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Height-Diameter Relationships and Stem Volume Equations in Young and Middle-Aged Forest Stands of Ukraine

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Abstract. Height-diameter (*h-d*) relationships in forest stands are commonly used in various scientific and practical forestry applications. Accurate *h-d* models combined with tree stem volume equations are recognised to be effective in growing stock volume estimation. *The purpose of the study* was threefold: 1) development of a set of mathematical models of the *h-d* relationship in young and middle-aged forest stands for ten forest-forming species in Ukraine; 2) modelling stem volume in above mentioned forest stands; 3) comparison of established mathematical models with corresponding ones for premature, mature, and overmature forest stands. The study was based on permanent and temporal sample plots data (about 600) established in forest stands during 1950s-2020s within the most forested regions of Ukraine (Polissia, Forest-Steppe, Carpathians). In total, about 10,000 sample trees were measured on the sample plots to accurately estimate their stem volume outside bark. The *h-d* models demonstrated very similar relationships between stem heights and diameters for most of our species except for spruce and fir in mountain Carpathian forests where the steeper *h-d* curves were obtained. The study revealed that birch and hornbeam tree stems had the lowest volumes among surveyed species. The results also indicated that tree species tend to have higher volumes (up to 7% for coniferous, and up 10% for aspen and birch forests) in young and middle-aged forest sands than in older ones. For the other species, a statistically significant difference between stem volumes of trees of different ages was not observed. The developed mathematical models can complement the corresponding models for older groups of forest stands since they revealed an important aspect of relationships between the key tree stem parameters. These models are also applicable for a more precise stem volume estimation during thinning operations in the young and middle-aged forests in Ukraine

Keywords: height curve, volume tables, height class, form factor, forecasting

Introduction

Relationships between tree heights and diameters (*h-d*) are widely used to determine tree stem volume in forest stands. Since most volume equations require measuring both diameter and height of trees [1], *h-d* models can be utilized to predict stem heights based on their diameters. From a practical perspective, *h-d* relationships are used to determine average heights by stem diameter classes which simplifies estimations of growing stock volume. In addition, the *h-d* relationships are applied in various stand growth models [2], in particular, to determine stand heights based on the corresponding average diameter.

The *h-d* relationships depend on tree species, growth conditions of forest stands [3; 4], and do not remain constant throughout time even in the same stand [5]. Differences in *h-d* relationships in even-aged and uneven-aged forest stands have been also investigated in the literature [6]. The shape of curves

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characterizing these relationships may change with age, i.e., have different steepness at different stages of the stand development. Sharma (2016) has revealed differences in *h-d* relationships for a number of coniferous tree species in natural forest stands and planted forests in Ontario, Canada [7].

Various mathematical models of *h-d* relationships have been tested in the literature. Most of them were developed to predict heights of individual trees based on their diameters. To characterize these relationships, various growth functions [8; 9] and some other nonlinear equations [10] were used. M. Liu et al. [11] tested 53 mathematical models and concluded that nonlinear functions have substantial advantages over linear ones. Such functions were found to be more flexible in describing various forms of relationship between tree heights and diameters. Additionally, D. Leduc & J. Goelz, [12] evaluated more than 40 mathematical equations and demonstrated that the highest accuracy of modelling could be achieved regardless of the form of the equations using relative heights and diameters instead of absolute values.

Generally, relationships of biometric parameters become more complex in uneven-aged and multi-layered stands which require more advanced approaches to characterize *h-d* relationships. For example, machine learning used in multi-layered tropical forest stands showed higher efficiency than parametric modelling methods [13]. Recent studies have also used mixed-effect models to fit *h-d* relationships [14]. Such models, in addition to the diameter of trees, which exhibits a fixed effect on the height of trees (i.e., thicker trees are systematically taller), incorporate random-effect factors (forest stand parameters). Random-effect parameters are believed to be effective in explaining a random *h-d* variation depending on the parameters of a particular stand. Many recent publications investigated the influence of density, productivity, and age of stands on the *h-d* relationships using mixed effect models [4; 15; 16].

Stem volumes are estimated using mathematical relationships between volume and other biometric parameters of stems which can be easily measured in a field [17]. Since diameters of growing trees can be measured more easily and with higher accuracy than height, models of *h-d* relationships are of great practical importance. Using such models for certain ecoregions, so-called local volume tables can be developed, which utilize only diameter of tree stems as an independent variable [18]. However, models of total stem volumes based on diameter and height of trees are considered to be more flexible [10].

The type of mathematical relationships between the diameter, height, and volume of stems can be quite diverse. Commonly, it is characterized using nonlinear equations [1; 19]. Models of stem taper are important for determining total and commercial wood volumes [20; 21]. In volume estimation, taper models can be used to predict diameters at specified heights and then determine the stem volume using well known formulae (Huber, Smalian, Newton, etc.) [17]. Additionally, some of taper models may have compatible volume equations, i.e. obtained via integration of taper equations [22-24]. As an alternative to these methods, a stem volume can be considered as a product of the volume of a solid (cylinder) and form factors. The form factor is the ratio of the stem volume to the cylinder volume which significantly simplifies estimation of the total stem volume [17]. This approach of volume modelling relies merely on adequately selected equations to characterize the variability of form factors depending on the diameter and height of trees [25].

