УДК 539.3

H. TAHARZADEH, PHD student, NTU «KPI»;
K. V. AVRAMOV, doctor of engineering science, NTU «KPI»

ON VIBRATION OF CLAMPED-FREE CYLINDRICAL SHELL

Для расчета собственных частот и форм колебаний консольных цилиндрических оболочек применяется метод Релея-Ритца. Анализируются свойства сопряженных собственных форм колебаний. Результаты расчетов сравниваются с данными конечноэлементного анализа.

Ключевые слова: метод Релея-Ритца, собственные частоты, собственные формы.

1 Introduction. Cylindrical shells are widely used in mechanical and aerospace engineering. Theory of linear vibration of cylindrical shells is treated by Liessa [1]. The modern state of art of shells free vibration is treated in the book of Amabili [2]. The applications of asymptotic methods to nonlinear vibrations of cylindrical shell are treated in the paper [3]. The linear and nonlinear vibrations of cantilever shell with rigid disk at the end are analyzed in the paper [4]. The forced vibrations of the cantilever cylindrical shell are analyzed in the paper [5]. The authors used the Chebyshev polynomials to approximate the shell displacements. Parametric vibrations of cylindrical shells and influence of initial imperfections on shell dynamics are treated by Kochurov and Avramov [6, 7]. Pellicano [8] used the orthogonal polynomials to expand longitudinal displacements. Thus, different boundary conditions are satisfied.

In this paper the Rayleigh –Ritz method is used to analyze the vibration of the cantilever cylindrical shells. The result are compared with finite element analyze. The properties of eigenmodes of free vibrations are analyzed .

2 Problem formulation and equations of motions. Thin, clamped-free cylindrical shell is considered (Fig.1). Shear is not taken into account. It is assumed, that the strains and displacements are small. Therefore, the strain-displacements relations are linear. It is assumed, that the cylindrical shell is without imperfections. Thus, the cylindrical shell performs linear vibrations. The position of the point on the shell middle surface is described by two coordinates (x,θ). The projections of the displacements of middle surface points on x, θ, z curves are denoted by $u(x,\theta,t)$, $v(x,\theta,t)$, $w(x,\theta,t)$ respectively. Then the elastic potential energy of the shell takes the following form [1]:

$$P=\frac{1}{2}\frac{Eh}{\left(1-\upsilon^2\right)}\int_0^{2\pi}\int_0^{L}\left(\varepsilon_{x,0}^2+\varepsilon_{\theta,0}^2+2\nu\varepsilon_{x,0}\varepsilon_{\theta,0}+\frac{1-\nu}{2}\gamma_{x\theta,0}^2\right)dxRd\theta+$$

$$+\frac{1}{2}\frac{Eh^3}{(1-\nu^2)}\int_0^{2\pi}\int_0^{L}\left(k_{x,0}^2+k_{\theta,0}^2+2\nu k_{x,0}k_{\theta,0}+\frac{1-\nu}{2}k_{x\theta,0}^2\right)dxRd\theta+ \qquad (1)$$

$$+\frac{1}{2}\frac{Eh^3}{6R(1-\nu^2)}\int_0^{2\pi}\int_0^{L}\left(\varepsilon_{x,0}k_x+\varepsilon_{\theta,0}k_\theta+\nu\varepsilon_{x,0}k_{x,\theta}+\frac{1-\nu}{2}\right)dxRd\theta+O\left(h^4\right),$$

where E is Young ‘s modulus; ν is the Poisson ratio; R is shell radius; h is the shell thickness. The first term of potential energy describes stretching and compression of the shell middle surface. The second and the third terms describe the shell bending. The strains of the shell middle surface and displacements satisfy the following equation:

$$\varepsilon_{x,0}=\frac{\partial u}{\partial x};\quad \varepsilon_{\theta,0}=\frac{\partial v}{R\partial\theta}+\frac{w}{R};\quad \varepsilon_{\theta,0}=\frac{\partial u}{R\partial\theta}+\frac{\partial v}{\partial x};$$

$$k_x=-\frac{\partial^2 w}{\partial x^2};\quad k_\theta=-\frac{\partial^2 w}{R\partial\theta^2};\quad k_{x\theta}=-2\frac{\partial^2 w}{R\partial x\partial\theta}.$$

