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Проблеми математичного моделювання
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THE APPROXIMATED METHOD OF SOLVING TWO-DIMENSIONAL NON-LINEAR PROBLEM OF THE CREEP THEORY FOR VISCOELASTIC BODIES WITH MOVING BOUNDARIES

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The approximated method of solving two-dimensional non-linear problem of the creep theory for viscoelastic bodies with moving boundaries is suggested. The problem of a stress-deformed state of viscoelastic hollow cylinder, which is being built up by virtue of inner pressure, is considered. It is assumed that the process of continuous build-up takes place outwards from the outer side. The case of non-liner creep law is viewed with calculation results presented as graphs, reflecting the dynamics of stress and deformation that occurs herewith.

Key words: viscoelastic cylinder, outward build-up, model of the build-up process.

1. Introduction

We face the problems with a moving limit (processes of build-up) while learning various technological and natural processes like coiling, sputtering, freezing-on, accretion, also in step-by-step constructing or loading of buildings and building constructions, crystal growing, phase changes of solid bodies and so on.

The theoretical ground for new production technologies concerning cases, pipes and other rotation workpieces through build-up requires development of calculation methods that fuller reflect the properties of the material used for a detail. It is known that polymeric materials and composite materials used for production of different details and construction elements have vivid creep properties. This leads to a redistribution of stress in the detail in the process of build-up, deformation in shape and size after production and during loading. Mechanics of similar processes can be investigated from different points of view. One of those is a model representation of build-up processes in real constructions based on the creep theory for heterogeneously ageing bodies [1,2]. Such bodied are characterized by consisting of different elements which, being created at different moments in time, are of different age that depends on spacial coordinates. Thus, along with traditional inconsistency there arises an inconsistency of a specific nature caused by the fact that the process of ageing in this elements runs differently for each element.

Solutions of boundary value problems for moving limits and methods of solving integral equations known from literature sources [3-17] model different processes of detail production. Some works [1,2] look into problems where the process of build-up takes place outwards from the outer side of the detail at given law of radius change and surface tension for the case of linear creep law.

This work suggests an approach to address the question of modeling the process of the hollow circular cylinder production of polymeric material through the method of build-up for the non-linear creep law. The process of build-up takes place from the outer side of the detail by virtue of the inner pressure.

2. The physical aspect of the problem

To produce the mathematical model of the viscoelastic hollow cylinder build-up, which is built up by virtue of inner tension, let us use the approach stated in [18].

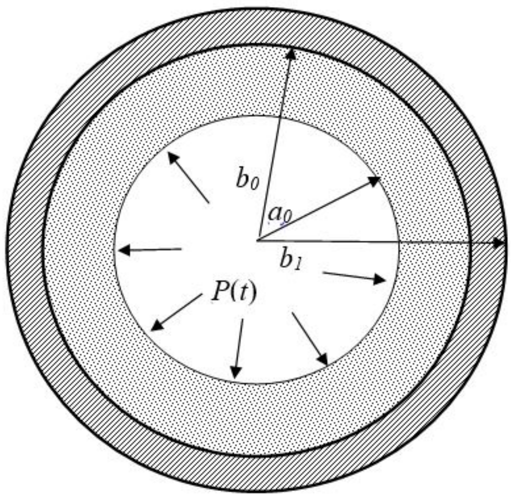

Fig. 1: The scheme of the outward build-up

Let the given viscoelastic hollow circular cylinder have the inner a_0 and outer b_0 radius at the moment of time $t = 0$ (Fig.1). The build-up process for this cylinder involves its outward thickening with homogeneous viscoelastic material by virtue of inner pressure $P(t)$. We will investigate this process in the time interval $[0, T]$ adhering to the following assumptions:

1. at the moment of time $t = 0$ there is a given initial value of $P(0) = P_0$;

2. the law is known that $b = b(t)$, according to which the outer cylinder radius changes with time;

3. the function $b = b(t)$ is a monotone decreasing one, where $b(0) = b_0$;

4. the functions $b(t)$ and $P(t)$ are continuously differentiable in the interval $0 < t < T$;

5. the build-up process stops at $t = T$, meaning $b(t) = b_1 = const$ when $t \geqslant T$.

