

V. M. Pirniak, P. D. Lezhniuk, Dc. Sc. (Eng.), Prof.;  
O. D. Demov, Cand. Sc. (Eng.), Assist. Prof.; Yu. Yu. Pivniuk

## CALCULATION OF ECONOMIC EQUIVALENTS OF REACTIVE POWER FOR THE NODES OF ELECTRIC GRID

*It is shown in the paper that economic equivalents of reactive power (EERP) of electric grid nodes can be found by means of the coefficients of losses distribution. It enables to determine EERP by the scheme of the grid and its parameters that corresponds to physical conditions of EERP formation.*

**Key words:** economic equivalents of reactive power, electric grid.

### Introduction

Main tool, intended for stimulation of the introduction of reactive power compensation installations in electrical grids of the consumers is the payment for reactive energy, determined according to “Technique for calculation of payment for reactive electric energy flow between energy transmission organization and its consumers” [1]. This payment is determined by the losses of active energy for the transmission of reactive power along the networks of energy supplying organization (EO) to the consumer. The networks of EO are equivalent source of reactive power, which is characterized by economic equivalent of reactive power (EERP).

With the development of market relations in electric energy sector of national economy the existing technique requires improvement. A number of papers, for instance [2, 3] are devoted to this problem. They are aimed at improvement and simplification of the given technique. The aim of the given paper is to determine economic equivalents of reactive power of electric grids nodes by means of the coefficients of power losses distribution in the branches of the grid. This enables to define EERP by the scheme of the grid and its parameters, that correspond to physical conditions of EERP formation.

### Problem statement

There are several methods of EERP determination. In [4] it is shown, that the value of EERP can be determined as:

$$D_i = \frac{d[\Delta P(Q_i)]}{dQ_i}, \quad i = \overline{1, n}, \quad (1)$$

where  $Q_i$  – is current value of reactive loading of  $i^{th}$  node;  $\Delta P(Q_i)$ - is the dependence of active power losses, caused by reactive load  $Q_i$  in the networks of EO;  $n$  – is the number of loading nodes of the networks.

The value of the increment of  $\Delta P(Q_i)$  function, when  $Q_i$  changes into  $\Delta Q_i$  can be determined as

$$\delta P_i = D_i \Delta Q_i, \quad \text{if } \Delta Q_i \ll Q_{pi}, \quad (2)$$

where  $Q_{pi}$  – is calculated reactive load of the  $i^{th}$  node.

Calculation of  $\delta P_i$  value, specified by  $Q_{pi}$  load, by the formula (1) gives great error. Besides, in this case the value of EERP  $D_i$  depends on reactive loads of all nodes of the networks [4], that makes the calculation more difficult.

In [5] it is recommended to calculate average value of EERP:

$$D_{ic} = \frac{\Delta P_{pi}}{Q_{pi}}, \quad (3)$$

where  $\Delta P_{pi}$  – active power losses, caused by  $Q_{pi}$  load

It is seen from the formula (3), that the value  $D_{ic}$  depends on the value  $Q_{pi}$ . That is, at different meanings of  $Q_{pi}$  for one and the same node  $D_{ic}$  will have different values. This is the disadvantage of such an approach, since EERP characterizes the conditions of  $Q_i$  power transfer and must not depend on the value of this power.

According to [6, 7], the value  $\Delta P_i$  can be presented as

$$\Delta P_i = \sigma_i Q_i + \delta_i Q_i^2, \quad (4)$$

where  $\sigma_i = \frac{2}{U_i^2} \cdot (Q_1 R_{1i} + Q_2 R_{2i} + \dots + Q_{i-1} R_{i-1,i} + Q_{i+1} R_{i+1,i} + \dots + Q_n R_{ni})$ ,

$\delta_i = \frac{R_{ii}}{U_i^2}$  – correspondingly, the first and the second derivatives from the function of losses  $\Delta P_i$ ; by

variable value  $Q_i$ ;  $U_n$  – is the nominal voltage of the network;  $R_{ii}$  – is the input resistance of the  $i^{\text{th}}$  node;  $R_{ij}$  – is mutual resistance of the  $i^{\text{th}}$  and  $j^{\text{th}}$  nodes ( $j = \overline{1, n}$ ,  $i \neq j$ );

The drawback of this approach is the dependence of  $\sigma_i$  characteristic on reactive loads of other nodes.