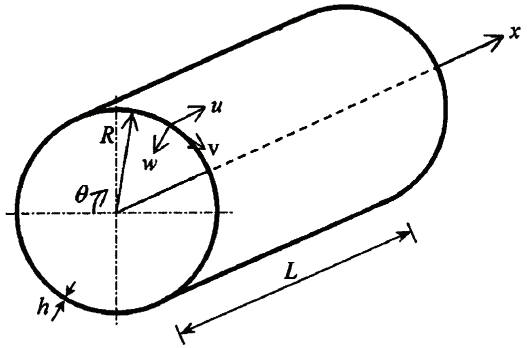

Figure 1 – Circular cylindrical shell

The kinetics energy of cylindrical shell can be written as

$$T_s=\frac{1}{2}\rho_s h\int_0^{2\pi}\int_0^{L}\left(\dot{w}^2+\dot{u}^2+\dot{v}^2\right)dxRd\theta\,, \qquad (2)$$

where ρ_s is material density; L is length of the shell.

The shell is clamped at the edge $x = 0$ and it is free at $x = L$. Thus, the following boundary conditions are true:

$$u=v=w=\frac{\partial w}{\partial x}=0 \ \text{ at } \ x=0$$

$$N_x=N_{x,\theta}+\frac{M_{x\theta}}{R}=M_x=Q_x+\frac{\partial M_{x,\theta}}{R\partial\theta}=0 \quad \text{at} \quad x=L,$$

where N_x, $N_{x,\theta}$ are membrane forces; M_x, $M_{x,\theta}$ are bending and torsional moments.

The boundary conditions at $x = 0$ are geometric and the boundary conditions at $x = L$ are natural. The method Rayleigh –Ritz is used to find free vibrations. Therefore, only geometric boundary conditions are satisfied and the natural boundary conditions are ignored.

The shell linear vibrations take the following form:

$$W(x,\theta,t) = \widetilde{W}(x,\theta)\sin\omega t\,;$$
$$U(x,\theta,t) = \widetilde{U}(x,\theta)\sin\omega t\,; \tag{3}$$
$$V(x,\theta,t) = \widetilde{V}(x,\theta)\cos\omega t\,.$$

The equations (3) are substituted into (1, 2). Then the kinetic and potential energies can be presented as:

$$T(x,\theta,t) = \omega^2 \sin^2(\omega t)\,\overline{T}\,(x,\theta)\,;$$
$$P\,(x,\theta,t) = \sin^2(\omega t)\overline{P}\,(x,\theta)\,.$$

As the shell is closed, the functions $\widetilde{W}(x,\theta)$, $\widetilde{U}(x,\theta)$, $\widetilde{V}(x,\theta)$ can be presented in the form of the double Fourier series:

$$W(x,\theta) = \sum_{m=1}^{N1}\sum_{n=1}^{N2} W_{m,n}\,\phi_m(x)\,\cos(n\theta);$$
$$U(x,\theta) = \sum_{m=1}^{N3}\sum_{n=1}^{N4} U_{m,n}\,\chi_m(x)\,\cos(n\theta)\,; \tag{4}$$
$$V(x,\theta) = \sum_{m=1}^{N5}\sum_{n=1}^{N6} V_{m,n}\,\chi_m(x)\sin(n\theta)\,,$$

where ϕ_m, χ_m, are beam functions; $W_{m,n}$, $U_{m,n}$, $V_{m,n}$ are unknown coefficients, which are determined by Rayleigh- Ritz method.

For the clamped- free cylindrical shell the beam functions $\chi_m(x)$ takes the following form:

$$\chi_m(x) = \sin\left(\frac{\pi\, x\,(2m-1)}{2L}\right).$$

The eigenmodes of cantilever beam are taken for the functions $\phi_m(x)$:

$$\phi_m(x) = \sin\left(\frac{\lambda_m x}{L}\right) + \alpha_m \cos\left(\frac{\lambda_m x}{L}\right) + \beta_m\left[\sin\left(\frac{\lambda_m x}{L}\right) + \eta_m \cosh\left(\frac{\lambda_m x}{L}\right)\right].$$

The following functional is used to calculate the eigenfrequencies and the eigenmodes:

$$\int_0^{2\pi/\omega} (P - \mathrm{T})\,dt = \frac{\pi}{\omega}\left[\overline{P}(W_{1,1},\ldots,V_{N5,N6}) - \omega^2 \mathrm{T}(W_{1,1},\ldots,V_{N5,N6})\right]. \tag{5}$$

Minimum of the functional (5) on the set of the variables $X = \{W_{1,1},\ldots,V_{N5,N6}\}$ is determined. The conditions of the minimum of the functional have the following form:

$$\frac{\partial}{\partial W_{mn}}(\overline{P}-\omega^2\overline{T})=0,(n=1...N_1,m=1..N_2);$$

$$\frac{\partial}{\partial U_{mn}}(\overline{P}-\omega^2\overline{T})=0,(n=1...N_3,m=1..N_4); \qquad (6)$$

$$\frac{\partial}{\partial V_{mn}}(\overline{P}-\omega^2\overline{T})=0,(n=1...N_5,m=1..N_6).$$

The equations (6) can be transformed into the following eigenvalue problem:

$$Det\left[C-\omega^2 M\right]=0, \qquad (7)$$

where C, M are stiffness and mass matrixes.

