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ABOUT EXACT SOLUTIONS OF MEASUREMENT'S INVERSE PROBLEMS

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The inverse problems of measurement are investigated in this paper. The main hypothesis was suggested for estimation from below of exact solutions of such problems. Two practical inverse problems have been considered as examples: inverse problem of Krylov, identification of moment of technological resistance on rolling mill.

Key words: measurement's inverse problem, main hypothesis, estimation of exact solutions.

1. Introduction

In a practice of inverse problems solutions are presented some inverse problems in which the error of a mathematical model of a real physical process is taken into account [1], [2]. Such type of inverse problems were named as measurement's inverse problems (interpretation's inverse problems or recognition's inverse problems) [3].

Let us now consider such inverse problem as solution of following equation with inexact given operator $\tilde{A}$:

$$\tilde{A}z = u_\delta, \tag{1.1}$$

where $z \in Z, u_\delta \in U$ (Z, U are functional spaces), $\tilde{A}$ is inexact given compact operator.

In work [4] regularizing algorithm was suggested for equation (1.1) with approximate linear operator $\tilde{A}$ for Banach spaces Z, U, which based on the regularization method [5].

Let us assume that the inaccuracy of operator $\tilde{A}$ and function u_δ are given:

$$\|\tilde{A} - A_{ex}\|_{Z\to U} \le h, \|u_\delta - u_{ex}\|_U \le \delta, \tag{1.2}$$

where A_{ex} is linear exact operator in (1.1), u_{ex} is exact right part of (1.1).

The solution of problem (1.1) is reduced to the solution of following extreme problem:

$$\inf_{z\in Z_1} M^\alpha[z, u_\delta, \tilde{A}] = \inf_{z\in Z_1} \|\tilde{A}z - u_\delta\|_U^2 + \alpha\Omega[z] = M^\alpha[z_\alpha, u_\delta, \tilde{A}], \tag{1.3}$$

where $\Omega[z]$ is stabilizing functional for equation (1.1).

Regularization parameter α is obtained from equation of general discrepancy:

$$\| \tilde{A}z_\alpha - u_\delta \|_U^2 = (\delta + h\|z_\alpha\|)^2 + \mu^2(u_\delta, \tilde{A}), \tag{1.4}$$

where $\mu^2(u_\delta, \tilde{A})$ is measure of discrepancy.

2. Main Hypothesis

But in some cases it is difficult to find the value of h. So in this paper is suggested the estimation of inverse problem solution instead of solution of equation (1.1). The following hypothesis is assumed for this purpose: the inequality is valid:

$$\Omega[z_{ex}] \geq \Omega[z_\alpha] \tag{2.1}$$

for any approximate operator $\tilde{A}$ in equation (1.1), z_{ex} is the solution of problem (1.1) with exact initial data u_{ex}.

The main hypothesis can be grounded on filtration properties of functional $\Omega[z]$.

Let us assume that space Z is reflexive B space. Suppose that the solution estimation of inverse problem is defined as minimum size of a functional $\Phi[z]$.

The following theorem is valid.

Theorem 2.1. *Let us consider the set of possible solutions Z_δ of equation (1.1) with compact operator $\tilde{A}$: $Z_\delta = \{z : z \in Z, \| \tilde{A}z - u_\delta \|_U \leq \delta\}$. If the functional $\Phi[z]$ is convex and lower semi-continuity on Z_δ, then the function $z_* \in Z_\delta$ exists, where*

$$\inf_{z \in Z_\delta} \Phi[z] = \Phi[z_*].$$

For illustration of such approach the solutions of two practical problems are given [1], [2]: Krylov's inverse problem, identification of technological resistance moment on rolling mill.

3. Krylov's Inverse Problem

In 1914 at tests of compressors of ships guns the significant excess of pressure in the compressors was found [1]. The measurement of pressure was carried out by the Vikkers indicator. The plot of pressure is presented on Fig. 1.

