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AMPLIFICATION OF Z' SIGNAL IN $e^+e^- \to \mu^+\mu^-$ PROCESS

One-parameter observables with the best value-to-uncertainty ratio are proposed to estimate possible signals of the Abelian Z' boson in the $e^+e^- \to \mu^+\mu^-$ scattering process. The value-to-uncertainty ratio is chosen as a natural criterion allowing the statistical amplification of the signal in experiment. The model independent relations between the Abelian Z' couplings to leptons are used in order to reduce the number of unknown parameters of the particle. The observables are constructed by angular integration with proper weight functions. In order to perform numeric optimization a set of orthogonal polynomials is introduced taking into account the kinematics of the process. The optimal weight functions are found to be smooth step-like functions close to the hyperbolic tangent shape. The observables are applied to data on differential cross-sections obtained in the LEP experiments. The Z' couplings to axial-vector and vector lepton currents are fitted and compared to other estimates.

Keywords: high energy physics, Z' bosons, differential cross-section, integrated cross-section.

Однопараметрические наблюдаемые с наилучшим отношением величины к неопределенности предложены для оценки возможных сигналов абелевого Z' бозона в процессе рассеяния $e^+e^- \to \mu^+\mu^-$. Отношение величины к неопределенности выбрано в качестве естественного критерия, позволяющего статистически усилить сигнал в эксперименте. Для уменьшения количества неизвестных параметров нового бозона применяются модельно-независимые соотношения между константами связи абелевого Z' с лептонами. Наблюдаемые построены угловым интегрированием с подходящей весовой функцией. Численная оптимизация выполняется при помощи системы ортогональных полиномов, введенных с учетом кинематики процесса. Оптимальные весовые функции выглядят сглаженными ступенчатыми, похожими на гиперболический тангенс. Наблюдаемые применены к дифференциальным сечениям из экспериментов LEP. Фитированы значения констант связи Z' с векторными и аксиально-векторными токами лептонов и сопоставлены с другими оценками.

Ключевые слова: физика высоких энергий, Z' бозоны, дифференциальные сечения рассеяния, интегральные сечения рассеяния.

Однопараметричні спостережувані з найкращим відношенням величини до невизначеності запропоновані для оцінки можливих сигналів абелевого Z' бозона в процесі розсіяння $e^+e^- \to \mu^+\mu^-$. Відношення величини до невизначеності обрано в якості природного критерію, який дозволяє статистично посилити сигнал в експерименті. Для зменшення кількості невідомих параметрів нового бозона застосовуються модельно-незалежні співвідношення між константами зв'язку абелевого Z' з лептонами. Спостережувані побудовані кутовим інтегруванням з доцільною ваговою функцією. Чисельна оптимізація виконується за допомогою системи ортогональних поліномів, введених з урахуванням кінематики процесу. Оптимальні вагові функції виглядають згладженими ступеневими, схожими на гіперболічний тангенс. Спостережувані застосовані до диференційних перерізів з експериментів LEP. Фітовані значення констант зв'язку Z' з векторними та аксіально-векторними струмами лептонів і порівняні з іншими оцінками.

Ключові слова: фізика високих енергій, Z' бозоны, диференційні перерізи розсіяння, інтегральні перерізи розсіяння.

Introduction

Electron-positron colliders provide possibility of precise measurements in high-energy physics. The history of LEP experiments showed that lepton processes can be sensitive to off-shell signals of physics beyond the standard model (SM). Unfortunately, the LEP statistics was not rich enough to detect clearly some signals of new heavy particles.

The special observables were designed to select probable signals of the Abelian Z' boson in various LEP processes [1, 2]. In particular, a one-parameter sign-definite observable was constructed as a generalized forward-backward cross-section of $e^+e^- \to \mu^+\mu^-$ process, and a hint of Z' boson was found at one standard deviation. However, the latest LHC experiments allow to conclude that the maximum likelihood values of Z' couplings from Ref. [1] seem to be overestimated. At the present time, the most powerful observables for Z' boson in $e^+e^- \to \mu^+\mu^-$ are found [3]. So, it is possible to revise the LEP data by means of the new approach.