The problem is to identify the stress-deformed state of the cylinder for each value of $t \in [0, T]$.

3. The mathematical aspect and problem solution

Let us introduce polar coordinates r, θ, z and consider the flat deformation of the cylinder (i.e. $u_z = 0$). We will further use the generally accepted symbols for movement, stress and deformation tensor components. Below listed are key relations that characterize the build-up process for the given problem:

• the condition of deformation commonality:

$$\varepsilon_r + \varepsilon_\theta = 0; \tag{3.1}$$

• the equation of balance:

$$\frac{\partial \sigma_r}{\partial r} = -\frac{\sigma_r - \sigma_\theta}{r}; \tag{3.2}$$

• Cauchy relation for the rate of deformation and movement:

$$\dot{\varepsilon}_r = \frac{\partial \dot{u}_r}{\partial r},\ \dot{\varepsilon}_\theta = \frac{\dot{u}}{r}. \tag{3.3}$$

For (3.3) a period symbolizes the partial derivative of time.

Since all components of deformation tensor except for ε_r and ε_θ equal zero, with due account of (3.1), we have the following relation:

$$\varepsilon_u = (2\varepsilon_{ij}\varepsilon_{ij})^{1/2} = 2|\varepsilon_r| = 2|\varepsilon_\theta|.$$

Then, taking into account the essential equations of the non-linear creep theory for heterogeneously ageing bodies (see f.e. [2]) and the condition $\varepsilon_\theta > 0$ the equation of state can be presented as follows:

$$\sigma_r(t,r) - \sigma_\theta(t,r) = 2G_i(t-\tau^*)(\varepsilon_r(t,r) - \varepsilon_\theta(t,r))\varepsilon_\theta^{m-1}(t,r) - \\ - \int_{\tau^*}^{t} R_i(t-\tau^*, \tau-\tau^*)(\varepsilon_r(\tau,r) - \varepsilon_\theta(\tau,r))\varepsilon_\theta^{m-1}(\tau,r)d\tau, i = 1,2. \tag{3.4}$$

Here $G_i = G \cdot 2^{m-1}$; $R_i = R \cdot 2^{m-1}$; $m = 1, 2, 3, ...$ – is given, a parameter which characterizes the level (degree) of the non-liner creep law; G – is a momentary elastic module of the material; $R = R(t, \tau)$ – is the relaxation kernel of the viscoelastic material; $\tau^*(r)$ – is the moment of elementary cylinder level formation; τ – is the age of material at the moment when stress is applied to it. The function $\tau^* = \tau^*(r)$ equals zero if $a_0 \leqslant r \leqslant b_0$ and coincides with the function inverse to the function $b(t)$ if $b_0 \leqslant r \leqslant b_1$, thus $\tau^*(b(t)) \equiv t$, $b(\tau^*(r)) \equiv r$, $\tau^*(b_0) = 0$, $b(0) = b_0$.

The boundary conditions look as follows:

$$\sigma_r|_{r=b_0,t=0} = 0, \sigma_r|_{r=a_0} = -P(t) \tag{3.5}$$

$$\sigma_{r,\theta}|_{r=b(t),0<t\leqslant T} = 0, \sigma_r|_{r=b_1,t>T} = 0 \tag{3.6}$$

Upon differentiation of the equation (3.1) with respect to the variable of time and applying it to the received representation (3.3), we come up with the following equation:

$$\frac{\partial \dot{u}_r}{\partial r} + \frac{\dot{u}_r}{r} = 0$$

Hence

$$\dot{u}_r = \frac{c(t)}{r}, \dot{\varepsilon}_r = -\dot{\varepsilon}_\theta = -\frac{c(t)}{r^2}, \tag{3.7}$$

where $c(t)$ is a function subject to definition.