The aim of the study is to develop mathematical models of tree stems *h-d* relationships in young and middle-aged forest stands of the main forest-forming species of Ukraine. Comparison of the results obtained with similar data for premature, mature, and over-mature stands would reveal main differences between these age groups of forest stands which represents the originality of this study.

Materials and Methods

The study is based on research materials collected in forest stands of Ukraine during 1950s-2020s. To develop mathematical models, the measurements of more than 10 thousand trees sampled on about 600 temporary and permanent plots were used. In general, the study covers 10 main forest-forming tree species in Ukraine: pine (*Pinus sylvestris* L.), spruce (*Picea abies* L.), fir (*Abies alba* L.), oak (*Querqus robur* L.), beech (*Fagus sylvatica* L.), ash (*Fraxinus excelsior* L.), hornbeam (*Carpinus betulus* L.), aspen (*Populus tremula* L.), birch (*Betula pendula* Roth.), and alder (*Alnus glutinosa* (L.) Gaerth). The sample plots were distributed among the most forested regions of Ukraine, i.e., Polissia, Forest-Steppe, and the Carpathians [26]. Considering the small size of trees in young and middle-aged stands, diameters of all trees on sample plots were measured using 2-cm diameter classes. To construct height curves, heights of 10-25 trees of dominant species selected from different diameter classes were measured. Average tree heights of the 2-cm diameter classes were used as a reference for sampling trees on the plot. According to the distribution of diameters and shape of height curve, 10-25 sample trees were selected and cut down on each plot. Their number was distributed proportionally to the total number of trees within 2-cm diameter classes. Diameters and bark thickness of felled trees were measured at mid-point of 2-m sections, then their volumes outside bark were estimated using the Newton's sectional formula.

Modelling h-d relationships

Relative values of tree heights and diameters were used to model the *h-d* relationships. Such approach allowed the development of a single mathematical model for forest stands of different ages, growth conditions, productivity levels, etc. The earlier studies [12] demonstrated that modelling *h-d* relationships using relative values can provide more accurate results. Thus, relative heights for all diameter classes at sample plots were calculated using the equation (1):

$$h_i^e = h_i / h_{ref}, \quad (1)$$

where h_i^e – relative height for the *i*-th diameter class; h_i – absolute height for the *i*-th diameter class, m; h_{ref} – height for the reference diameter class, m.

The diameter class of 16 cm was used as a reference based on its prevalence as average diameter on sampled forest stands. The authors believe that using the height that corresponds to the average diameter provides a more accurate calculation of relative heights as h_{ref} values in equation (1) are obtained with higher precision. The obtained data were summarised for each tree species and the average values of relative heights were calculated for all diameter classes observed in the study material. Equations (2-4) were used as mathematical models to characterize the *h-d* relationships. In particular, for pine, spruce, oak, beech, and ash, the following equation was used:

$$h_i^e = a_0 + a_1 d^{a_2}, \quad (2)$$

where a_0, a_1, a_2 – equation parameters; d – tree stem diameter, cm.

For fir, aspen, birch, and alder, the relative height model had the form:

$$h_i^e = a_0 + \frac{a_1}{d + a_2}. \quad (3)$$

The following mathematical model was used for hornbeam:

$$h_i^e = a_0 \cdot d^{a_1} \cdot \exp(a_2 \cdot d). \quad (4)$$

Predicted relative height values were used to calculate the absolute heights for each diameter classes using the reference height h_{ref}:

$$h_i = h_i^e \cdot h_{ref} \quad (5)$$

Obtained mathematical models of relative heights allow predicting the height of trees of different levels of productivity and growth conditions. This paper did not aim at developing the *h-d* relationships for stands of different site indices or density, but used standard stand height classes of the existing volume tables for the corresponding species [27]. Thus, 23.0 m was taken as the reference height of the I^a height class. The reference heights corresponding to the reference diameter class of 16 cm for other height classes were calculated using the equation (6):

$$h_{ref} = h_{16} = 23.0 - 2 \cdot I_R. \quad (6)$$

Height classes indices (I_R) of stands in accordance with the officially accepted numbering in forest industry of Ukraine [27] are provided in Table. 1.

Table 1. Indices and numbering of stand height classes in Ukraine

R	I^c	I^b	I^a	I	II	III	IV	V	V^a	V^b
I_R	-2	-1	0	1	2	3	4	5	6	7

Estimating stem volume

Stem volume was estimated using the equation (7):

$$V = g \cdot h \cdot f. \quad (7)$$

where g – cross-sectional area of stems at a height of 1.3 m above ground level (basal area), m²; f – cylindrical form factor; h – stem height, m.

Based on equation (7), the key methodological issue in stem volume estimation is modelling relationships between form factors and other tree stem parameters (e.g., diameter, height). In this study, a mathematical model of form factors as a function of stem diameters and heights $f=\psi(d, h, \ldots)$ was used for such tree species as pine, spruce, oak, beech, and alder. Form factors for the rest of species were predicted based just on stem diameters, i.e. $f=\psi(d,\ldots)$.

