)}{x^2}\right],\ t_k\in\mathbf{Z},\ u_k\in\mathbf{R},\ k=\overline{1,m}.$$

In the same way like for CLRP with continuous time, it is obvious that the probability distribution of discrete time CLRP belongs to the class of mixtures of infinitely divisible distributions [13].

Strict sense stationarity. Stationary random processes are an important class of random signal models that can be used to construct more complex mathematical objects, for example, piecewise stationary, cyclostationary [14–16], etc. Let us consider the properties that are required for kernel and generating white noise in representation (2) in order for discrete time CLRP to be stationary in the strict sense. For this purpose, we will use the expressions obtained above for the characteristic functions of CLRP.

Let us assume that for any $s\in\mathbf{Z}$ the following conditions are met:

- random matrices $\varphi_{\tau,t}(\omega)$, $\tau,t\in\mathbf{Z}$ and $\varphi_{\tau+s,t+s}(\omega)$ are stochastically equivalent in the wide sense, that is, their finite-dimensional distributions are equal:

$$\mathbf{P}\left(\bigcap_{i=1}^{n}\bigcap_{j=1}^{m}\left\{\omega:\varphi_{\tau_j,t_j}(\omega)<x_{ij}\right\}\right)=\mathbf{P}\left(\bigcap_{i=1}^{n}\bigcap_{j=1}^{m}\left\{\omega:\varphi_{\tau_j+s,t_j+s}(\omega)<x_{ij}\right\}\right),\ x_{ij}\in\mathbf{R}, \tag{5}$$

- white noise $\zeta_{\tau}(\omega)$, $\tau\in\mathbf{Z}$ is strict sense stationary:

$$a_{\tau}=a_{\tau+s}=a,\quad d_xK(x;\tau)=d_xK(x;\tau+s)=dK(x).$$

Then characteristic function (4) of the discrete time CLRP satisfies the condition:

$$f_{\xi}(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=f_{\xi}(u_1,u_2,...,u_m;t_1+s,t_2+s,...,t_m+s),\ \forall s\in\mathbf{Z}. \tag{6}$$

that is, the discrete time CLRP is stationary in the strict sense.

The above mentioned statement can be proved by means of analyses the properties of multidimensional characteristic function (4). Further, if the random variables $\xi(\omega)$ and $\eta(\omega)$ have the same distributions, then we write $\mathrm{Law}(\xi(\omega))=\mathrm{Law}(\eta(\omega))$. Thus,

$$\mathrm{Law}\left(a\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k}(\omega)\right)=\mathrm{Law}\left(a\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau+s,t_k+s}(\omega)\right)=\mathrm{Law}\left(a\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k+s}(\omega)\right),$$

$$\text{Law}\left(\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)\right)\frac{dK(x)}{x^2}\right)=$$

$$=\text{Law}\left(\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau+s,t_k+s}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{\tau+s,t_k+s}(\omega)\right)\frac{dK(x)}{x^2}\right)=$$

$$=\text{Law}\left(\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k+s}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k+s}(\omega)\right)\frac{dK(x)}{x^2}\right).$$

Therefore, the distribution of *m*-dimensional F_{φ}-characteristic function $f_{\xi}^{\mathsf{F}_{\varphi}}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m)$ of the discrete time CLRP does not depend on the shift in the set of its time arguments, i.e.

$$\text{Law}(f_{\xi}^{\mathsf{F}_{\varphi}}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m))=\text{Law}(f_{\xi}^{\mathsf{F}_{\varphi}}(\omega,u_1,u_2,...,u_m;t_1+s,t_2+s,...,t_m+s)).$$

Since $f_{\xi}(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=\mathbf{E}f_{\xi}^{\mathsf{F}_{\varphi}}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m)$, then (6) holds, i.e., discrete time CLRP is stationary in the strict sense.