Thus, the value of EERP of the  $i^{\text{th}}$  node, calculated by the existing methods, depends on reactive loads of this node and other nodes. This makes the calculation of the charge for reactive energy and its forecast by the consumer more difficult. Proceeding from this fact, it is suggested, for instance, in [2] to refuse from EERP and establish relations between energy supplier and energy consumer, using the ratio of reactive power to active one in controlled points. Increase or decrease of the tariff for the consumption of reactive power is determined by the value of the deviation of the real value of  $tg\varphi_\delta$  from the fixed value of  $tg\varphi_i$ .

However, if EERP is still applied, then it is expedient to improve the algorithm of its determination. One of the ways is to return to its physical value. The value of  $D$  is the derivative, determined as the relation of the increment of active power losses while the change of the reactive power. In [8] this value, as specific transport costs, was used for assessment and planning of measures, aimed at reduction of technological power consumption (electric energy) in electric networks. As the sensitivity factor, it is used for determination of the place of installation and power sources of reactive power (RPS) in electric networks. It is possible to determine EERP, calculating it by means of the coefficients of losses distribution [9].

### Determination of EERP by means of the coefficients of losses distribution

Let us consider the possibility of this approach for elementary circuit, equivalent circuit of which is shown in Fig. 1.

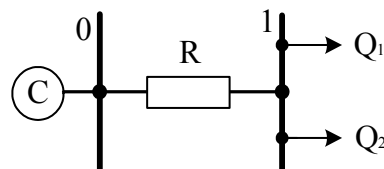


Fig. 1. Equivalent circuit of calculation network: C – energy grid

We will find losses, which create, correspondingly, reactive loads  $Q_1$  and  $Q_2$

$$\Delta P_1 = \Delta U_1 \cdot \frac{Q_1}{U_1}, \quad \Delta P_2 = \Delta U_2 \cdot \frac{Q_2}{U_1}, \quad (5)$$

where  $\Delta U_1, \Delta U_2$  – are voltage drops at the section 01 accordingly, due to the flow of reactive loads  $Q_1$  and  $Q_2$ ;  $U_1$  – is the voltage in the node 1.

If we assume, that  $\Delta U_{1*} = \frac{\Delta U_1}{U_1}$  and  $\Delta U_{2*} = \frac{\Delta U_2}{U_2}$  – are relative voltage drops, then (5) will be written as:

$$\Delta P_1 = \Delta U_{1*} \cdot Q_1, \quad \Delta P_2 = \Delta U_{2*} \cdot Q_2.$$

The latter expressions will be written, taking into account the distribution coefficients of total loading  $Q = Q_1 + Q_2$ :

$$\Delta P_1 = \Delta U_{1*} \cdot c_1 \cdot Q; \quad \Delta P_2 = \Delta U_{2*} \cdot c_2 \cdot Q, \quad (6)$$

where  $c_1 = \frac{Q_1}{Q}$  and  $c_2 = \frac{Q_2}{Q}$ .

In general form, power losses in  $s^{th}$  branch as a result of reactive power of  $i^{th}$  node can be determined:

$$\Delta P_{si} = \Delta U_{s*} \cdot c_{si} \cdot Q_i. \quad (7)$$

Since the value  $\Delta U_{s*} \cdot c_{si}$  is determined in the same way as the coefficient of losses distribution  $t_{si}$ , as the element of matrix  $\mathbf{T}$  of the coefficients of power losses distribution in branches of electric networks from the powers in its nodes [9], then for calculation of EERP it is possible and expedient to apply the method of calculation of losses distribution coefficients. In this case losses created by  $Q_i$  load in  $s^{th}$  branch of the electric networks can be presented in the following way:

$$\Delta P_{si} = t_{si} Q_i, \quad (8)$$

where  $t_{si}$  – is the  $s^{th}$  element of the matrix  $\mathbf{T}$  of losses distribution coefficients.

It is seen from the formula (8), that the coefficient  $t_{si}$  shows the share of active power losses in  $s^{th}$  branch as a result of the reactive load of  $i^{th}$  node  $Q_i$ . The share of the losses  $\Delta P_i$  in electric networks, caused by reactive load  $Q_i$ , is determined as

$$\Delta P_i = Q_i \sum_{s=1}^m t_{si} \quad \text{or} \quad \Delta P_i = T_i Q_i, \quad (9)$$

where  $T_i = \sum_{s=1}^m t_{si}$  – is the sum of elements of  $i^{th}$  column of the matrix  $\mathbf{T}$  of the coefficients of power losses distribution in the network;  $m$  – is the number of branches in electric networks.