The parameters of mathematical models were calculated based on the nonlinear least squares (NLS) regression. In the modelling, several alternative equations were considered for each species. The final model was selected based on a combination of its simplicity and accuracy. The adequacy of models was evaluated using the Fisher's *F*-statistics and determination coefficient R^2 [28]. If several models had a statistically insignificant difference in accuracy, the preference was given to equations with fewer parameters.

Results and Discussion

The parameters of mathematical models (2-4) of relative heights for young and middle-aged stands of the main forest-forming tree species of Ukraine were established in this study (Table 2).

Table 2. Parameters of mathematical models of relative heights depending on stem diameters

No.	Tree species	Parameters of equations (2-4)		
		a_0	a_1	a_2
1.	Pine	3.812	-3.692	-0.0982
2.	Spruce	-0.635	0.6781	0.3174
3.	Fir	2.301	-51.76	23.79
4.	Oak	5.252	-5.178	-0.0710
5.	Beech	9.514	-9.418	-0.0364
6.	Ash	2.021	-2.260	-0.2866
7.	Hornbeam	0.288	0.5225	-0.0128
8.	Aspen	1.426	-8.982	5.109
9.	Birch	1.435	-9.549	5.930
10.	Alder	1.587	-14.75	9.137

Figure 1 shows an example of relative height curves for individual tree species. Among species, the *h-d* relationships are distinguished only for spruce and fir growing in the mountain forests of the Carpathians. For the rest of the main forest-forming tree species in Ukraine, the shapes of relative height curves were quite similar. The developed models of the tree *h-d* relationships in tabular form had been published in the new Forest inventory handbook (Table 2.1-2.10) [27].

Models of tree stem volumes outside bark for young and middle-aged stands of the main forest-forming tree species of Ukraine are given in Table 3.

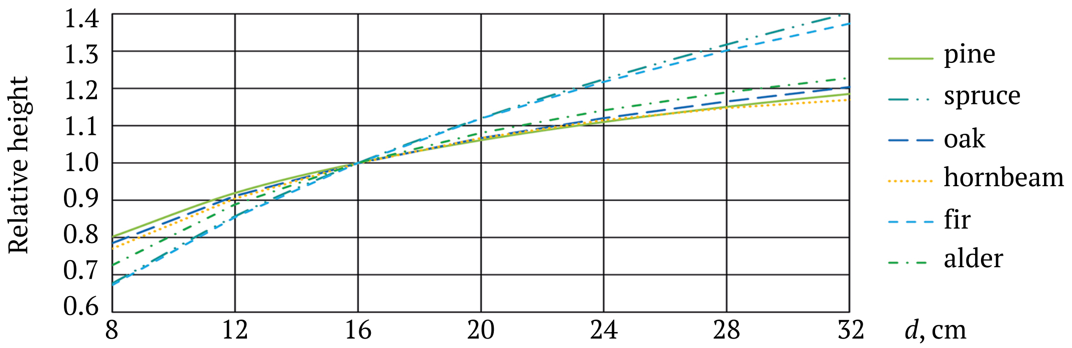

Figure 1. The relationships between tree heights and diameters in forest stands of different tree species

Table 3. Mathematical models of tree stem volumes outside bark

No.	Tree species	Mathematical model of stem volume outside bark
1.	Pine	$V_{ob}=(576.5+9059\cdot h^{-2,207}-19.47\cdot d^{0.5451})\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
2.	Spruce	$V_{ob}=\left(521.0+4.300\cdot h+\frac{301.8}{h-2.93}-10.39\cdot d^{0.811}\right)\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
3.	Fir	$V_{ob}=\left(588.4-2.864\cdot d+\frac{62.75}{d-3.663}\right)\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
4.	Oak	$V_{ob}=\left(h^{0.0957}\cdot \exp\,(5.830+\frac{0.9721}{d}+\frac{1.804}{h}-0.00230\cdot d)\right)\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
5.	Beech	$V_{ob}=\left(0.541+\frac{1.570}{d^2}+\frac{6.963}{h^2}-\frac{3.915}{d\cdot h}-0.07841\cdot \frac{d}{h}\right)\cdot d^2\cdot h\cdot 7.854\cdot 10^{-5}$
6.	Ash	$V_{ob}=(435.2+685.2\cdot d^{-0.9495})\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
7.	Hornbeam	$V_{ob}=(371.3+1947/(d+4.585))\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$

Table 3, Continued

No.	Tree species	Mathematical model of stem volume outside bark
8.	Aspen	$V_{ob}=(-104.7+700.9\cdot d^{-0.0581})\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
9.	Birch	$V_{ob}=(654.4-40.02\cdot d^{0.538})\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$
10.	Alder	$d<16$ cm $V_{ob}=(1677\cdot d^{-0.0991}\cdot h^{-0.5583}\cdot \exp\cdot(0.03624\cdot h))\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$ $d\geqslant 16$ cm $V_{ob}=(398.1+314.3\cdot d^{-0.4555})\cdot d^2\cdot h\cdot 7.854\cdot 10^{-8}$

Note: V_{ob} – tree stem volume outside bark, m^3

The developed models of relative heights (2-4) and the mathematical models of tree stem volumes (Table 3) were used to construct volume tables by height classes for young and middle-aged stands [27; Table 2.42-2.51]. These tables are used for wood volume estimation extracted during thinning or salvage logging in young and middle-aged forests in Ukraine. In addition, the developed set of models significantly simplifies methods of growing stock volume estimation during sample-based forest inventory, since there is no need to measure heights of all tally trees on sample plots. Accordingly, the value of the variable *h* in the stem volume models (Table 3) can be estimated using equations (2-6).