Conclusions. In this paper, discrete time CLRP is defined, the relationship with the corresponding model with continuous time is analyzed. The expressions for one-dimensional and multi-dimensional characteristic functions of the process are substantiated, which makes it possible to characterize the conditions under which CLRP will be stationary in the strict sense. On the basis of the obtained expressions for characteristic functions, it is also possible to determine the conditions under which the investigated discrete-time process will be cyclostationary.

References

1. Babak V. P., Babak S. V., Eremenko V. S., Kuts Yu. V., Myslovych M. V., Scherbak L. M., Zaporozhets A. O. Models of Measuring Signals and Fields. Models and Measures in Measurements and Monitoring, volume 360 of Studies in Systems, Decision and Control. Springer, Cham, 2021. P. 33–59. https://doi.org/10.1007/978-3-030-70783-5_2
2. Pierre P. A. Central Limit Theorems for Conditionally Linear Random Processes. SIAM Journal on Applied Mathematics. 1971. Vol. 20. Issue 3. P. 449–461. https://doi.org/10.1137/0120048
3. Fryz M. Properties of conditional linear random processes and their applications in the applied problems of stochastic signal mathematical modelling. Matematychne ta kompiuterne modeliuvannia. Seriia: Tekhnichni nauky: zbirnyk naukovykh prats. 2012. Vol. 6. P. 228–238. Doi: https://doi.org/10.32626/2308-5916.2012-6.228-238. [In Ukraine].
4. Fryz M., L. Scherbak Statistical analysis of random coefficient periodic autoregression and its application for short-term electricity consumption forecasting. Tekhnichna elektrodynamika. K.: Institute of Electrodynamics National Academy of Science of Ukraine. 2019. Vol. 2. P. 38–47. [In Ukraine]. https://doi.org/10.15407/techned2019.02.038
5. Fryz M., Mlynko B. Properties of Stationarity and Cyclostationarity of Conditional Linear Random Processes. Proceedings of the 2020 IEEE 15th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET). Lviv, Slavske, Ukraine, 2020. P. 166–170. https://doi.org/10.1109/TCSET49122.2020.235415
6. Iwankiewicz R. Dynamical mechanical systems under random impulses. Singapore: World Scientific Publishing Co. Pte. Ltd., 1995. 176 p. https://doi.org/10.1142/2767
7. Barndorff-Nielsen O. E., Benth F. E., Veraart A. E. D. Ambit Stochastics. Volume 88 of Probability Theory and Stochastic Modelling. Springer Nature Switzerland AG, 2018. 402 p. https://doi.org/10.1007/978-3-319-94129-5

8. Fryz M., Scherbak L., Karpinski M., Mlynko B. Characteristic Function of Conditional Linear Random Process. Proceedings of the 1st International Workshop on Information Technologies: Theoretical and Applied Problems. Ternopil, Ukraine, 2021. –P. 129–135. URL: https://ceur-ws.org/Vol-3039/short40.pdf.
9. Chen P., Zhang T., Sung S. H. Strong laws for randomly weighted sums of random variables and applications in the bootstrap and random design regression. Statistica Sinica. 2019. Vol. 29. No. 4. P. 1739–1749. https://doi.org/10.5705/ss.202017.0106
10. Krizmanić D. Maxima of linear processes with heavy-tailed innovations and random coefficients. Journal of Time Series Analysis. 2022. Volume 43. Issue 2. P. 238–262. https://doi.org/10.1111/jtsa.12610
11. Demei Yuan, Lan Lei. Some results following from conditional characteristic functions. Communications in Statistics. Theory and Methods. 2013. Volume 45. Issue 12. P. 3706–3720. https://doi.org/10.1080/03610926.2014.906614
12. Bulinski A. V. Conditional central limit theorem. Theory of Probability & Its Applications. 2017. Volume 6. Issue 4. P. 613–631. https://doi.org/10.1137/S0040585X97T98837X
13. F. W. Steutel, Klaas van Harn Infinitely Divisibility of Probability Distributions on the Real Line. Boca Raton: CRC Press, 2003. 550 p. https://doi.org/10.1201/9780203014127
14. Lytvynenko I. Method of the quadratic interpolation of the discrete rhythm function of the cyclical signal with a defined segment structure. Scientific Journal of TNTU. 2016. No. 4 (84). P. 131–138. https://doi.org/10.33108/visnyk_tntu2018.03.134
15. Gotovych V., Nazarevych O., Shcherbak L. Mathematical modeling of the regular-mode electric power supply and electric power consumption processes of the organization. Scientific Journal of TNTU. 2018. Vol. 91. No. 3. P. 134–142. https://doi.org/10.33108/visnyk_tntu2021.04.087
16. Lytvynenko I., Lupenko S., Nazarevych O., Shymchuk H., Hotovych V. Additive mathematical model of gas consumption process. Scientific Journal of TNTU. Vol. 104. No. 4. P. 87–97.