Let us describe briefly main checkpoints of the present investigation. We use common phenomenological parameterization of Z' couplings with SM fermions as well as the model-independent relations between the Z' couplings [4]. The optimal one-parameter observables are constructed as the cross-sections integrated over the scattering angle with proper weight functions maximizing the value-to-uncertainty ratio for the observable. Then, we fit LEP data using the observables in order to estimate Z' coupling.

The low-energy phenomenology of the Abelian Z' boson

The Abelian Z' boson [5-7] is usually described by its couplings to vector and axial-vector currents. In general, there is also the mixing between Z and Z' bosons. The corresponding Lagrangian is

$$L_{\bar{f}fZ} = \frac{1}{2} Z_\mu \bar{f}\gamma^\mu[(v_{fZ}^{SM} + \gamma^5 a_{fZ}^{SM})\cos\theta_0 + (v_f + \gamma^5 a_f)\sin\theta_0]f\,,$$

$$L_{\bar{f}fZ'} = \frac{1}{2} Z'_\mu \bar{f}\gamma^\mu[(v_f + \gamma^5 a_f)\cos\theta_0 - (v_{fZ}^{SM} + \gamma^5 a_{fZ}^{SM})\sin\theta_0]f \qquad (1)$$

where we omit effective interactions inspired by loop corrections and next-to-leading order terms in inverse heavy mass scales.

Not all the coupling constants in (1) are independent, if we assume the Abelian Z' boson associated with an effective U(1) gauge symmetry at low energies. If we consider the single neutral vector boson with a mass of order TeVs, the following relations arise [4]

$$v_{f[T_3=1/2]} = v_{f[T_3=-1/2]} = -2a\,,\ a_{f[T_3=-1/2]} = -a_{f[T_3=1/2]} = a\,,$$

$$\theta_0 = -a\frac{\sin(2\theta_W)}{\sqrt{4\pi\alpha_{em}}}\left(\frac{m_Z}{m_{Z'}}\right)^2 \qquad (2)$$

where T_3 is the third component of the weak isospin, and the fermions are taken from the same SM doublet. The relations can be motivated by general theoretical reasons (gauge symmetry, renormalizability at energies of the Z' decoupling) which are described in details in Ref. [4]. Let us note that the relations (3) cover a wide set of popular Z' models. In this regard, they can be called model-independent. Considering the cross-sections at energies below the Z' mass, it is convenient to use couplings

$$\bar{a}_f = \frac{m_Z}{\sqrt{4\pi}m_{Z'}}a_f\,,\quad \bar{v}_f = \frac{m_Z}{\sqrt{4\pi}m_{Z'}}v_f\,. \qquad (3)$$

The virtual Z' boson state contributes to the differential cross-section of $e^+e^- \to \mu^+\mu^-$ process. In the lowest order in the inverse Z' mass the cross-section deviates from its SM value as

$$\frac{d\sigma}{dz} - \frac{d\sigma^{SM}}{dz} = F_1(\sqrt{s}, z)\bar{a}^2 + F_2(\sqrt{s}, z)\bar{a}\bar{v}_e +$$

$$+ F_3(\sqrt{s}, z)\bar{a}\bar{v}_\mu + F_4(\sqrt{s}, z)\bar{v}_e\bar{v}_\mu + \ldots \quad (4)$$

where $z = \cos\theta_0$ is the cosine of the scattering angle and dots stand for higher corrections in the inverse Z' mass. Factors F_i arise from the interference between the SM scattering amplitude and the Z' exchange amplitude. They have to be computed numerically taking into account both the tree-level contribution and loop corrections.

Being measured in experiments, the cross-section (4) allows to estimate the Z' couplings $\bar{a}$, $\bar{v}_e$, and $\bar{v}_\mu$. A non-zero value of some coupling mentioned can be called the Z' signal.

Minimal number of unknown parameters is preferable in fitting data. Therefore, one-parameter observable is the most prominent from the statistical point of view. Moreover, sign-definite observable is more informative, since it can also reject the hypothesis, whereas sign-indefinite one can only accept the signal. These properties are especially important in case of statistics which is not rich enough to detect clear signals at high confidence levels. Fortunately, the cross-section (4) contains one sign-definite term with $\bar{a}^2$. If we could select this term in the cross-section, we would obtain a powerful observable to detect Z' signals in experiments. In case of lepton universality the term with $\bar{v}_e\bar{v}_\mu$ also becomes sign-definite.

