# Stem volume by height classes of immature, mature and overmature stands of the main forest-forming species of Ukraine 

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#### Abstract

Generally, it is impossible to measure diameters and heights of all trees in a forest stand. Therefore, models of relationships between heights ( $h$ ) and diameters ( $d$ ) of trees are commonly used in practice for stem volume estimation. This study aimed at developing models of tree height-diameter ( $h-d$ ) relationships as well as corresponding models of the tree stem volume for immature, mature and overmature stands of the main forest-forming species of Ukraine. This paper is a aggregation of long-term studies of the stem volume, which are based on the results of measuring about 10 thousand sample trees. Modelling of the tree height-diameter relationships was performed using relative height values. The methodology used in this study allowed generalising the measurements of sample trees collected in stands of various forest site types, productivity levels, and age categories. The average height of trees with a diameter of 24 cm was taken as the reference during modelling relative heights, while the diameter of 40 cm was chosen as the reference for overmature Scots pine stands. As a result, the parameters of a unified mathematical model of relative heights for immature, mature, and overmature stands of the main forest-forming tree species of Ukraine were established. Based on these models, heightdiameter relationships in forest stands of different height classes were predicted. The authors demonstrated that the developed mathematical models substantially simplify the methodology of field work during timber surveys. The paper also presents models of the tree stem volume. These models predict the stem volume outside the bark based on diameters and heights of trees or using the developed models of $h-d$ relationships. In this study, a unified system of mathematical models of stem volume by height classes were created for immature, mature, and overmature stands of the main forest-forming species of Ukraine. The results of the study are introduced to the National Forest Inventory of Ukraine for growing stock volume calculation at sample plot level using measurements of individual trees. The developed models can be used both by operation forestry (estimation of the timber volume during harvesting), and forest management (forecasting the future structure of forests and estimating the growing stock volume), as well as in the forest ecology


Keywords: volume tables, relative height, form factor, mathematical model

## Introduction

Accurate assessment of individual tree stem and growing stock volume of forest stands is essential for sustainable forest resource management. For this purpose, volume models based on diameter ( $d$ ) and height ( $h$ ) of individual trees are used. Since measuring the height of all trees is impractical for financial reasons and often involves significant errors, the growing stock volume in practice is calculated based on sample trees measurements. The heights of trees out-of-sample are calculated based on tree height-diameter $(h-d)$ relationships models.

Modelling the $h$-d relationships has important scientific and practical significance. From the standpoint of forest ecology, this issue starts a discussion about the
tree competition in forest stands during their growth [1], namely the estimation of competition indices [2]. Temesgen et al. [3] showed that larger trees (with a higher rank by diameter) reach a higher height in the forest stand. The $h-d$ relationships can be used in predictive models of stand development [4], e.g., to simulate the future structure of forest stands in a result of thinning. In practical terms, the $h$ - $d$ relationship is more widely used to find the volume of individual trees and the growing stock volume of forest stands [5; 6]. Methods used absolute values of height to predict the $h-d$ relationships of trees are quite common in the literature [7]. They use some stand parameters (average diameter, basal area, etc.) as factors in corresponding models.

## Suggested Citation:

Bilous, A., Myroniuk, V., Svynchuk, V., Kashpor, S., \& Lesnik, O. (2022). Stem volume by height classes of immature, mature and overmature stands of the main forest-forming species of Ukraine. Ukrainian Journal of Forest and Wood Science, 13(3), 7-12.