Список використаних джерел

1. Babak V. P., Babak S. V., Eremenko V. S., Kuts Yu. V., Myslovych M. V., Scherbak L. M., Zaporozhets A. O. Models of Measuring Signals and Fields. Models and Measures in Measurements and Monitoring, volume 360 of Studies in Systems, Decision and Control. Springer, Cham, 2021. P. 33–59. https://doi.org/10.1007/978-3-030-70783-5_2
2. Pierre P. A. Central Limit Theorems for Conditionally Linear Random Processes. SIAM Journal on Applied Mathematics. 1971. Vol. 20. Issue 3. P. 449–461. https://doi.org/10.1137/0120048
3. Фриз М. Є. Властивості умовних лінійних процесів та їх застосування в прикладних задачах математичного моделювання стохастичних сигналів. Математичне та комп'ютерне моделювання. Серія: Технічні науки: збірник наукових праць. 2012. Вип. 6. С. 228–238. Doi: https://doi.org/10.32626/2308-5916.2012-6.228-238.
4. Фриз М. Є., Щербак Л. М. Статистичний аналіз періодичної авторегресії з випадковими коефіцієнтами в задачах оперативного прогнозування електроспоживання підприємств. Технічна електродинаміка. К.: Інститут електродинаміки Національної академії наук України, 2019. Вип. 2. С. 38–47. https://doi.org/10.15407/techned2019.02.038
5. Fryz M., Mlynko B. Properties of Stationarity and Cyclostationarity of Conditional Linear Random Processes. Proceedings of the 2020 IEEE 15th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET). Lviv, Slavske, Ukraine, 2020. P. 166–170. https://doi.org/10.1109/TCSET49122.2020.235415
6. Iwankiewicz R. Dynamical mechanical systems under random impulses. Singapore: World Scientific Publishing Co. Pte. Ltd., 1995. 176 p. Doi: https://doi.org/10.1142/2767.
7. Barndorff-Nielsen O. E., Benth F. E., Veraart A. E. D. Ambit Stochastics. Volume 88 of Probability Theory and Stochastic Modelling. Springer Nature Switzerland AG, 2018. 402 p. https://doi.org/10.1007/978-3-319-94129-5
8. Fryz M., Scherbak L., Karpinski M., Mlynko B. Characteristic Function of Conditional Linear Random Process. Proceedings of the 1st International Workshop on Information Technologies: Theoretical and Applied Problems. Ternopil, Ukraine, 2021. –P. 129–135. URL: https://ceur-ws.org/Vol-3039/short40.pdf.
9. Chen P., Zhang T., Sung S. H. Strong laws for randomly weighted sums of random variables and applications in the bootstrap and random design regression. Statistica Sinica. 2019. Vol. 29. No. 4. P. 1739–1749. https://doi.org/10.5705/ss.202017.0106
10. Krizmanić D. Maxima of linear processes with heavy-tailed innovations and random coefficients. Journal of Time Series Analysis. 2022. Volume 43. Issue 2. P. 238–262. https://doi.org/10.1111/jtsa.12610