It is also worth to note that factors $F_{2,3}$ are small with respect to $F_{1,4}$. Their contributions to the cross-section are about 1%, and their existence does not affect the key ideas of the present investigation. So, the Z' signal in $e^+e^- \to \mu^+\mu^-$ can be discussed as two-parametric.

The observables

The differential cross-section (4) contains two leading terms at $\bar{a}^2$ and $\bar{v}_e\bar{v}_\mu$. The corresponding factors $F_i(\sqrt{s}, z)$ are the functions of energy and scattering angle. We can use angular integration in order to suppress one factor comparing to another. Actually, this means that we will construct some integrated cross-section with specific properties.

In general, integrated cross-sections are well known in the literature. The most popular integration schemes are based on bin summation with equal weights but opposite signs. As examples, we can mention the total cross-section, the forward-backward cross-section, the center-edge cross-section, etc. However, the equal weight of bins is just a possible option. The most general integration scheme can be described by weight function p(z):

$$\sigma = \int_{-1}^{1} dz p(z)\left(\frac{d\sigma}{dz} - \frac{d\sigma^{SM}}{dz}\right). \quad (5)$$

In these notations, the popular mentioned cross-sections correspond to step-like weight functions. The observables used in previous analysis of LEP data are also based on step-like weight functions.

The statistical uncertainty of the observable (5) can be estimated taking into account that the actual number of events in bin is distributed under the Poisson distribution. This means the variance of events coincides with the average number of events. Then, the standard deviation of the observable is [3]

$$\delta\sigma \cong \sqrt{\frac{1}{L}\int_{-1}^{1} dz p^2(z)\frac{d\sigma^{SM}}{dz}} \tag{6}$$

where L is the integrated luminosity of the experiment.

Let us consider the observable which amplifies the Z' signal as much as possible. This aim can be reached by maximizing the value-to-uncertainty (signal-to-uncertainty) ratio where the weight function is assumed to be varied in the optimization procedure. The general algorithm to find the optimal weight function is described in details in [3]. In the present paper we mention briefly just the main steps of the algorithm.

$$abs\left(\frac{\sigma}{\delta\sigma}\right) \propto abs\left(\frac{\int_{-1}^{1} p(z)\left(\frac{d\sigma}{dz}-\frac{d\sigma^{SM}}{dz}\right)dz}{\sqrt{\int_{-1}^{1} p^2(z)\frac{d\sigma}{dz}dz}}\right) \to \max \tag{7}$$

In fact, the optimization (7) has to be performed under additional constraints. First of all, the normalization of the weight function must be taken into account, since (7) is evidently invariant under the rescaling of the weight function. We choose the normalization

$$\int_{-1}^{1} dz p^2(z) = 1 . \tag{8}$$

Second, the weight function is chosen to suppress all the factors in the differential cross-section (4) except for either F_1 or F_4. The most general scheme takes into account both the contributions of leading factors $F_{1,4}$ and small factors $F_{2,3}$ in the differential cross-section (4). In order to select the factor F_1 we can minimize the cumulative relative contribution of the factors $F_{2,3,4}$:

$$\frac{\sum_{i=2}^{4} abs\left(\int_{-1}^{1} dz p(z) F_i(\sqrt{s}, z)\right)}{\sum_{i=1}^{4} abs\left(\int_{-1}^{1} dz p(z) F_i(\sqrt{s}, z)\right)} \to \min \tag{9}$$

The factor F_4 is selected in a similar way using $F_{1,2,3}$ in the nominator. Eq. (9) does not specify a unique weight function, it defines a subspace in the Hilbert space of $p(z)$. It is clearly seen from the fact that Eq. (9) does not change when a function orthogonal to $F_{1,2,3,4}$ is added to $p(z)$.

The optimization (7) with the constraints (8) and (9) has to determine uniquely the weight function *p*(*z*) for the most amplified Z' signal in the considered process. These calculations require choosing some basis in the Hilbert space of weight functions.