As it was noted in earlier publications, the height curves that characterize the relationship between the heights and diameters of stems in forest stands of a certain tree species can be variable and depend primarily on age. Accordingly, the shape of the relative height curves obtained in this study was compared with the corresponding curves obtained for older stands [29]. To properly compare the shape of the curves, relative heights for both groups of stands were calculated using the common reference height obtained for diameter class of 24 cm. The relative height curves in both young and middle-aged, and premature, mature, and over-mature stands had similar shape for spruce, fir, and beech which predominate in the forests of the Ukrainian Carpathians. The relative height curves among these two age groups differ most for pine, oak, ash, birch, and aspen. Notably, significant differences in tree height growth of different age groups were noted by many other studies [7; 30]. As an example, Figure 2 shows relative height curves for individual tree species in stands of different ages.

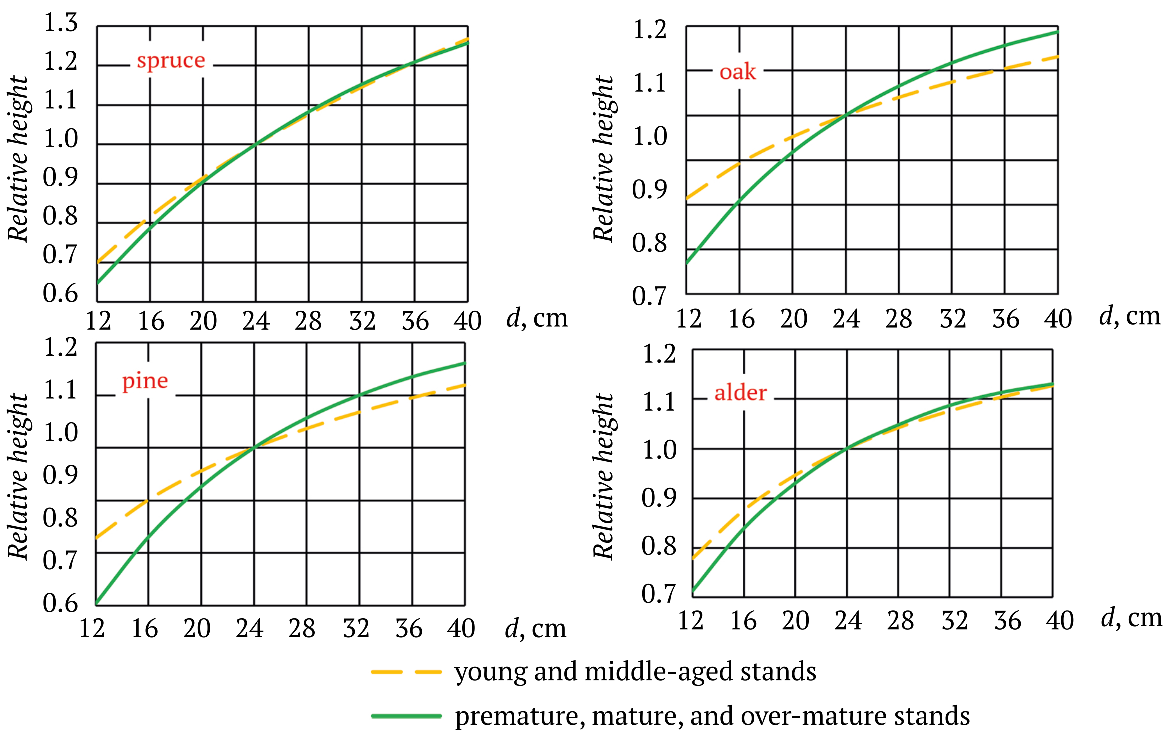

Figure 2. Relationships between tree heights and diameters in forest stands of different age groups

It is well known in forest mensuration that tree stem volumes of the same size, i.e., diameter and height, can vary due to differences in their shapes. Accordingly, there are distinguished tree species in the forests of Ukraine that have smaller stem taper, thus larger volumes. Based on the developed models (Table 3), stems of fir, oak, and aspen in young and middle-aged stands reach the largest volumes for fixed values of diameter and height. Hornbeam and birch are characterized by the smallest volume of tree stems. Regarding the volume of stems of the same size in stands of different age groups (young and middle-aged versus premature, mature, and over-mature), systematically higher (5-7%) volumes are observed in young and middle-aged stands for coniferous tree species. In some cases, this difference can surpass even 10%. Similar trends were also observed for some deciduous tree species of Ukrainian forests, in particular, stem volumes in young and middle-aged aspen and birch stands are 3-7% higher than in premature, mature, and over-mature forests. For other deciduous tree species, there is no systematic difference in tree stem volumes of the same size in different age groups. As an example, Figure 3 compares for selected tree species the volume of tree stems of typical sizes in forest stands of two age groups.