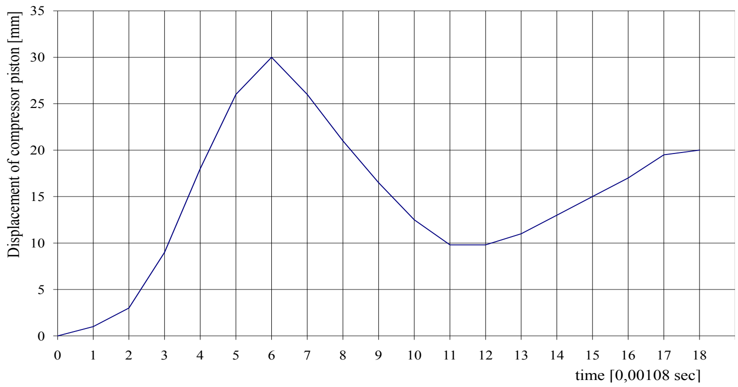

Fig. 1: Plot of displacement of Vikkers indicator piston during tests.

Instead of normal pressure 30 MPa the device has shown pressure 45 MPa. If the data of indicator correspond to the real situation then the replacement of ready guns by new guns is necessary. Such replacement had cost in that time about 2 millions gold Russian rubles.

Krylov has assumed that the records of Vikkers indicator do not correspond to real situation because of defects of the indicator design. For confirmation of this conclusion Krylov has considered an inverse problem for dynamic model of the indicator [1].

During this research A. N. Krilov the following inverse problem was originally considered: using a curve of movement of the piston of the indicator Vikkers and motion equation of indicator model (weight on a spring) to determine the valid pressure in the cylinder of the compressor [1]. Mathematical model of motion of piston on spring was chosen as following:

$$\ddot{x}(t) + \omega^2 x(t) = \frac{G(t)}{M} = \frac{S}{M} P(t), \tag{3.1}$$

where $P(t)$ is the pressure to piston;

M is the mass of piston, S is the surface square of piston;

$x(t)$ is the piston motion during examination;

ω is the frequency own fluctuations of the piston on a spring;

M $= 0.472$ kg, $\omega^2 = 3.18.10^6 sec^{-2}$.

Dependence of piston motion of the indicator has been approximated by Krylov by three dependencies:

$$x_1(t) = [1.5(1 - \cos 2\pi/\tau) - 0.25(1 - \cos 4\pi/\tau)].10^{-2}[m], at \quad t \in [0, 0.5\tau];$$

$$x_2(t) = \{2.0 + \cos 2\pi(t - 0.5\tau)/\tau - 0.25[1 - \cos 4\pi(t - 0.5\tau)/\tau]\}.10^{-2}[m],$$
$$at \quad t \in [0.5\tau, \tau];$$

$$x_3(t) = \{1.5[1 - 0.5\cos 2\pi(t - \tau)/\tau]\}.10^{-2}[m], \qquad at\ t \in [\tau, 1.5\tau]; \tag{3.2}$$

where (τ is the period of own fluctuations of the piston on a spring), $\tau = 0.00353$ sec. This approximation was executed so well, that the error in the uniform metrics did not surpass thickness of a pencil line on the diagram of piston motion of the indicator. Using this information A. N. Krylov originally has considered the following inverse problem: to determine the valid pressure in the cylinder of the compressor $P(t)$, believing, that the known mathematical model of moving of the piston on a spring makes program motion (3.2). As a result of the solution of such inverse problem the following dependencies of pressure on time were received:

$$P_1(t) = 18.76 + 1499 \cos 3560t [MPa], \qquad for \quad t \in [0, 0.5\tau];$$

$$P_2(t) = 26.25 + 2.87 \cos 1780(t - 0.5\tau) + 2.43 \cos 3560(1 - 0.5\tau) [MPa],$$

$$for \quad t \in [0.5\tau, \tau];$$

$$P_3(t) = 22.5 [MPa], \qquad for \quad t \in [\tau, 1.5\tau]. \tag{3.3}$$

Functions $P_k(t)(k = 1,2,3)$ as a whole on a piece $t \in [0, 1.5\tau]$ represent explosive function. Such result does not correspond to physical sense of a problem and A. N. Krylov has refused such method of its solution.