The most natural basis takes into account the kinematics of $e^+e^- \to \mu^+\mu^-$ process. Due to the absence of the flavor-changing neutral currents, there are no virtual bosons in the t-channel. Moreover, all the leptons can be considered as massless. This leads to the well-known two-polynomial structure of all the factors in the differential cross-sections:

$$F_i(\sqrt{s}, z) = a_i(\sqrt{s}) p_1(z) + b_i(\sqrt{s}) p_2(z) \tag{10}$$

where $p_1(z) \sim z$, $p_2(z) \sim (1+z^2)$. In this regard, it is convenient to use orthogonal polynomials as a basis in the Hilbert space of weight functions. We define orthogonal normalized polynomials in the standard way,

$$\int_{-1}^{1} dz p_i(z) p_j(z) = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases} \tag{11}$$

The full set of polynomials can be reconstructed starting from p_1 and p_2 and increasing the largest power of the polynomial [3]:

$$p_1 = \sqrt{\frac{3}{2}} z\,,\ p_2 = \frac{1}{2}\sqrt{\frac{15}{14}}(1+z^2)\,,$$

$$p_3 = \sqrt{\frac{6}{7}}(1 - \frac{5}{2}z^2)\,,\ p_4 = \frac{1}{2}\sqrt{\frac{7}{2}}(5z^3 - 3z)\,. \qquad \ldots(12)$$

Weight function *p*(*z*) can be expanded by p_i:

$$p(z) = \sum_{i=1}^{\infty} c_i p_i(z)\,. \tag{13}$$

Then, the normalization condition (8) becomes

$$\sum_{i=1}^{\infty} c_i^2 = 1\,. \tag{14}$$

Since the Z' contributions to the cross-section are described by two polynomials $p_{1,2}$, we use the fixed direction in the functional subspace based on $p_{1,2}$ in order to suppress either F_1 or F_4 factor:

$$k = c_2 / c_1\,. \tag{15}$$

This can be done by means of (9). The numerical analysis shows that the corresponding relative weight of F_1 or F_4 is 0.98. Thus, we can estimate the systematic error of the variable as 2 %.

There is also the normalization condition (14) allowing to determine one of the coefficients through the others. For instance,

$$c_1 = \sqrt{\frac{1 - c_3^2 - c_4^2 - \ldots}{1 + k^2}} \tag{16}$$

Thus, two coefficients c_1 and c_2 are explicitly expressed by the other coefficients. As a result, c_3, c_4,... are to be varied to find the maximum (7).

In fact, the usage of orthogonal polynomials is just calculation tool to perform optimization (7) to find the most effective weight function. However, a convenient 'natural' basis helps us to obtain results in the most quick and simple way.

The signal-to-uncertainty ratio is maximized to find the coefficients at polynomial expansion (13). Increasing the number of polynomials in (13) we can observe asymptotic behavior of $p(z)$. We can estimate the relative accuracy of the result comparing the weight functions at the current and previous steps of the calculation:

$$\eta = \sqrt{\int_{-1}^{1} dz (p_{current} - p_{previous})^2} \ . \qquad (17)$$

Using eight polynomials from the basis, we find η<0.01 at all the considered energies, which is below the systematic theoretical error (2%) of the observables. The results of optimization are shown in Tables 1, 2.

Table 1

The results of optimization of the weight function to select $\bar{a}^2$ for LEP energies. The parameter $k = c_2 / c_1$ is computed in accordance with (9), the coefficients c_i in (13) are found by (7)

$\sqrt{s}$, GeV	k	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
130	-0.567	0.770	-0.437	-0.416	0.193	-0.011	-0.050	-0.044	0.019
136	-0.524	0.802	-0.420	-0.392	0.141	0.025	-0.061	-0.040	0.014
161	-0.425	0.863	-0.367	-0.330	0.036	0.076	-0.056	-0.017	0.000
172	-0.402	0.876	-0.352	-0.314	0.014	0.083	-0.052	-0.011	-0.003
183	-0.385	0.885	-0.340	-0.302	-0.002	0.086	-0.048	-0.007	-0.004
189	-0.377	0.889	-0.335	-0.296	-0.009	0.088	-0.046	-0.006	-0.005
192	-0.374	0.891	-0.333	-0.294	-0.012	0.088	-0.045	-0.005	-0.005
196	-0.369	0.893	-0.330	-0.291	-0.016	0.089	-0.044	-0.004	-0.005
200	-0.365	0.894	-0.327	-0.288	-0.019	0.089	-0.043	-0.003	-0.006
202	-0.363	0.895	-0.325	-0.287	-0.020	0.089	-0.042	-0.003	-0.006
205	-0.361	0.897	-0.323	-0.285	-0.023	0.090	-0.041	-0.002	-0.006
207	-0.359	0.897	-0.322	-0.284	-0.024	0.090	-0.041	-0.002	-0.006