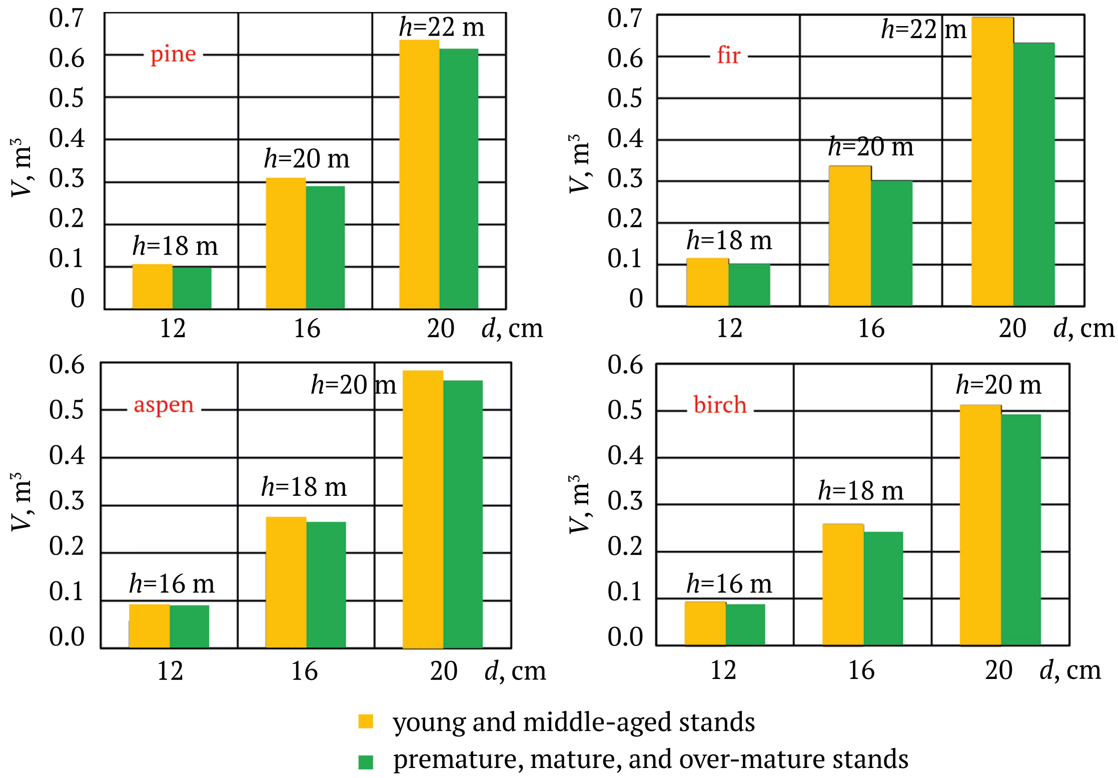

Figure 3. Comparison of stem volume in stands of different age groups

A comparison of the developed models of tree stem volumes in forest stands of Ukraine with data obtained in other European countries [29], showed distinct differences for some tree species. Mathematical models of the volume of tree stems in the forests of Germany, the Czech Republic, Poland, the Netherlands, Belgium, Great Britain, Finland, Norway, and Sweden were used. Stem volumes in these countries were modelled directly from the diameter and height of the stems using nonlinear regression equations, mainly with power or exponential functions. It was found that deviations in stem volumes for coniferous tree species generally do not exceed ±5% and only for the thinnest trees could reach 10% and higher. Among the tested models, the Swedish volume model for pine stems is characterized by the lowest differences (mainly up to ±2%) compared to the developed one in this study. The volume prediction for stems of spruce trees using various equations is characterized by nearly equal accuracy, with the exception of the volume model developed by Finnish scientists. Thus, the volume of spruce tree stems with heights of more than 15 m and diameters up to 20 cm in Finland is systematically lower (10-15%) than in Ukraine, Poland,

and Sweden. As an example, Figure 4 shows models of tree stem volumes as a function of diameter using two fixed values of height of 12 and 18 m for the predominant tree species in the forests of Ukraine. The choice of these values is explained by average tree heights in young and middle-aged forests of Ukraine.