With due account of the initial condition $\varepsilon_r(\tau^*(r), r) = \varepsilon_\theta(\tau^*(r), r) = 0$, (3.7) can be transformed to

$$u_r(t,r) = \frac{A(t) - A(\tau^*(r))}{r}, \tag{3.8}$$

$$-\varepsilon_r(t,r) = \varepsilon_\theta(t,r) = \frac{A(t) - A(\tau^*(r))}{r^2} \text{ when } b_0 < r \leqslant b(t), \tag{3.9}$$

$$u_r(t,r) = \frac{A(t)}{r}, \tag{3.10}$$

$$-\varepsilon_r(t,r) = \varepsilon_\theta(t,r) = \frac{A(t)}{r^2} \text{ when } a_0 \leqslant r \leqslant b_0, \tag{3.11}$$

$$A(t) = -\int_0^t c(\tau)d\tau. \tag{3.12}$$

Let us evaluate stress σ_r through the function $A(t)$. For that two areas will be considered

1. Area $a_0 \leqslant r \leqslant b_0$, the initial cylinder. By applying the formula for the deformation component (3.11) to (3.4), we receive the following:

$$\sigma_r(t,r) - \sigma_\theta(t,r) = -\frac{2}{r^{2m}}\left[2G_1(t)A^m(t) - \int_0^t R_1(t,\tau)A^m(\tau)\,d\tau\right]. \tag{3.13}$$

By integrating (3.2) in the limits from a_0 to r with the account of the boundary conditions (3.5-3.6) and the formula (3.13), the following is received:

$$\sigma_r(t,r) = -P(t) + \left[4G_1(t)A^m(t) - 2\int_0^t R_1(t,\tau)A^m(\tau)d\tau\right]\int_{a_0}^r \frac{dr}{r^{2m+1}}. \tag{3.14}$$

2. Area $b_0 < r \leqslant b(t)$ – the zone of build-up

Let us apply the deformation components of (3.9) to the equation of state (3.4), the result is:

$$\sigma_r(t,r) - \sigma_\theta(t,r) = -\frac{2}{r^{2m}} \left[2G_1(t-\tau^*(r))(A(t) - A(\tau^*(r)))^m - \right.$$
$$\left. - \int_{\tau^*(r)}^{t} R_1(t-\tau^*(r), \tau - \tau^*(r)) \left(A(\tau) - A(\tau^*(r))\right)^m d\tau \right]. \quad (3.15)$$

By integrating (3.2) in the limits of r to $b(t)$ with the account of the boundary conditions (3.5-3.6) and the formula (3.15), the following presents itself:

$$\sigma_r(t,r) = -2 \int_r^{b(t)} \frac{1}{r^{2m+1}} \left[2G_1(t-\tau^*(r))(A(t) - A(\tau^*(r)))^m - \right.$$
$$\left. - \int_{\tau^*(r)}^{t} R_1(t-\tau^*(r), \tau - \tau^*(r))(A(\tau) - A(\tau^*(r)))^m d\tau \right] dr. \quad (3.16)$$

The equation for the function definition $A(t)$ comes out of the condition of stress continuity $\sigma_r(t,r)$ on the border of the two areas considered above, that is for $r = b_0$. By applying the value $r = b_0$ to (3.14) and (3.16) and equating their right parts in the course of certain transformation the following is received:

$$H_1 \left(2G_1(t)A^m(t) - \int_0^t R_1(t,\tau)A^m(\tau)d\tau \right) + \int_0^t H_2(t,\tau)(A(t) - A(\tau))^m d\tau -$$
$$- \int_0^t \int_0^\tau H_3(t,\tau,s)(A(\tau) - A(s))^m ds d\tau = P(t), \quad (3.17)$$

where

$$H_1 = \int_{a_0}^{b_0} \frac{2dr}{r^{2m+1}}, \; H_2(t,\tau) = \frac{4\dot{b}(\tau)}{b^{2m+1}(\tau)} 2G_1(t-\tau),$$
$$H_3(t,\tau,s) = \frac{2\dot{b}(s)}{b^{2m+1}(s)} R_1(t-s, \tau - s).$$