3 Free linear vibration modal analysis. In this section a linear vibration analysis of shell is carried out and numerical results are compared with the result, obtained by software ANSYS. The calculations of eigenfrequencies and eigenmodes are carried out for the shell with the following numerical values of parameters:

$$L = 0{,}48 \text{ m};\quad h = 0{,}178\cdot 10^{-3} \text{ m};\quad E = 6{,}8258\cdot 10^{10} \text{ Pa};$$
$$\rho = 2712{,}2 \text{ kg/m}^3;\quad \nu = 0{,}3;\quad R = 0{,}073914 \text{ m}.$$

The eigenvalue problem (7) is solved to calculate eigenfrequencies and eigenmodes of the shell. The obtained results are compared with the data, obtained by software ANSYS, and with the results, published by Kurilov and Amabili [5] and by Leissa [1]. The results of the eigenfrequency analysis are shown on Table. The data, obtained by the Rayleigh- Ritz method, is published in the first row of the Table. The results, obtained by the software ANSYS, are published in the second row of the Table. The data, which are published in the papers [5] and [1], are shown on the third and fourth rows of the Table, respectively. As you can see from the Table, the results, obtained by different methods, are close.

Wave number	1	2	3	4	5
Rayleight-Ritz	177,2	204,8	243,3	294,1	384,3
Ansys	176,6	208,63	234,03	286,81	391,83
Kurylov	175,7	205,3	233,9	279,4	377,6
Liessa	181	207	246	280	378

Fig.2 shows the eigenmodes of the shell bending vibrations. They are arranged in ascending order of the eigenfrequency. The shell bending vibrations in the shell longitudinal direction are governed by the eigenmode of the cantilever beam. Note, that the first seven eigenmodes of the bending vibrations are governed by the first eigenmode of the cantilever beam. The first and the second eigenmodes of the bending vibrations contain eight and ten nodes in the circumference direction. The node number of the eigenmodes can be followed from Fig. 2. Let us