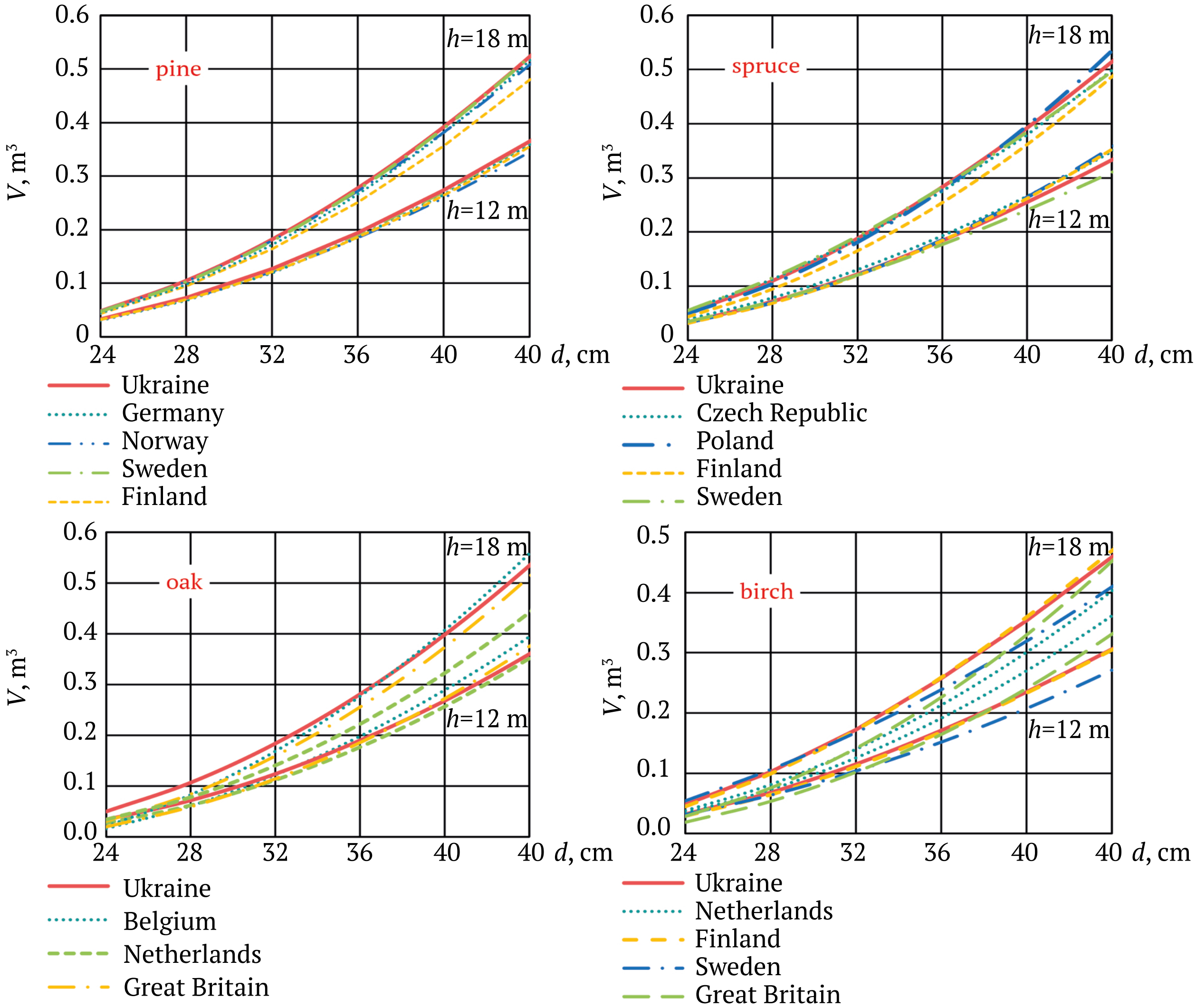

Figure 4. Comparison of the volume of tree stems as a function of diameter at a constant height

For deciduous tree species, volume curves are characterized by higher variability compared to conifers species (Fig. 4). Deviations in the stem volume for trees with diameters less than 16 cm often exceed 10%, and in some cases can reach more than 20%. However, the differences between developed in this study and published volume models [1] are not systematic. The stem volumes in Finland, Sweden, Great Britain, Belgium, and the Netherlands are systematically lower only for the thinnest trees (usually less than 20 cm in diameter) in comparison to the presented ones in this paper.

A detailed analysis of the developed mathematical models of tree stem volume and comparison of them with similar models for different European countries [1] indicates a high variability of volume estimates. Based on the identified differences in models for stands of different age groups (Fig. 2 and Fig. 3), the development of a set of national-scale mathematical models of stem volumes is reasonable. In Ukraine, two age categories of forest stands are proposed to consider for which systematic differences between stem volumes and *h-d* relationships were identified: 1) young and middle-aged stands, and 2) premature, mature, and over-mature stands.

Besides the practical aspect aimed at more accurate assessment of forest resources, the developed set of models has sufficient scientific importance. For example, mathematical models of *h-d* relationships in forest stands of different ages are widely used in forest ecology. This provides an understanding of tree competition in a stand, which can be used to justify effective treatment scenarios to increase forest productivity and sustainability. Stem volume models along with growth models are important for a reliable assessment of forest ecosystem functions (e.g., carbon sequestration) and forecasting forest vulnerability in response to climate change. The practical significance of the developed mathematical models is explained by their utility in wood volume estimation harvested in young and meddle-aged stands during thinning or salvage logging.