That is  $T_i = D_i$ .

Let us consider in details the connection of the matrix of losses distribution with EERP. Matrix of the coefficients of losses distribution for random electric circuit has the following form [9]:

$$T = U_n^T \cdot M_E \cdot C \cdot U_d^{-1}, \quad (10)$$

where  $U_n^T$  – is the transposed matrix of nodal voltages of the networks with balance node;  $M_E$  – is the first matrix of networks connection with balance node;  $C$  – is the matrix of current distribution coefficients;  $U_d^{-1}$  – is inverse diagonal matrix of nodal voltages.

Matrix of current distribution coefficients for the circuit with active resistances is determined as [10]

$$C = R_v^{-1} \cdot M^T \cdot (M \cdot R_v^{-1} \cdot M^T)^{-1}, \quad (11)$$

where  $R_v^{-1}$  – is inverse diagonal matrix of active resistances of networks branches,  $M^T$  – is the transposed matrix of networks connection without balance node.

Let us substitute (11) into (10) and we obtain EERP matrix:

$$D = T = U_n^T \cdot M_E \cdot R_v^{-1} \cdot M^T (M \cdot R_v^{-1} \cdot M^T)^{-1} \cdot U_d^{-1} \quad (12)$$

It is seen from the formula (12), that unlike the existing methods [4 – 6], the values of EERP do not depend on reactive loads of other nodes. They are determined by the conditions of reactive power transfer (circuit, active resistances of elements, voltage in the nodes of the networks), that corresponds to physical conditions of the formation of active power losses from reactive power transfers. As these conditions are predictable, then it enables to forecast and control EERP values and, correspondingly, charges for reactive energy.

**Example.** For the networks, equivalent circuit and parameters of which are show in Fig. 2, find total losses of active power and losses, created by reactive loads of each node. Use EERP, found on the basis of the coefficients of power losses distribution.

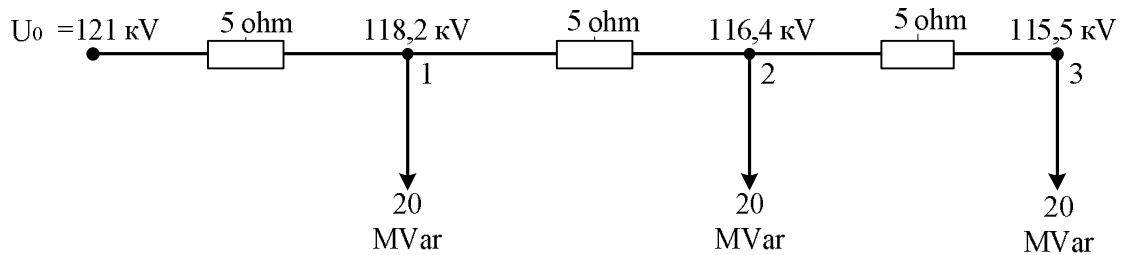


Fig. 2. Equivalent circuit of calculated grids

**Solution.**

According to [10], we find matrices  $M_E$ ,  $M$ ,  $Q$ ,  $U_y$ ,  $U_d$ ,  $R_v$ :

$$M_E = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}; \quad M = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}; \quad Q = \begin{bmatrix} -20 \\ -20 \\ -20 \end{bmatrix}, \text{ Mvar};$$

$$U_y = \begin{bmatrix} 118,2 \\ 116,4 \\ 115,5 \\ 121 \end{bmatrix}, \text{ kV}; \quad U_d = \begin{bmatrix} 118,2 & 0 & 0 \\ 0 & 116,4 & 0 \\ 0 & 0 & 115,5 \end{bmatrix}, \text{ kV}; \quad R_v = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \text{ ohm}.$$

According to (12), we find matrix EERP:

$$D = \begin{bmatrix} 118,2 \\ 116,4 \\ 115,5 \\ 121 \end{bmatrix}^T \cdot \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}^T \times$$

$$\times \left( \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}^T \right)^{-1} \cdot \begin{bmatrix} 118,2 & 0 & 0 \\ 0 & 116,4 & 0 \\ 0 & 0 & 115,5 \end{bmatrix}^{-1} = \begin{bmatrix} -0,023 & -0,039 & -0,047 \end{bmatrix} \frac{KW}{k \text{ var}}.$$