First of all, let us present some theoretical aspects of the possibility to solve the integral equation (3.17)

Definition 3.1. The integral operator $I(x)$, performing in the Banach space X, is called the compressing operator if there is a value $0 \leqslant \lambda < 1$, that

$$\|I(x_1) - I(x_2)\| \leqslant \lambda \|x_1 - x_2\|$$

for any element x_1 and x_2 of the X space, belonging to the range of definition for the operator I.

Theorem 3.1. *If the integral operator $I(x)$ transforms a closed set F of the Banach space into itself, it has a single stationary point. Successive approximations $x_n = I(x_{n-1})$, $(n = 1, 2, ...)$ converge to this point at any given point x_0.*

Proof. By equation $x_n = I(x_{n-1}),\ (n = 1, 2, ...)$ and with the account of definition of the compressing operator, the following is received

$$\|x_{n+1} - x_n\| \leqslant \lambda\|x_n - x_{n-1}\|. \tag{3.18}$$

If $n = 1$ than

$$\|x_2 - x_1\| \leqslant \lambda\|x_1 - x_0\|,$$

if $n = 2$

$$\|x_3 - x_2\| \leqslant \lambda\|x_2 - x_1\| \leqslant \lambda^2\|x_1 - x_0\|.$$

Moving forward,

$$\|x_{n+1} - x_n\| \leqslant \lambda^n\|x_1 - x_0\|. \tag{3.19}$$

Let m and n be natural numbers, with $m > n$. Then the inequation (3.19) leads to

$$\|x_m - x_n\| \leqslant \|x_m - x_{m-1}\| + \|x_{m-1} - x_{m-2}\| + ... + \|x_{n+1} - x_n\| \leqslant$$
$$\leqslant (\lambda^{m-1} + \lambda^{m-2} + ... + \lambda^n)\|x_1 - x_0\| =$$
$$= \frac{\lambda^n - \lambda^m}{1 - \lambda}\|x_1 - x_0\| \leqslant \frac{\lambda^n}{1 - \lambda}\|x_1 - x_0\|. \tag{3.20}$$

Since $\lambda < 1$, for the given $\varepsilon > 0$ the number N can be selected so big that the inequation $n > N$ will be true

$$\frac{\lambda^n}{1 - \lambda}\|x_1 - x_0\| < \varepsilon.$$

With such selected N and (3.20) the following is actualized: with $m > n > N$

$$\|x_m - x_n\| < \varepsilon.$$

Thus, the sequence $\{x_n\}$ is fundamental and due to the completeness of the X space, it is also convergent. Let $x = \lim_{n\to\infty} x_n$. Since the set F is closed and $x_n \in F\ (n = 1, 2, ...)$, $x \in F$.

Let us demonstrate that x is a stationary point of the operator $I(x)$. Hence,

$$\|x - I(x)\| \leqslant \|x - x_{n+1}\| + \|x_{n+1} - I(x)\|.$$

Since $x_n = I(x_{n-1}),\ \ (n = 1, 2, ...)$, the previous inequation can be transformed in the following way

$$\|x - I(x)\| \leqslant \|x - x_{n+1}\| + \|I(x_n) - I(x)\| \leqslant$$
$$\leqslant \|x - x_{n+1}\| + \lambda\|x_n - x\|. \tag{3.21}$$

Since $x_n \to x$, for any $\varepsilon > 0$ there is such n, that

$$\|x - x_{n+1}\| + \lambda\|x_n - x\| < \varepsilon.$$

From (3.21) the following derives

$$\|x - I(x)\| \leqslant \varepsilon.$$

As ε was selected randomly, $x = I(x)$.