Table 2

The results of optimization of the weight function to select $\bar{v}_e \bar{v}_\mu$ for LEP energies. The parameter $k = c_2 / c_1$ is computed in accordance with (9), the coefficients c_i in (13) are found by (7)

$\sqrt{s}$, GeV	k	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
130	-1.258	-0.597	0.751	0.230	-0.150	0.009	0.039	0.034	-0.015
136	-1.362	-0.579	0.788	0.174	-0.102	-0.018	0.044	0.029	-0.010
161	-1.678	-0.510	0.856	0.052	-0.021	-0.045	0.033	0.010	0.000
172	-1.775	-0.490	0.870	0.025	-0.008	-0.046	0.029	0.006	0.002
183	-1.856	-0.474	0.879	0.006	0.001	-0.046	0.025	0.004	0.002
189	-1.894	-0.466	0.883	-0.003	0.005	-0.046	0.024	0.003	0.003
192	-1.912	-0.463	0.885	-0.006	0.006	-0.046	0.023	0.002	0.003
196	-1.934	-0.459	0.887	-0.011	0.008	-0.046	0.022	0.002	0.003
200	-1.955	-0.455	0.889	-0.015	0.010	-0.045	0.022	0.002	0.003
202	-1.965	-0.453	0.890	-0.017	0.010	-0.045	0.021	0.001	0.003
205	-1.980	-0.450	0.891	-0.020	0.011	-0.045	0.021	0.001	0.003
207	-1.989	-0.449	0.892	-0.022	0.012	-0.045	0.020	0.001	0.003

In Figs. 1, 2 we show how the optimal weight functions depend on the collision energy. As it is seen, the result is stable for different LEP energies.

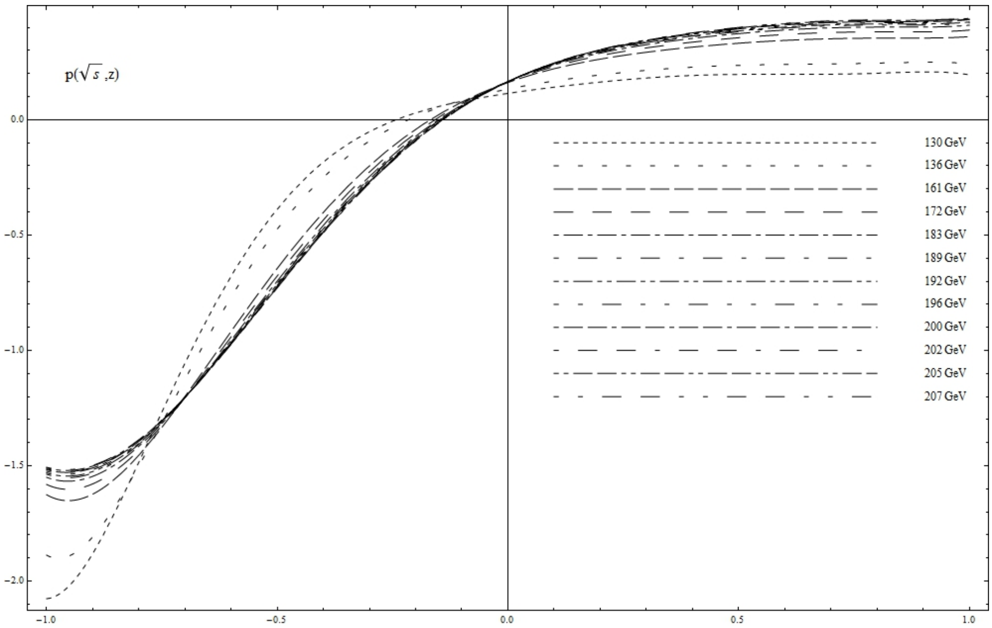

Fig. 1. The optimal weight functions to select $\bar{a}^2$ for different LEP energies.