Conclusions

Accurate estimation of tree stem volumes is essential for addressing current and strategic forest management challenges. In particular, the differences in the growth of forest stands of different age categories should be explicitly explained by relevant mathematical models. This paper presents the modelling results of *h-d* relationships in young and middle-aged forest stands as well as the corresponding stem volume models. The study identified differences (up to 10%) between the volumes of trees of the same size in young and middle-aged forest stands compared to the older stands. Considering the differences in *h-d* relationships for individual tree species, the developed mathematical models allow refining the forecast of growing stock volume in young and middle-aged forest stands of Ukraine. Besides theoretical significance, the obtained results have practical applications in evaluation (including monetary valuation) of log-grade distribution of harvested wood during thinning operations. The use of mathematical models of *h-d* relationships could also significantly reduce labour costs of fieldwork operations during the national forest inventory of Ukraine, while the developed stem volume models could be used for accurate estimation of sample plot-level growing stock volumes.

References

[1] Zianis, D., Muukkonen, P., Mäkipää R., & Mencuccini, M. (2005). *Biomass and stem volume equations for tree species in Europe*. Tampere: Tammer-Paino Oy.

[2] Tian, X., Sun, S., Mola-Yudego, B., & Cao, T. (2020). Predicting individual tree growth using stand-level simulation, diameter distribution, and Bayesian calibration. *Annals of Forest Science*, 77(2), article number 57. doi: 10.1007/s13595-020-00970-0.

[3] Saunders, M.R., & Wagner, R.G. (2008). Height-diameter models with random coefficients and site variables for tree species of Central Maine. *Annals of Forest Science*, 65(2), 203-203. doi: 10.1051/forest:2007086.

[4] Zhang, X., Fu, L., Sharma, R.P., He, X., Zhang, H., Feng, L., & Zhou, Z. (2021). A nonlinear mixed-effects height-diameter model with interaction effects of stand density and site index for larix olgensis in Northeast China. *Forests*, 12(11), article number 1460. doi: 10.3390/f12111460.

[5] Mehtätalo, L. (2005). Height-diameter models for Scots pine and birch in Finland. *Silva Fennica*, 39(1), 55-66. doi: 10.14214/sf.395.

[6] de-Miguel, S., Pukkala, T., Assaf, N., & Bonet, J.A. (2012). Even-aged or uneven-aged modelling approach? A case for *Pinus brutia*. *Annals of Forest Science*, 69(4), 455-465. doi: 10.1007/s13595-011-0171-2.

[7] Sharma, M. (2016). Comparing height-diameter relationships of boreal tree species grown in plantations and natural stands. *Forest Science*, 62(1), 70-77. doi: 0.5849/forsci.14-232.

[8] Duan, G., Gao, Z., Wang, Q., & Fu, L. (2018). Comparison of different height–diameter modelling techniques for prediction of site productivity in natural uneven-aged pure stands. *Forests*, 9(2), article number 63. doi: 10.3390/f9020063.

[9] Temesgen, H., Hann, D.W., & Monleon, V.J. (2007). Regional height–diameter equations for major tree species of Southwest Oregon. *Western Journal of Applied Forestry*, 22(3), 213-219. doi: 10.1093 wjaf/22.3.213.

[10] Burkhart, H.E., & Tomé, M. (2012). *Modeling forest trees and stands*. Dordrecht: Springer. doi: 10.1007/978-90-481-3170-9.

[11] Liu, M., Feng, Z., Zhang, Z., Ma, C., Wang, M., Lian, B., Sun, R., & Zhang, L. (2017). Development and evaluation of height diameter at breast models for native Chinese Metasequoia. *PLOS ONE*, 12(8), article number e0182170. doi: 10.1371/journal.pone.0182170.

[12] Leduc, D., & Goelz, J. (2009). A height–diameter curve for longleaf pine plantations in the Gulf Coastal Plain. *Southern Journal of Applied Forestry*, 33(4), 164-170. doi: 10.1093/sjaf/33.4.164.