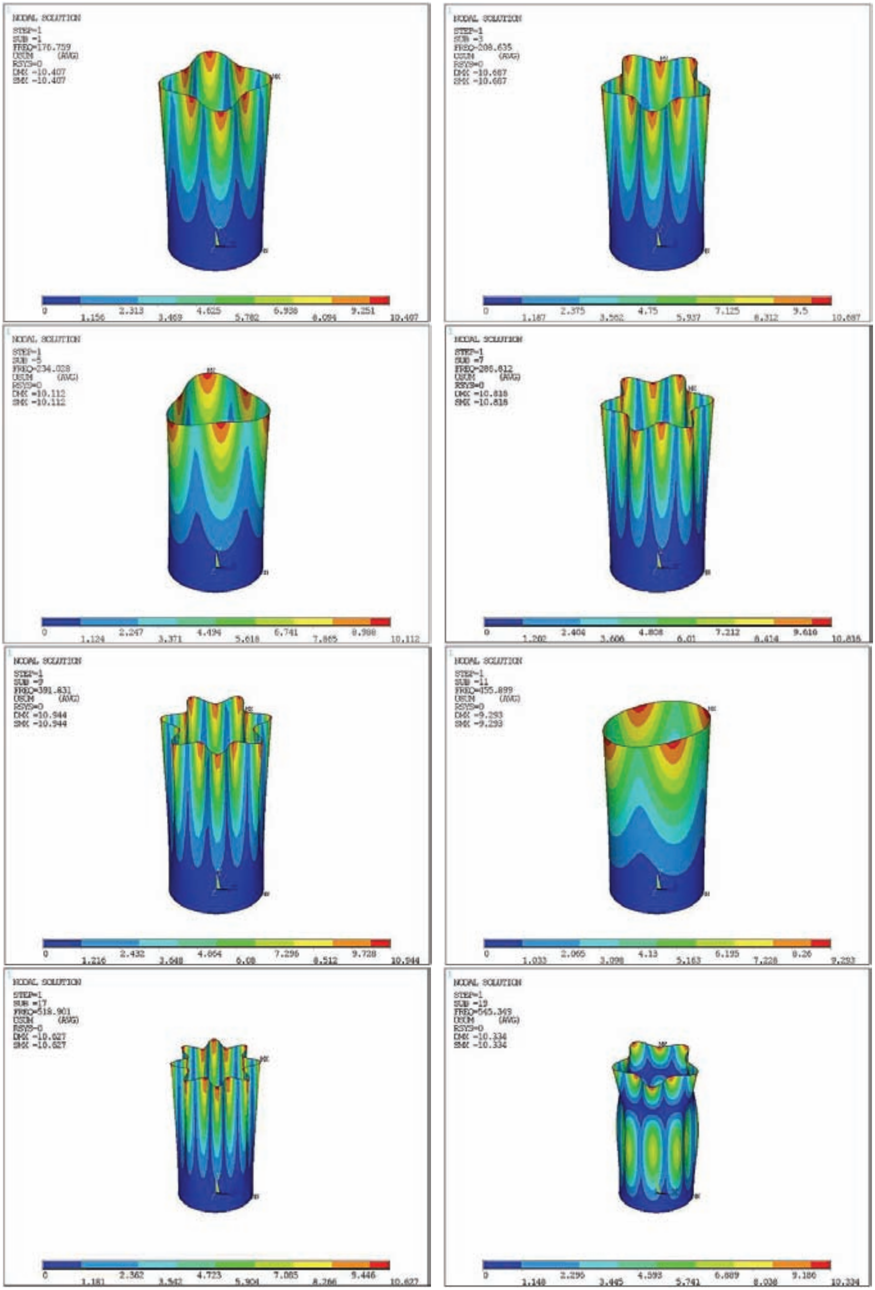

Figure 2 – Eigenmodes of the shell vibrations

consider the six eigenmode. It consists of mostly longitudinal shell vibrations. The bending component is absent on this eigenmode.

Conclusion. The version of Rayleigh- Ritz method is suggested to study the vibrations of the cantilever cylindrical shells. The numerical calculations confirm the effectiveness of the suggested approach for analysis of the cantilever cylindrical shells.

The properties of eigenmodes have been analyzed.

References. 1. *Leissa A.W.* Vibration of Shells // NASA SP-288, Government Printing Office, Washington DC, 1973. **2**. *Amabili M.* Nonlinear vibrations and Stability of Shells and Plates // Cambridge University Press, New York, 2008. **3.** *Avramov K.V, Mikhlin Yu.V, Kurilov. E.* Asymptotic analysis of nonlinear dynamic of simply supported cylindrical shells. – 2007. **4.** *Pellicano F, Avramov K.V.* Linear and nonlinear dynamic of a circular cylindrical shell connected to rigid disk // Journal of Sound and Vibration. – 2007. **5.** *Kurylov Ye, Amabili M.* Polynomial versus trigonometric expansions for nonlinear vibrations of circular cylindrical shells with different boundary conditions // Journal of Sound and Vibration. – 329 (2010). – P. 1435–1449. **6**. *Kochorov R., Avramov K.A.* nonlinear modes and traveling waves of parametrically excited cylindrical shells // Journal of Sound and Vibration. – 2010. **7.** *Kochorov R., Avramov K.V.* Parametric vibration of cylindrical shells in the region of combination resonances under geometrically nonlinear deformation // Journal of Sound and Vibration. – 2011. **8.** *Pellicano F.* Vibrations of circular cylindrical shells: theory and experiments // Journal of Sound and Vibration 303. – 2007.

Has gone to editorial board 16.10.2013

УДК 539.3

On vibration of clamped-free cylindrical shell / H. Taharzadeh, K. V. Avramov // Вісник НТУ «ХПІ». Серія: Динаміка і міцність машин. – Х.: НТУ «ХПІ», 2013. – № 63 (1036). – С. 147-153. – Бібліогр.: 8 назв.

Для розрахунку власних частот і форм коливань консольних циліндрових оболонок застосовується метод Релея-Рітца. Аналізуються властивості зв'язаних власних форм коливань. Результати розрахунків порівнюються з даними скінченноелементного аналізу.

Ключові слова: метод Релея-Рітца, власні частоти, власні форми.

The Rayleigh-Ritz method is applied to analyze the eigenfrequencies and the eigenmodes of the cantilever cylindrical shells. The properties of the conjugate eigenmodes are analyzed. The results of the analysis are compared with the data of finite element calculations.

Key words: Rayleigh-Ritz method, eigenfrequencies, eigenmodes.