Results of the earlier studies [3; 8] indicate that the parameters of nonlinear models that characterise the $h-d$ relationship in forest stands of a certain tree species are variable. They depend primarily on age of trees. In remove younger stands, height curves are characterised by greater steepness than in older stands. Therefore, parameters of height curves are estimated independently for different age groups (competition levels) of forest stands [1]. The $h-d$ relationship in even- and uneven-aged forest stands is also important issue [9]. Some recent publications indicate that $h-d$ relationships of trees depend on the origin of forest stands [10] and are closely related to their density [11]. Site conditions directly impact the $h-d$ relationships of trees [12]. In general, characteristics of individual forest stand (e.g., site index, density, relative stocking, etc.) are usually considered as factors that introduce random variations in the $h-d$ relationships in forest stands of the same age. While fixed-effect models characterise the most typical relationship for a wide group of forest stands, mixed-effect models explain the random component of covariance of these characteristics for a particular forest stand [11]. Such models, apart from fixed-effect terms of the equation (the typical stand), also contain random-effect terms that describes remove the differences between each individual forest stand in relation to the typical one, thus revealing specific patterns of heights and diameters depending on random factors.

There are several methodological approaches used in forest mensuration to modelling stem volume ( $V$ ) of trees, i.e., the development of multidimensional volume equations ( $V=\psi(d, h, \ldots)$ ), the derivation of stem volumes based on the stem profile equations, and the use of specific coefficients, namely, the form factor [2]. Multidimensional mathematical models of the stem volume based on diameter and height of trees are often used in different countries: Germany, Austria, Finland, Sweden, Romania, Norway, etc. [15]. Estimating coefficients of such equations using modern computer technologies is a simple task, while accuracy of the models meets the needs of practice. Stem profile equations are used when in addition to the total stem volume, it is necessary to characterise its taper or estimate the volume of stem zones with certain qualitative characteristics (e.g., commercial timber) [16]. In Ukraine, this approach has been used relatively recently [18]. Much more often, the stem volume is considered as the product of three factors: the basal area (g), height, and form factor (f), where $f$ is a conversion coefficient between the volume of cylinder (gh) and stem volume.

Mathematical models of the tree stem form factors for major species of Ukraine were published in earlier studies [19]. However, during the preparation of a new edition of the forest inventory handbook [20], some mathematical models have been improved.

## Materials and Methods

The data for modelling stem volumes by height classes were collected during 1950-2020 on test areas distributed over the territory of Ukraine. In total, measurements about 10 thousand sample and tally trees on more than 700 temporary and permanent experimental plots for 13 main for-est-forming tree species in Ukraine were used. Experimental plots characterise immature, mature, and overmature forest stands of such natural and climatic zones of Ukraine
as Polissia, Forest-Steppe, Carpathians [21]. At experimental plots, the diameters of all trees were measured by $4-\mathrm{cm}$ diameter classes. Then, heights of 9-20 trees of the dominant species (selected from different diameter classes) were measured to plot the height curve for the corresponding forest stand. Based on the distribution of diameters and heights (according to height curves), 5-20 sample trees were selected and cut down on each site. The diameters were measured at the midpoint of $2-\mathrm{m}$ sections, while the stem volume was calculated using Newton's sectional formula.

## Modelling the tree height-diameter relationships

The study utilized an approach according to which the $h-d$ relationships of stems was investigated using relative values. This allows summarising the research data from a wide age range and from different forest conditions based on a unified shape of height curve. According to this, the relative heights for each diameter classes at an individual experimental plot were calculated according to the following formula:

$$
\begin{equation*}
h_{i}^{e}=h_{i} / h_{r e f}, \tag{1}
\end{equation*}
$$

where $h_{i}^{e}$ is the relative height value of the $i^{\text {th }}$ diameter class; $h_{i}$ is the absolute height of the $i^{\text {th }}$ diameter class, m; $h_{\text {ref }}$ is the tree height of the reference diameter class, $m$.

For the majority of stands, a dimeter class of 24 cm was taken as the reference, while for overmature Scots pine stands it was 40 cm .