Let us showcase the unity of solution. Let's assume that there are two stationary points x and $\overline{x}$, then

$$\|x - \overline{x}\| = \|I(x) - I(\overline{x})\| \leqslant \|x - \overline{x}\|.$$

Since $\lambda < 1$, this inequation is possible only on condition that $x = \overline{x}$. □

To solve the integral Volterra equation of the second kind (3.17) the Simpson's rule was applied. Let us consider

$$y(x) - \int_a^x K(x, t, y(t))dt = f(x) \tag{3.22}$$

on the interval $a \leqslant x \leqslant b$. We shall assume that $K(x, t, y(t))$ and $f(x)$ are continuous functions.

Based on the equation (3.22), let us find $y(a) = f(a)$. Let us take the constant step of integration h and view the discrete set of points $x_i = a + h(i - 1)$, where $i = 1, 2, 3, ..., n$. For the points $x = x_i$, the equation (3.22) looks as follows:

$$y(x_i) - \int_a^{x_i} K(x_i, t, y(t))dt = f(x_i). \tag{3.23}$$

By picking x_i as quadrature nodes for the formula, we shall neglect the approximation error and by replacing the integral in the equation (3.23) with the formula, the following system of non-linear algebraic (or transcendent) equations is received:

$$y_1 = f_1, \quad y_i - \sum_{j=1}^{i} A_{ij} K_{ij}(y_i) = f_i, \quad i = 2, 3, ..., n, \tag{3.24}$$

where A_{ij} is the coefficient of the quadrature formula for the interval $[a, x_i]$, y_i are the approximated values of the solution $y(x)$ in the nodes x_i, $f_i = f(x_i)$ $K_{ij}(y_j) = K(x_i, t_j, y_j)$.

The correlation (3.24) can be noted as a sequence of recurrent non-linear equations.

$$y_1 = f_1, \quad y_i - A_{ii} K_{ii}(y_i) + \sum_{j=1}^{i-1} A_{ij} K_{ij}(y), \quad i = 2, 3, ..., n, \tag{3.25}$$

$$A_1 = A_{2m+1} = \frac{h}{3}, \; A_2 = ... = A_{2m} = \frac{4h}{3}, \; A_3 = ... = A_{2m-1} = \frac{2h}{3}, \tag{3.26}$$

$$h = \frac{b-a}{n-1},\ x_i = a + h(i-1)\ \ (n = 2m+1,\ i = 1, 2, ..., n), \tag{3.27}$$

where $m \in N$ for the definition of the approximated value of the sought for solution in the node points.

By defining the function $A(t)$ from the non-linear Volterra equation (3.17), the movement u_r and deformation components ε_r and ε_θ can be defined by the following formulae:

$$u_r(t,r) = \frac{A(t) - A(\tau^*(r))}{r},\ -\varepsilon_r(t,r) = \varepsilon_\theta(t,r) = \frac{A(t) - A(\tau^*(r))}{r^2},\ b_0 < r \leqslant b(t);$$

$$u_r(t,r) = \frac{A(t)}{r},\ -\varepsilon_r(t,r) = \varepsilon_\theta(t,r) = \frac{A(t)}{r^2},\ a_0 \leqslant r \leqslant b_0;$$

$$A(t) = -\int_0^t c(\tau) d\tau$$

and stress σ_r and σ_θ by the formulae:

$$\sigma_r(t,r) - \sigma_\theta(t,r) = -\frac{2}{r^{2m}} \left[2G_1(t) A^m(t) - \int_0^t R_1(t,\tau) A^m(\tau)\, d\tau \right],\ a_0 \leqslant r \leqslant b_0;$$

$$\sigma_r(t,r) = -P(t) + \left[4G_1(t) A^m(t) - 2\int_0^t R_1(t,\tau) A^m(\tau) d\tau \right] \int_{a_0}^{r} \frac{dr}{r^{2m+1}},\ b_0 \leqslant r \leqslant b(t);$$