We find losses, created by reactive loads of the first, second and third nodes, correspondingly, and, also, total, losses:

$$\Delta P_1 = D_1 \cdot Q_1 = 0.023 \cdot 20 = 0.46 \text{ MW}; \quad \Delta P_2 = D_2 \cdot Q_2 = 0.039 \cdot 20 = 0.78 \text{ MW};$$

$$\Delta P_3 = D_3 \cdot Q_3 = 0.047 \cdot 20 = 0.94 \text{ MW};$$

$$\Delta P_{\Sigma} = \Delta P_1 + \Delta P_2 + \Delta P_3 = 0.46 + 0.78 + 0.94 = 2.18 \text{ MW}.$$

We will find total losses in the networks, applying classic method [6]:

$$\Delta P_{\Sigma} = \frac{1}{U_n^2} \cdot \sum_1^m Q_s^2 \cdot R_s = \frac{(60)^2}{115^2} \cdot 5 + \frac{(40)^2}{115^2} \cdot 5 + \frac{(20)^2}{115^2} \cdot 5 = 2,1 \text{ MBm},$$

where  $Q_s$ ,  $R_s$  – reactive flow in  $s^{\text{th}}$  branch of the networks and active resistance of this branch correspondingly.

Since the value of losses, found by both methods, practically coincide, then EERP can be found by means of losses distribution coefficients.

### Conclusions

1. The existing methods of calculation of economic equivalents of reactive power for the nodes of the networks of electric transmission company depend on reactive loads of other nodes, that complicates their determination and, correspondingly, forecasting of charges for reactive energy.

2. The suggested method of calculation of economic equivalents of reactive power is based on the data, regarding the scheme of the network of electric energy transmission company and its parameters, corresponds to physical conditions of the formation of active power losses due to transfers of reactive power and enables to forecast the charges for reactive energy.

### REFERENCES

1. Методика обчислення плати за перетікання реактивної електроенергії між електропередавальною організацією та її споживачами / Міністерство палива та енергетики України. – К. – 2002. – 29 с. [затверджено наказом № 19 від 17.01.2002 р.]
2. Железко Ю. С. Новые нормативные документы, определяющие взаимоотношения сетевых организаций и покупателей электроэнергии в части условий потребления реактивной мощности / Ю. С. Железко // Новини енергетики. – 2008. – № 8. – С. 45 – 49.
3. Методика розрахунків плати за перетоки реактивної електроенергії та реактивну потужність між енергопостачальною організацією та її споживачами і суб'єктами оптового ринку електроенергії / Б. С. Рогальський, О. М. Нанака, А. В. Праховник [та ін.] // Промислова електроенергетика та електротехніка. Промелектро. – 2009. – № 5. – С. 10 – 20.
4. Економічні еквіваленти реактивної потужності. Математичний та чисельний аналіз / Д. Б. Банін, О. С. Яндульський, М. Д. Банін [та ін.] // Промелектро. – 2004. – № 1. – С. 22 – 33.
5. Рогальський Б. С. Економічні еквіваленти реактивної потужності (ЕЕРП) та їх використання / Б. С. Рогальський, О. М. Нанака // Вісник Вінницького політехнічного інституту. – 2005. – № 6. – С. 126 – 129.
6. Карпов Ф. Ф. Компенсация реактивной мощности в распределительных сетях / Ф. Ф. Карпов. – М.: Энергия, 1975. – 184 с.
7. Демов О. Д. Про розрахунок економічного еквівалента реактивної потужності / О. Д. Демов, О. П. Паламарчук, І. О. Бандура, Ю. А. Григораш // Промелектро. – 2010. – № 1. – С. 3 – 6.
8. Экономия энергии в электрических сетях / [И. И. Магда, С. Я. Меженный, В. Н. Сулейманов и др.] ; под ред. Н. А. Качановой и Ю. В. Щербины. – К.: Техніка, 1986. – 167 с.
9. Лежнюк П. Д. Взаємовплив електричних мереж і систем / Лежнюк П. Д., Кулик В. В., Бурикін О. Б. – Вінниця: ВНТУ, 2008. – 122 с.
10. Мельников Н. А. Матричный метод анализа электрических цепей / Н. А. Мельников. – М.: Энергия, 1972. – 231 с.

**Pirniak Viktor** – Deputy Chief of Vinnytsia Regional Inspection of State Energy Control.

**Lezhnuik Petro** – Head of the Chair of Electric Stations and Systems.

**Demov Olexandre** – Assistant Professor, Chair of Electric Engineering Systems of Energy Consumption and Energy Management.

**Pivniuk Yuriy** – Student master, Chair of Electric Stations and Systems.  
Vinnytsia National Technical University.