Using Equation (1), the average values of relative heights $\left(h_{i}^{e}\right)$ in forest stands by the diameter classes were calculated for each of the 13 main forest-forming tree species in Ukraine. A unified mathematical model of relative height depending on the stem diameter was used for all species:

$$
\begin{equation*}
h_{i}^{e}=a_{0}+a_{1} \cdot \exp \left(a_{2} \cdot d\right) . \tag{2}
\end{equation*}
$$

The following formula was used to convert relative height into absolute height values:

$$
\begin{equation*}
h_{i}=h_{i}^{e} \cdot h_{r e f} . \tag{3}
\end{equation*}
$$

To unify the reference data for determining tree stem and growing stock volumes, the heights of the reference diameter classes, numbering of height classes $(R)$, and the interval between them were harmonized with the volume tables developed by height classes [20] that have been legally acknowledged in forestry of Ukraine. Accordingly, a height of 31.78 m was taken as the reference height in $\mathrm{I}^{\mathrm{a}}$ height class for overmature Scots pine stands, and 27.40 m - for the rest of species and age categories. The reference heights in other height classes for overmature Scots pine stands (the reference diameter class of 40 cm ) were calculated using the following formula:

$$
\begin{equation*}
h_{r e f}=h_{40}=31.78-2.55 \cdot I_{R}, \tag{4}
\end{equation*}
$$

where $I_{R}$ is the index of the height class of tree stands (Table 1).
In other cases (the reference diameter class of 24 cm ), reference heights were calculated according to the formula:

$$
\begin{equation*}
h_{r e f}=h_{24}=27.4-2.2 \cdot I_{R} . \tag{5}
\end{equation*}
$$

Table 1 presents the coding of height classes according to the volume tables that are legally acknowledged in forestry of Ukraine.

Table 1. Indices and numbering of tree stand height categories in Ukraine

| $R$ | $\mathrm{I}^{\mathrm{c}}$ | $\mathrm{I}^{\mathrm{b}}$ | $\mathrm{I}^{\mathrm{a}}$ | I | II | III | IV | V | $\mathrm{V}^{\mathrm{a}}$ | $\mathrm{V}^{\mathrm{b}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{R}$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

## Modelling the stem volume

In the stem volume modelling, the form factors were predicted using their relationship with the diameter and height of the trees. Then, the stem volume was calculated based on the following equation:

$$
\begin{equation*}
V=g \cdot h \cdot f \tag{6}
\end{equation*}
$$

A mathematical model of a form factors as a function of diameters and heights $f=\psi(d, h, \ldots)$ was used for such species as pine and spruce of the middle and upper mountain belts of the Carpathians. Dependence of the form factors of tree trunks exclusively on diameters $f=\psi(d, \ldots)$ was studied for spruce (on the plain), silver-fir, and maple. For the rest of the tree species, the form factor models depending on the stem heights $f=\psi(h, \ldots)$ were used.

A direct regression of the stem volume against diameter and height was applied for birch and aspen using the allometric equation:

$$
\begin{equation*}
V=a_{0} \cdot d^{a_{1}} \cdot h^{a_{2}} \tag{7}
\end{equation*}
$$

To characterize the relationship between the form factors of different tree species with diameter, height, or both, the optimal set of regression coefficients in the model (6) was found based on a multivariate examination of multiple candidate equations. In general, such a variety of methodological approaches is explained by the fact that mathematical models were developed in different periods, trying to achieve maximum accuracy of models while maintaining their simplicity.

The parameters of mathematical models were calculated based on regression analysis using the nonlinear least squares method (NLS) [22].

## Results and Discussion

Based on the results of the research, the parameters of a unified mathematical model of relative height for immature, mature, and overmature stands of the main forest-forming tree species of Ukraine were established (Table 2).