$$\sigma_r(t,r) - \sigma_\theta(t,r) = -\frac{2}{r^{2m}} \left[2G_1(t - \tau^*(r))(A(t) - A(\tau^*(r)))^m - \right.$$

$$\left. - \int_{\tau^*(r)}^{t} R_1(t - \tau^*(r), \tau - \tau^*(r)) \left(A(\tau) - A(\tau^*(r))\right)^m d\tau \right],\ a_0 < r \leqslant b_0;$$

$$\sigma_r(t,r) = -2\int_r^{b(t)} \frac{1}{r^{2m+1}} \left[2G_1(t - \tau^*(r))(A(t) - A(\tau^*(r)))^m - \right.$$

$$\left. - \int_{\tau^*(r)}^{t} R_1(t - \tau^*(r), \tau - \tau^*(r))(A(\tau) - A(\tau^*(r)))^m d\tau \right] dr,\ b_0 < r \leqslant b(t)$$

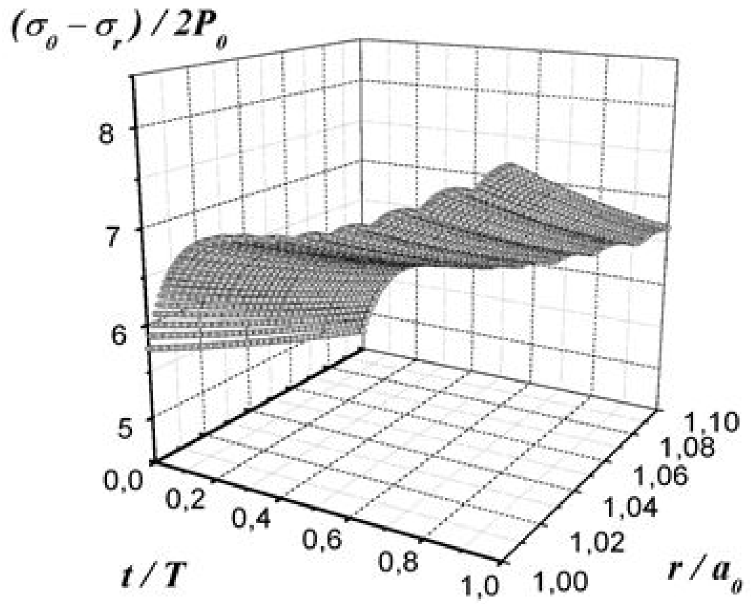

Fig. 2: The distribution of stress for the initial cylinder at T=50 hours

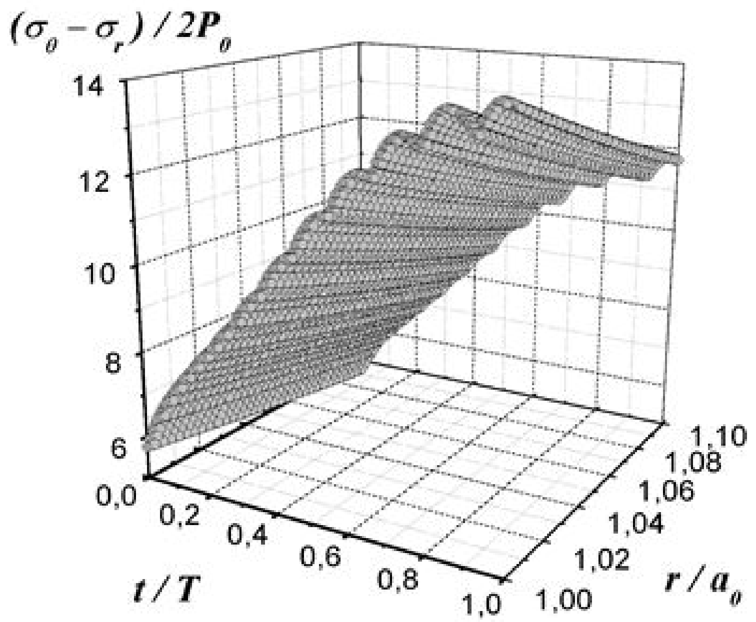

Fig. 3: The distribution of stress for the initial cylinder at T=100 hours

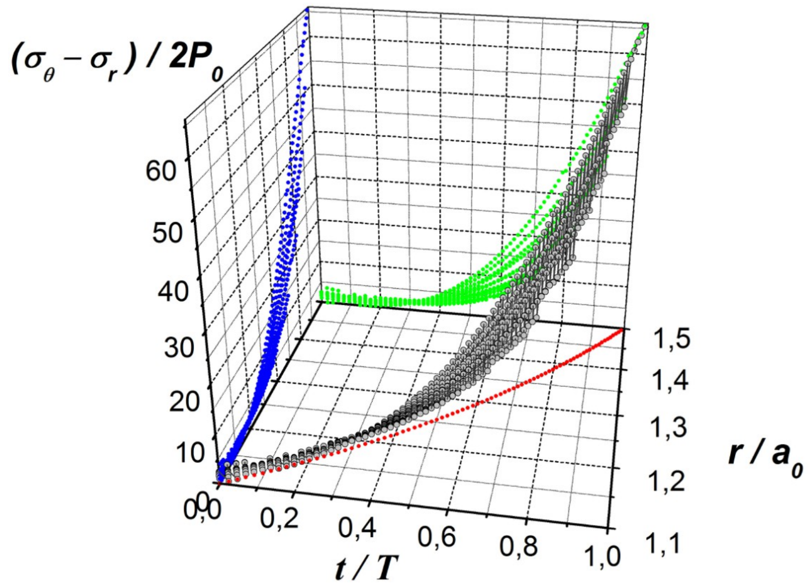

Fig. 4: The distribution of stress for the built-up cylinder at T=50 hours

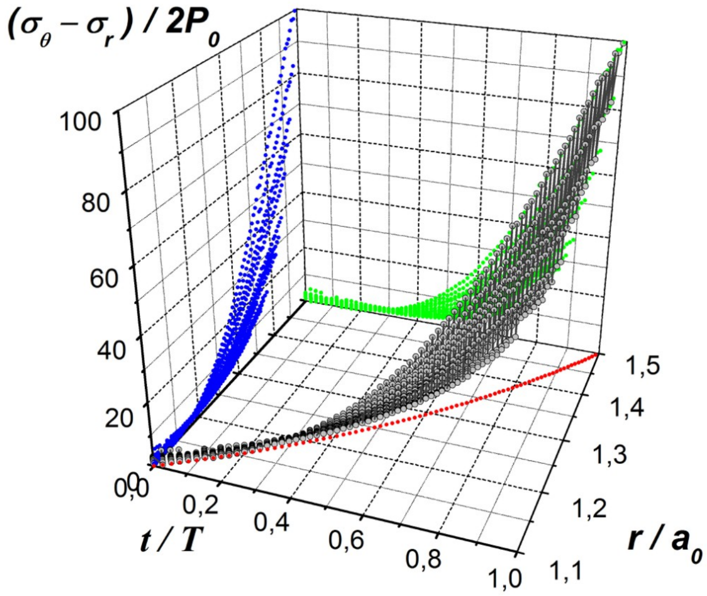

Fig. 5: The distribution of stress for the built-up cylinder at T=100 hours

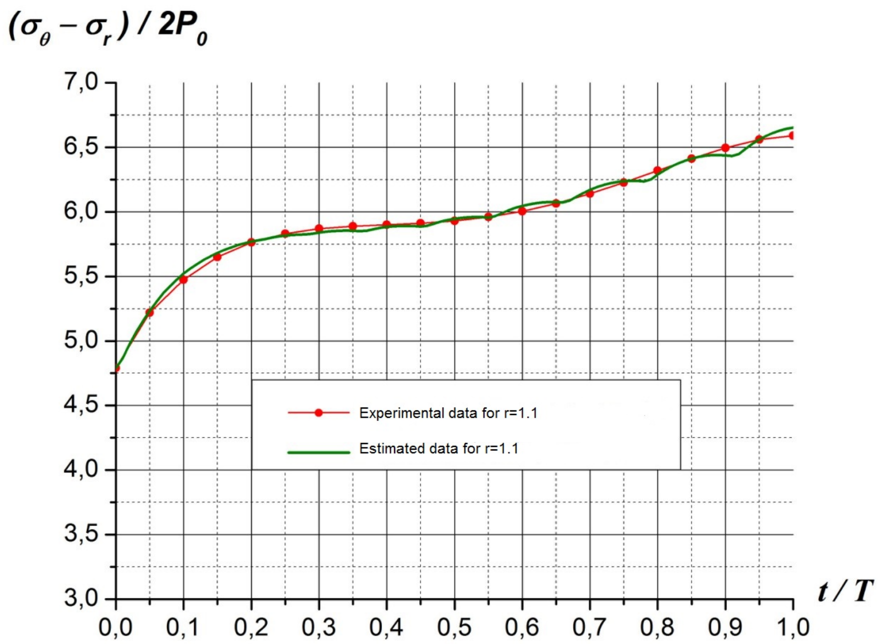

Fig. 6: The distribution of stress for the initial cylinder at T=100 hours

4. The result analysis

Let us consider the case when the function $\mu(t,\tau)$ looks as follows:

$$R(t,\tau) = \frac{\partial \mu(t,\tau)}{\partial \tau},$$

$$\mu(t,\tau) = 2G(\tau) - \varphi(\tau)(1 - e^{-\gamma(t-\tau)})$$

on condition that

$$G = const,\ \varphi(\tau) = 2G(C_0 + A_0 e^{-\beta\tau}).$$

Let us assume that the outer radius of cylinder $b(t)$ changes according to the law:

$$\frac{1}{b^2(t)} = \frac{1}{b_0^2} + \left(\frac{1}{b_1^2} - \frac{1}{b_0^2}\right)\frac{t}{T},\ 0 \leqslant t \leqslant T,$$

and the inner pressure decreases proportionally on the interval $[0,T]$ down to the value that is two times smaller than the initial one, and at the moment the build-up is over $t = T$, the pressure drops to zero.

In the figures 2, 3 dependencies of the dynamics of the maximum tangent stress for the following points of cylinder is given $1 - r = b_0$; $2 - r = a_0 - 0 = 0,9b_0 - 0$; $3 - r = 0,79b_0$. In the course of calculations, the parameters

C_0, A_0, β, γ and dimensions a_0, b_0 were selected this way: $C_0 = 0,05$, $A_0 = 0,07$, $\beta = 0,02^{-1}$, $\gamma = 0,1^{-1}$. Since stress does not depend on the value of the momentary elastic module G, and movement along with deformations are inversely related to it, the calculations assumed that $G = 1$.

The figures 2 and 3 show the dependence of the maximum tangent stress both on time and the coordinate of the different cylinder zones ($a_0 \leqslant r \leqslant b_0$ for the initial cylinder and $b_0 < r \leqslant b(t)$ for the build-up zone) for different build-up duration. The results are given for polyvinyl chloride for the following values of the geometrical parameters: $b_0 = 1,1a_0$, $b_1 = 1,5a_0$, $b(t) = \frac{b_0 b_1 \sqrt{T}}{\sqrt{b_1^2 T + t(b_0^2 - b_1^2)}}$.

Thus, the presented calculation method enables to evaluate the influence of build-up duration on the distribution of stress and movements in a round viscoelastic hollow cylinder made of homogenous viscoelastic material.

The probability of the results is based on the correct mathematic problem statement, efficacy of the analysis methods and verified by satisfactory matches of numeric data and experimental data known from literature [1, 2]. Therefore the suggested model can be utilized by research and project organizations to model technological processes connected to rotary workpiece production and construction elements production by means of build-up.

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