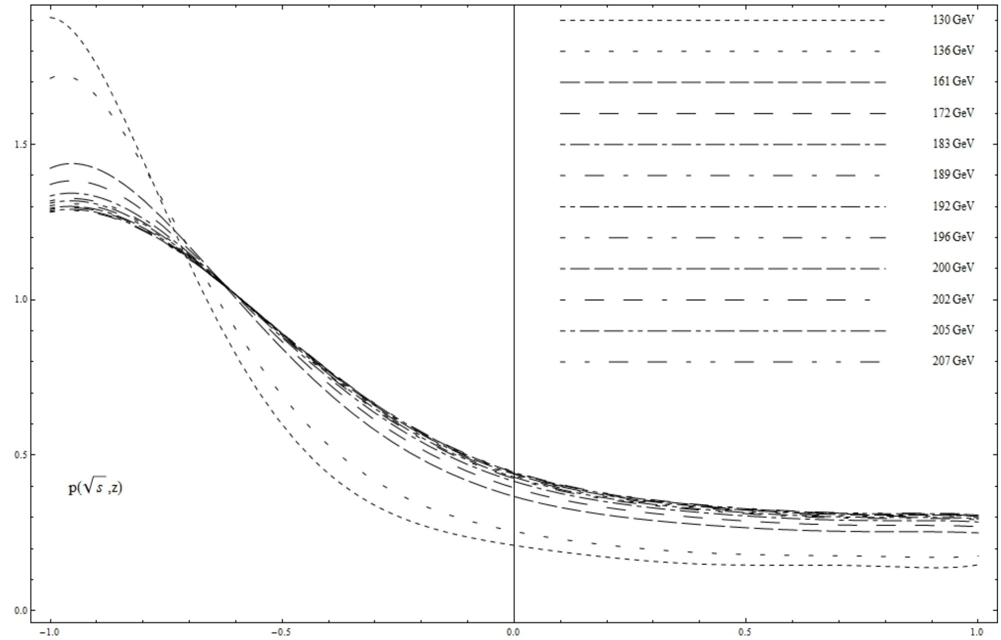

Fig. 2. The optimal weight functions to select $\bar{v}_e \bar{v}_\mu$ for different LEP energies.

Data fit

Data fit is performed in the standard way using chi-square function to combine different scattering energies together. First, we calculate both the mean values and the statistical uncertainties of our observables at different energies taking data on differential cross-sections published by the LEP Collaborations [8-10]. Dividing the values by a known numeric factor, we compute the experimental estimate of either $\bar{a}^2$ or $\bar{v}_e \bar{v}_\mu$. In this way we obtain 21 data points for each type of observables. After that, we combine all the data points altogether by means of the standard chi-square technique obtaining the mean values and the uncertainties of the *Z'* couplings:

$$\bar{a}^2 = (1.4369 \pm 4.8614)\times 10^{-5},\ \bar{v}_e \bar{v}_\mu = (-7.5890 \pm 6.0377)\times 10^{-5}$$

Conclusions

Let us discuss the obtained results. First of all, the uncertainty of $\bar{a}^2$ is close to the uncertainty within the indirect measurement of the axial-vector coupling by the total cross-sections and forward-backward asymmetries [1]. However, we use less data points, since the differential cross-sections were not published for some LEP energies depending on the collaboration. This reflects the fact that the new observables are more statistically powerful with respect to the observables used in [1].

Second, the mean value of $\bar{a}^2$ decreases comparing to the indirect estimates [1]. This is in accordance with the latest constraints from the LHC [11] showing that this coupling should be about 10^{-6} rather than 10^{-5}.

Finally, the mean value and the uncertainty of the vector coupling are quite large, so we cannot interpret them as some signal of the particle.

The new observables for searching for Z' signals in $e^+e^- \to \mu^+\mu^-$ process show they can be useful in data fitting. They have good perspectives in future experiments at lepton colliders such as the ILC.

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