Table 2. Parameters of the mathematical model of relative height as a function of stem diameter

| No. | Tree species | Latin name of a tree species | Parameter of Equation (2) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $a_{0}$ | $a_{1}$ | $a_{2}$ |
| 1 | Scots pine | Pinus sylvestris L. | 1.247 | -1.183 | -0.06527 |
| 2 | Scots pine (overmature stands) |  | 1.202 | -0.9769 | -0.03933 |
| 3 | Norway spruce (on the plain) | Picea abies L. | 1.391 | -1.486 | -0.05563 |
| 4 | Norway spruce (middle mountain belt of the Carpathians) |  | 1.501 | -1.459 | -0.04454 |
| 5 | Norway spruce (upper mountain belt of the Carpathians) |  | 1.711 | -1.739 | -0.03727 |
| 6 | Silver fir | Abies alba (L.) | 1.397 | -1.374 | -0.05173 |
| 7 | Common oak | Querqus robur L. | 1.295 | -1.320 | -0.06243 |
| 8 | European beech | Fagus sylvatica (L.) | 1.212 | -0.9445 | -0.06226 |
| 9 | Common ash | Fraxinus excelsior L. | 1.340 | -1.343 | -0.05725 |
| 10 | Norway maple | Acer platanoides L. | 1.348 | -1.228 | -0.05254 |
| 11 | Common hornbeam | Carpinus betulus L. | 1.144 | -0.9224 | -0.07738 |
| 12 | Common aspen | Populus tremula L. | 1.290 | -1.199 | -0.05914 |
| 13 | Silver birch | Betula pendula Roth. | 1.302 | -1.080 | -0.05310 |
| 14 | Black alder | Alnus glutinosa (L.) Gaerth | 1.186 | -1.209 | -0.07799 |
| 15 | Small-leaved linden | Tilia cordata Mill. | 1.229 | -1.252 | -0.07069 |
| 16 | Black locust | Robinia pseudoacacia L. | 1.210 | -0.923 | -0.06170 |

The developed tree $h$ - $d$ relationships were published in a tabular form in the new Forest inventory handbook (Tables 2.11-2.26) [20]. Stem volume models for immature, mature, and overmature stands of the main forest-forming tree species in Ukraine can be utilized using direct measurements of the diameter and
height of individual trees (Tables 1.18-1.31) [20] or in combination with $h$-d models (Table 3). Finally, based on the developed models of relative height (2) and mathematical models of the stem volume presented in Table 3, volume tables by height classes were developed (Table 2.272.41) [20].

Table 3. Mathematical models of stem volumes outside bark

| No. | Tree species | Mathematical model of stem volume in bark |
| :---: | :---: | :---: |
| 1 | Scots pine | $V=\exp (7.767-0.04235 \cdot \ln (d+8)-0.6374 \cdot \ln (h+2)+0.02158 \cdot(h+2)) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-8}$ |
| 2 | Scots pine (overmature stands) | $V=\left(0.3521+0.5343 \cdot d^{-0.4546}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 3 | Norway spruce (on the plain) | $V=(0.4217+1.023 /(h-0.723)) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |

Table 3, Continued

| No. | Tree species | Mathematical model of stem volume in bark |
| :---: | :---: | :---: |
| 4 | Norway spruce (middle mountain <br> belt of the Carpathians) | $V=\left(438-2.3 \cdot \mathrm{~h}+0.0934 \cdot h^{2}-(d-40) /(0.163+0.00874 \cdot d)\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-8}$ |
| 5 | Norway spruce (upper mountain <br> belt of the Carpathians) | $V=\left(402.5-1.601 \cdot h+0.0572 \cdot h^{2}-(d-42) /(0.142+0.00545 \cdot d)\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-8}$ |
| 6 | Silver fir | $V=(0.4118+1.176 /(h-0.289)) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 7 | Common oak | $V=\left(0.3812+0.4955 \cdot d^{-0.4811}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 8 | European beech | $V=\left(0.5007-0.001507 \cdot d+2.106 / d^{2}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 9 | Common ash | $V=\left(-0.1634+0.8110 \cdot d^{-0.07708}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 10 | Norway maple | $V=(0.3948+1.246 /(h+0.966)) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 11 | Common hornbeam | $V=\left(0.103+0.5889 \cdot d^{-0.1702}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 12 | Common aspen | $V=0.3081 \cdot d^{1.8708} \cdot h^{1.1932} \cdot 10^{-4}$ |
| 13 | Silver birch | $V=0.5631 \cdot d^{1.755} \cdot h^{1.073} \cdot 10^{-4}$ |
| 14 | Black alder | $V=(0.4105+1.285 /(d+1.084)) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 15 | Small-leaved linden | $V=\left(-4.166+4.849 \cdot d^{-0.01390}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |
| 16 | Black locust | $V=\left(0.4024+1.170 \cdot h^{-1.036}\right) \cdot d^{2} \cdot h \cdot 7.854 \cdot 10^{-5}$ |