[13] Ogana, F.N., & Ercanli, I. (2022). Modelling height-diameter relationships in complex tropical rain forest ecosystems using deep learning algorithm. *Journal of Forestry Research*, 33(3), 883-898. doi: 10.1007/s11676-021-01373-1.
[14] Zuur, A.F. (Ed.). (2009). *Mixed effects models and extensions in ecology with R*. Dordrecht: Springer.
[15] Bronisz, K., & Mehtätalo, L. (2020). Mixed-effects generalized height–diameter model for young silver birch stands on post-agricultural lands. *Forest Ecology and Management*, 460, article number 117901. doi: 10.1016/j.foreco.2020.117901.
[16] Temesgen, H., Zhang, C.H., & Zhao, X.H. (2014). Modelling tree height–diameter relationships in multi-species and multi-layered forests: A large observational study from Northeast China. *Forest Ecology and Management*, 316, 78-89. doi: 10.1016/j.foreco.2013.07.035.
[17] Kershaw, J.A., Ducey, M.J., Beers, T., & Hush, B. (2016). *Forest mensuration*. Hoboken: Wiley-Blackwell.
[18] Brooks, J.R., & Wiant, H.V. (2008). Ecoregion-based local volume equations for Appalachian hardwoods. *Northern Journal of Applied Forestry*, 25(2), 87-92. doi: 10.1093/njaf/25.2.87.
[19] Muukkonen, P. (2007). Generalized allometric volume and biomass equations for some tree species in Europe. *European Journal of Forest Research*, 126(2), 157-166. doi: 10.1007/s10342-007-0168-4.
[20] McTague, J.P., & Weiskittel, A. (2021). Evolution, history, and use of stem taper equations: A review of their development, application, and implementation. *Canadian Journal of Forest Research*, 51(2), 210-235. doi: 10.1139/cjfr-2020-0326.
[21] Salekin, S., Catalán, C.H., Boczniewicz, D., Phiri, D., Morgenroth, J., Meason, D.F., & Mason, E.G. (2021). Global tree taper modelling: A review of applications, methods, functions, and their parameters. *Forests*, 12(7), article number 913. doi: 10.3390/f12070913.
[22] Fang, Z. (2000). Compatible volume-taper models for Loblolly and Slash Pine based on a system with segmented-stem form factors. *Forest Science*, 46(1), article number 12.
[23] McClure, J.P., & Czaplewski, R.L. (1986). Compatible taper equation for loblolly pine. *Canadian Journal of Forest Research*, 16(6), 1272-1277. doi: 10.1139/x86-225.
[24] Özçelik, R., & Brooks, J.R. (2012). Compatible volume and taper models for economically important tree species of Turkey. *Annals of Forest Science*, 69(1), 105-118. doi: 10.1007/s13595-011-0137-4.
[25] Tenzin, J., Wangchuk, T., & Hasenauer, H. (2016). Form factor functions for nine commercial tree species in Bhutan. *Forestry*, 90, 359-366. doi: 10.1093/forestry/cpw044.
[26] Gensiruk, S.A. (1992). *Forests of Ukraine*. Kyiv: Naukova dumka.
[27] Bilous, A., Kashpor, S, Myroniuk, V., Svynchyk, V., & Lesnik, O. (2021). *Forest inventory handbook*. Kyiv: Vinichenko.
[28] Draper, N.R., & Smith, H. (1998). *Applied regression analysis*. Hoboken: Wiley-Interscience.
[29] Bilous, A., Myroniuk, V., Svynchuk, V., Kashpor, S., & Lesnik, O. (2022). Tree stem volumes by height classes in premature, mature and overmature stands of main sorest species of Ukraine. *Ukrainian Journal of Forest and Wood Science,* 13(3), 7-12. doi: 10.31548/forest.13(3).2022.7-12.
[30] Soshenskiy, O.M. (2016). *Peculiarities of biometrics structure, wood sort and merchantability structure of linden tree stands in the Lisostep of Ukraine* (Doctoral thesis, National University of Life and Environmental Sciences of Ukraine, Kyiv, Ukraine).

Співвідношення між висотами і діаметрами та рівняння об'єму стовбурів дерев у молодняках і середньовікових лісових насадженнях України

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Анотація. Співвідношення висот і діаметрів дерев (*h-d*) у лісових насадженнях зазвичай використовується в різних наукових і практичних задачах лісового господарства. При цьому точні моделі *h-d* у поєднанні з рівняннями об'єму стовбурів дерев ефективні в оцінці об'єму запасів. *Мета роботи* полягає в 1) розробці системи математичних моделей співвідношення висот і діаметрів у молодняках і середньовікових деревостанах для десяти лісоутворювальних видів Україні; 2) моделюванні об'єму стовбурів у зазначених категоріях лісових насаджень; 3) порівнянні виявлених залежностей із аналогічними даними для пристигаючих, стиглих і перестійних деревостанів. Дослідження ґрунтувалося на даних постійних і тимчасових пробних площ (близько 600), закладених у лісових насадженнях протягом 1950–2020-х років у найбільш лісистих регіонах України (Полісся, Лісостеп, Карпати). З метою точної оцінки об'єму стовбурів у корі на пробних площах було обміряно близько 10 тис. модельних дерев. Моделі співвідношення *h-d* були підібрані з використанням відносних значень висоти дерев та абсолютних значень діаметра, які продемонстрували дуже схоже співвідношення між висотою та діаметром дерев для більшості деревних видів, за винятком ялини та ялиці в гірських лісах Карпат, де були отримані більш круті криві висот. Встановлено, що стовбури берези та граба мають найменший об'єм серед досліджуваних деревних видів. Результати досліджень також засвідчили, що стовбури дерев, як правило, мають більший об'єм (до 7 % для хвойних і до 10 % для осики та берези) у молодняках і середньовікових лісових насадженнях, ніж у деревостанах старшого віку. Для інших деревних видів автори не виявили статистично значущої різниці між об'ємами стовбурів дерев різного віку. Розроблені математичні моделі можуть доповнювати відповідні моделі для старших груп лісових насаджень, оскільки вони розкрили важливий аспект взаємозв'язків між ключовими таксаційними показниками стовбурів дерев. Вони також можуть застосовуватися для точнішої оцінки об'єму стовбурів дерев під час рубок догляду в молодняках та середньовікових насадженнях України з практичної точки зору

Ключові слова: крива висот, об'ємні таблиці, розряд висот, видове число, прогнозування