Confirmation of his assumption he has received from investigation of physical features of Vikkers indicator.

In a modern variant Krylov inverse problem is reduced to the solution of the equation such as (1.1) with the approximate operator $\tilde{A}$ [1]:

$$\int_0^t \sin \omega_1 (t - \tau) \exp(-0.5b(t - \tau)) z(\tau) d\tau = \tilde{A}z = u(t), \tag{3.4}$$

where $u(t) = \omega_1 M S^{-1} \{x(t) - \exp(-0.5bt)[x(0) \cos \omega_1(t) + \omega_1^{-1}(-0.5bx(0) + \dot{x}(0)) \sin \omega_1(t)]\}$,

$x(t)$ is given function (experiment), $\omega_1 = \sqrt{\omega^2 - 0.25b^2}$,

$\tilde{A}$ is integral operator.

The Tikhonov's regularization method was used for solution of instable problem (3.4). Initial problem (3.4) is reduced to following extreme problem [5]:

$$\Omega[z_0] = \inf_{\tilde{A}} \inf_{z \in Q_\delta} \Omega[z], \tag{3.5}$$

where $Q_\delta = \{z : z \in Z, \|\tilde{A}z - \tilde{u}\|_U \leq \delta\}$, $\|u_{ex} - \tilde{u}\|_U \leq \delta, \delta$ is inaccuracy of initial data, u_{ex} is exact initial data. The solution of extreme problem (3.5) shown on Fig.2. (lower line).

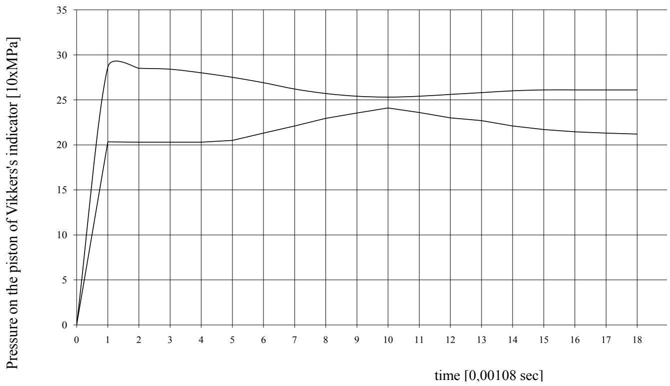

Fig. 2: Graphics of pressure change on piston indicator during tests obtaining regularization method.

However, the physical sense of an initial inverse problem does not correspond to the sense of the solution z_0 of an extreme problem (3.5). The solution of the following extreme problem is more suitable in this case [1]:

$$\Omega[z_{est}] = \inf_{\tilde{A}} \sup_{z \in Q_\delta} \Omega[z]. \tag{3.6}$$

In work [1] the solution of an extreme problem (3.6) is obtained. The graphics of this function was shown on Fig. 2. (upper line).

The function z_{est} has the maximal amplitude equal 29 MPa. By virtue of suggested hypothesis (an inequality (2.1)) the exact solution z_{ex} can have amplitude smaller 30 MPa. Hence, for replacement of a set of ready ships guns there are no objective basis. This result corresponds to Krylov's conclusion which was received by other way.

4. Identification of moment of technological resistance on rolling mill

As the second of measurement's inverse problems the problem of definition of the moment of technological resistance on the rolling mill is considered [2].

The important characteristic of process of rolled metal is the moment of technological resistance on the working barrels of the rolling mill.

The curve of change of technological resistance moment which was obtained by help of plasticity theory was shown on Figure 3. as dotted line.

In paper the problem of definition of the technological resistance moments by a method of identification is considered [2], i.e. method of indirect measurements: on basis of results of measurement of fluctuations of the moments in the main mechanical line of the rolling mill it is necessary to determine the real character of change of the moments of technological resistance. In this case it is necessary to take into account an error of the mathematical description of process of fluctuations.