The use of stem volume models by height classes substantially simplifies the method of forest inventory, since there is no need to measure the height of all tally trees. Therewith, in the stem volume models (Table 3), the value of the variable $h$ is obtained based on the equations (2-3).

In the literature, there are various approaches to modelling the stem volumes. In most cases, direct regression of the volume of stems against the diameter and height is used [15]. It is worth noting, the attitude towards this approach is ambiguous. Although it is possible to obtain a high value of the determination coefficient, the parameters of the equation are very sensitive to the data set of experimental information (e.g., incorrect maximum values can substantially increase/decrease the volume forecast for extreme tree diameters). The high determination coefficient is also explained by the presence of functional dependencies between the volume of solids and their size. In this regard, another approach based on the analysis of random variations in the stem volume of trees of the same size ( $d$ and $h$ ) using form factors has considerable practical advantages. For example, the stem volume has a fixed component, i.e., the volume of the cylinder (can be directly obtained based on $d$ and $h$ ), and a variable correction factor (form factor). The accuracy of modelling the stem volume depends exclusively on the adequacy of the description of the variation in their shape depending on the diameter
and height, i.e., the variability of the form factors which is lower than for the corresponding stem volumes.

In general, the stem volume of trees of the same size ( $d$ and $h$ ) is determined by the stem form. In this regard, there are forest tree species in Ukraine that reach a larger volume, i.e., they have a smaller stem taper. Based on the models presented in Table 3, it was found that for the same diameter and height, the stems of spruce, fir, and oak have the largest volumes (Fig. 1a). At the same time, the stems of birch trees have a substantially smaller volume.

The altitudial zonation of the Ukrainian Carpathians impacts the productivity, morphology, and wood quality characteristics of spruce stands. Optimal conditions for spruce growth in mountains are observed at altitudes from 700 to 1100 m above sea level. At higher altitudes, significant changes in the morphology and productivity of stands are observed, namely more sparse forest stands are mainly formed here, the trunks of which have larger taper [23]. Considering these features, the decision to develop mathematical models of the volume for the flat part, the middle (up to 1100 m ) and the upper mountain (over 1100 m ) belt of the Carpathians allowed more accurately characterising the volume of spruce trunks in typical growth conditions (Fig. 1b). According to our data, spruce trunks growing in the high-altitude belt of the Carpathian Mountains have the greatest stem taper (i.e., the smallest volume at constant values of diameter and height).


Figure 1. Comparison of the stem volumes as a function of diameter at a constant height ( $h=23 \mathrm{~m}$ ): a) typical examples of tree species of Ukraine that have different stem taper; b) the volume of spruce stems in different growth conditions

Similar to spruce, the study reported two groups of models for Scots pine stands (Table 2, Table 3). This can be explained by the specific $h-d$ relationship in overmature stands, i.e., described by shallow curves at an older age. Furthermore, in overmature pine stands (over 130 years old), the yield of merchantable timber substantially decreases. This is explained by the specific features of the dimensional and qualitative characteristics of trunks and the structure of stands approaching the age of natural maturity. Thus, if the goal is to develop tables for the distribution of stem volume by size and quality categories, the availability of different models for the specified age groups of forest stands is critical.