11. Demei Yuan, Lan Lei. Some results following from conditional characteristic functions. Communications in Statistics. Theory and Methods. 2013. Volume 45. Issue 12. P. 3706–3720. https://doi.org/10.1080/03610926.2014.906614
12. Bulinski A. V. Conditional central limit theorem. Theory of Probability & Its Applications. 2017. Volume 6. Issue 4. P. 613–631. https://doi.org/10.1137/S0040585X97T98837X
13. F. W. Steutel, Klaas van Harn Infinitely Divisibility of Probability Distributions on the Real Line. Boca Raton: CRC Press, 2003. 550 p. https://doi.org/10.1201/9780203014127
14. Lytvynenko I. Method of the quadratic interpolation of the discrete rhythm function of the cyclical signal with a defined segment structure. Scientific Journal of TNTU. 2016. No. 4 (84). P. 131–138. https://doi.org/10.33108/visnyk_tntu2018.03.134
15. Gotovych V., Nazarevych O., Shcherbak L. Mathematical modeling of the regular-mode electric power supply and electric power consumption processes of the organization. Scientific Journal of TNTU. 2018. Vol. 91. No. 3. P. 134–142. https://doi.org/10.33108/visnyk_tntu2021.04.087
16. Lytvynenko I., Lupenko S., Nazarevych O., Shymchuk H., Hotovych V. Additive mathematical model of gas consumption process. Scientific Journal of TNTU. Vol. 104. No. 4. P. 87–97.

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ВИЗНАЧЕННЯ ХАРАКТЕРИСТИЧНОЇ ФУНКЦІЇ УМОВНОГО ЛІНІЙНОГО ВИПАДКОВОГО ПРОЦЕСУ З ДИСКРЕТНИМ ЧАСОМ ТА ЇЇ ЗАСТОСУВАННЯ

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Резюме. *Лінійні та умовні лінійні випадкові процеси (УЛВП) поширені у задачах математичного моделювання стохастичних інформаційних сигналів, зображуваних у вигляді суми великого числа випадкових імпульсів, що виникають у випадкові моменти часу. Для УЛВП ці імпульси є стохастично залежними (для лінійних процесів – незалежні), а моменти їх виникнення утворюють неоднорідний пуассонівський потік. Прикладами таких сигналів можуть бути електрофізіологічні сигнали (сума постсинаптичних потенціалів), процеси споживання енергоресурсів (електро-, газо-, водоспоживання), вібраційні сигнали енергооб'єктів, радіолокаційні сигнали та ін. Розвиток методології математичного моделювання інформаційних сигналів із використанням УЛВП, а також побудова методів їх статистичного аналізу й прогнозування в інформаційних системах технічного й медичного призначення є актуальною науково-прикладною проблемою. Проаналізовано варіант моделі УЛВП із дискретним часом, що може бути отриманий шляхом дискретизації відповідного сигналу з неперервним часом (в інформаційних системах і технологіях цифрового опрацювання даних) або ж шляхом його агрегації (наприклад, у системах моніторингу споживання енергоресурсів). Отримано вирази для одновимірної та багатовимірної характеристичної функції досліджуваного процесу, що дає можливість здійснювати теоретичний аналіз ймовірнісних властивостей модельованих сигналів. Встановлено, що ймовірнісний розподіл умовного лінійного випадкового процесу з дискретним часом належить до класу сумішей безмежно подільних розподілів. Також у роботі обґрунтовано умови, яким повинні задовольняти ядро та породжуючий безмежно подільний білий шум у зображенні УЛВП для того, щоб він був стаціонарним у вузькому сенсі. Перспективним напрямком досліджень є аналіз властивостей циклостаціонарних умовних лінійних випадкових процесів із дискретним часом, який можна здійснити на основі результатів, викладених у даній роботі.*

Ключові слова: *математична модель, інформаційний сигнал, умовний лінійний випадковий процес, характеристична функція, стаціонарний у вузькому сенсі випадковий процес.*

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