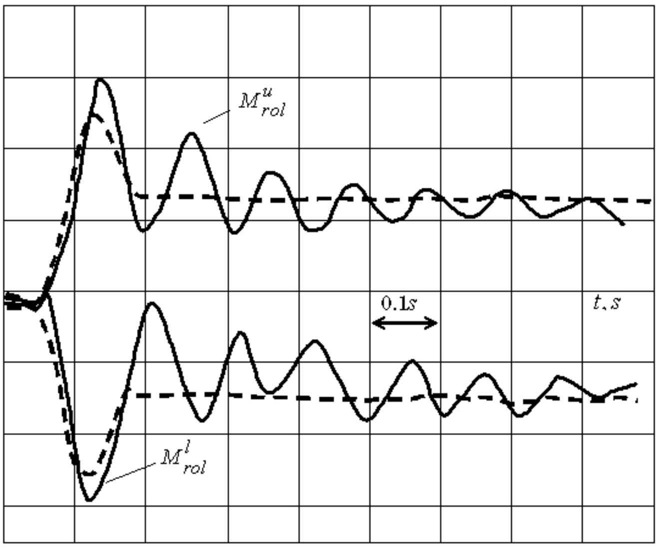

Fig. 3: Graphics of moments which are acting to worked barrels of list rolling mill at experiment.

Mathematical model of motion of the main mechanical line of the rolling mill of sheets was chosen in form:

$$\begin{aligned} &\ddot{M}_{12} + a_{12}M_{12} + a_{13}M_{23} + a_{14}M_{24} = b_1 J_u, \\ &\ddot{M}_{23} + a_{23}M_{23} + a_{22}M_{12} + a_{24}M_{24} = b_2 M^u_{rol}, \\ &\ddot{M}_{24} + a_{34}M_{24} + a_{32}M_{13} + a_{33}M_{23} = b_3 M^l_{rol}. \end{aligned} \tag{4.1}$$

where functions $J_u(t), M_{23}(t), M_{24}(t)$ were obtained by experiment [2]. The integral equations such as (1) with the inexact operators are received for definition of unknown functions M^u_{rol}, M^l_{rol}.

The regularization method is used for inverse problem solution of technological resistance moment identification [2]. The initial problem (1.1) was replaced with the solution of following extreme problem:

$$\Omega[z_*] = \inf_{z \in Q_\delta} \Omega[z], \tag{4.2}$$

where $Q_\delta = \{z : z \in Z, \|\tilde{A}z - \tilde{u}\|_U \leq \delta\}, \|u_{ex} - \tilde{u}\|_U \leq \delta, \delta$ is inaccuracy of initial data, u_{ex} is exact initial data.

The functional of kind

$$\Omega[z_{est}] = \int_0^T \dot{z}^2 dt \tag{4.3}$$

was chosen as stabilizing functional.

The solutions of equation (1.1) with the exact operators were obtained by regularization method. Functions M^u_{rol}, M^l_{rol} are shown on Fig. 3 (continuous lines).

On basic of main hypothesis we can take the conclusion that exact solutions have more oscillating characters of solutions as for regularized solutions the following inequalities are valid:

$$\Omega[M^u_{rol,ex}] = \int_0^T \{\dot{M}^u_{rol,ex}\}^2 dt \geq \Omega[M^u_{rol}] = \int_0^T \{\dot{M}^u_{rol}\}^2 dt, \tag{4.4}$$

$$\Omega[M^l_{rol,ex}] = \int_0^T \{\dot{M}^l_{rol,ex}\}^2 dt \geq \Omega[M^l_{rol}] = \int_0^T \{\dot{M}^l_{rol}\}^2 dt, \tag{4.5}$$

where $M^u_{rol,ex}, M^l_{rol,ex}$ are real moments of technological resistance on working barrels of rolling mill of sheets.

Thus as a result of an estimations of the exact solutions the useful information concerning the real moments of technological resistance was received which cannot have character of change as shown on Fig. 3 (dotted line).

5. Conclusion

In paper the main hypothesis was suggested for estimation of exact solution of measurement inverse problems. As examples two practical problems were considered: inverse problem of Krylov, identification of external moment loads to working barrels of rolling mill of sheets.

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