The developed system of mathematical models is integrated into the system of the National Forest Inventory of Ukraine for growing stock volume calculation at sample plot level. The use of mathematical models of the $d-h$ relationships simplifies the methodology of field work, since there is no need to measure the height of all tally trees on the sample plot.

## Conclusions

The methods used to calculate the stem and growing stock volume of forest stands should be accurate and
financially efficient to use in forest resource management over large areas. Since measuring diameters and heights of all trees in a stand is impractical or unfeasible, mathematical models of the $h-d$ relationships are used in the forest inventory. This paper provides the system of mathematical models of the stem volume and the tree $h-d$ relationship of the main forest-forming species of Ukraine. The presented models can be used in operational forestry (estimation the timber volume during harvesting), forest management (forecasting the future structure of forests and estimating the growing stock volume of stem wood), as well as in forest ecology, forest ecosystem services studies, etc. The developed set of mathematical models creates the basis for harmonising methods of forests assessment in Ukraine at various phases of forest management. In addition to the total stem volume calculation, developed volume equations have been integrated into tables that provide an estimate of the distribution of timber volume by thickness classes.

The results of this study promote further investigation of the distribution of the stem volume by quality classes, developing reference data, and software for improved forest recourse assessment.

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# Об’єм стовбурів за розрядами висот пристиглих, стиглих і перестійних деревостанів основних лісоутворювальних видів України 

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#### Abstract

Анотація. У більшості випадків забезпечити вимірювання діаметра та висот усіх дерев неможливе, тому на практиці під час визначення об'єму стовбурів прийнято застосовувати моделі співвідношення висоти та діаметра дерев у насадженні. Мета роботи полягає в розробці моделей співвідношення висоти та діаметра дерев для пристиглих, стиглих і перестійних деревостанів основних лісоутворювальних видів України, а також відповідних моделей об'єму стовбурів дерев зазначених категорій лісових насаджень. Дослідження базується на узагальненні багаторічних досліджень об'єму стовбурів, в основу яких покладено результати обміру близько 10 тис. модельних дерев. Моделювання співвідношення висоти і діаметра дерев у деревостанах виконувалося за допомогою відносних значень висоти. Використаний методичний підхід дозволив узагальнювати матеріали обміру модельних дерев у деревостанах різних лісорослинних умов, рівнів продуктивності та вікових категорій. За базове значення висоти під час моделювання приймалася середня висота дерев діаметром 24 см, для перестійних соснових деревостанів як базовий обирався діаметр 40 см. Встановлено параметри єдиної математичної моделі відносної висоти для пристиглих, стиглих і перестиглих деревостанів основних лісотвірних деревних видів України. На основі неї спрогнозованоможливіспіввідношення висотиідіаметрівунасадженняхзарізнимирозрядамивисот.Обгрунтована можливість використання розроблених математичних моделей, що значно спрощує методику польових робіт. У роботі також представлено моделі об'єму стовбурів дерев. Зазначені моделі дозволяють спрогнозувати об'єми стовбурів у корі залежно від діаметра та висоти дерев, або, використовуючи розроблені співвідношення висот і діаметрів дерев. На основі проведеного дослідження було створено єдину систему математичних моделей об’єму стовбурів за розрядами висот для пристиглих, стиглих і перестиглих деревостанів основних лісоутворювальних видів України. Результати дослідження запропоновано використовувати під час національної інвентаризації лісів України для визначення запасу лісових насаджень на рівні пробних площ, використовуючи обміри окремих дерев. Розроблені моделі можуть використовуватися як для вирішення практичних задач лісового господарства (визначення об’єму деревини під час рубок), лісоуправління (прогнозування майбутньої структури лісів і оцінки запасів стовбурової деревини), так і під час дослідження екології лісових екосистем


Ключові слова: таблиці об’єму стовбурів, відносна висота, видове число, математична модель

