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ANALYSIS OF POWER TRANSFORMER IMPACT ON STEADY-STATE MODE OF ELECTRIC GRID BASED ON CAUCHY EQUATION

Paper analyses the sensitivity of power transformer output characteristics change to variations of its parameters. Sensitivity calculation is performed using Cauchy differential equations. Spatial magnetic system is proposed on the base of the analysis, the system enables to decrease leakage inductance in the transformer, that results in the reduction both of no-load losses and load losses in the process of their usage in electric grids.

Key words: *electric grid, power transformers, parametric sensitivity, spatial magnetic system, active power losses.*

Introduction

Power transformers are one of the most important and complex elements of electric energy system (EES). They are directly involved in the processes of generation, transport, distribution and consumption of electric energy. In the United Energy System of Ukraine (UES) the installed power of the transformers is several times greater than the installed power of generators at electric power plants. It is obvious that the transformers greatly influence the quality of electric grids (EG) of energy systems operation. In particular, they not only influence but also determine reliability and economic efficiency of their modes. Efficiency factor of modern transformer is close to 1 ($(\eta \geq 99\%)$. However, due to numerous transformations on the way from generation to consumption of electric energy, the number of the transformations could reach 4 – 5, technological losses of electric energy take place, they may be referred to conventionally constant.

The objective of the paper is the construction of the algorithm of parametric sensitivity assessement on the base of Cauchy differential equations of state that enables to synthesize new constructive solutions in the process of power transformers development.

Cauchy differential equations of power transformer state

In the research [1, 2] the possibility of the formation of differential equations of electromagnetic processes, taking place in power transformer on the base of currents analysis is shown. Application of such approach gave the possibility to pass to differential equations in Cauchy form:

$$\frac{d\mathbf{X}}{dt} = \mathbf{B} \cdot \mathbf{Z}(t). \tag{1}$$

where $\mathbf{B} = \begin{pmatrix} g_1\, g_2 & & \\ & a_{21}\, a_{22} & \\ & & \mathbf{C}^{-1} \end{pmatrix}$ – matrix of the coefficients, characterizing transformers parameter and loading character; $\mathbf{X} = [\psi, i_2, u_c]^T$ – vector of state variables; $\mathbf{Z}(t) = [\mathbf{U} - \mathbf{RI}, i_2]^T$; $\psi = [\Psi_1, \Psi_2']^T$ – vector of complete magnetic-flux linkages of windings; $\mathbf{I} = [i_1, i_2]^T$ – vector of winding currents; $\mathbf{U} = [u_1, -u_c]^T$ – vector of voltages; $\mathbf{R} = diag[r_1, r_2 + R_H]$ – diagonal matrix of resistances.

We will write down the system of equations (1) by power transformer parameters:

$$\frac{d\psi}{dt}=\frac{1/L_{\sigma1}}{1/L_{\sigma1}+(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)+\alpha_2''}\cdot(u_1-r_1\cdot i_1)+$$
$$+\frac{(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)}{1/L_{\sigma1}+(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)+\alpha_2''}\cdot(-u_c-(r_2+R_H)\cdot i_2);$$
$$\frac{di_2}{dt}=-(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)\frac{1/L_{\sigma1}}{1/L_{\sigma1}+(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)+\alpha_2''}(u_1-r_1\cdot i_1)+ \quad (2)$$
$$+(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)(1-\frac{(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)}{1/L_{\sigma1}+(1/L_{\sigma2})/(1+(1/L_{\sigma2})L)+\alpha_2''})(-u_c-(r_2+R_H)\cdot i_2);$$
$$\frac{du_c}{dt}=\frac{1}{C}\cdot i_2;$$

where $L_{\sigma1}, L_{\sigma2}$ – leakage inductances; $\alpha''=\frac{d\varphi(\psi)}{d\psi}=\alpha''(\psi)$ – reversed differential inductance of the transformer, determined by magnetization curve $\phi(\psi)=i_1+i_2$.

Reduction of the system of power transformer state equations to Cauchy form allows to apply for their solution Neuton method and avoid rotating of coefficients matrix. Such approach enables to simplify the procedure of parametric sensitivity determination.

Assessment of parametric sensitivity on the base of Cauchy differential equation

Sensitivity theory includes the set of methods, aimed at determination of the degree of object parameters impact on its output characteristics and usage of the data for object analysis.Most commonly, calculation of parametric sensitivity is realized by means of variational methods and this procedure is rather complicated.

The problem is that the degree of object specific parameter impact on its specific output characteristic can be different. In the process of object parameters optimization from the point of view of steady-state modes, transient processes and the degree of impact on output characteristics change, it is necessary to choose the parameter, change of which to largest extent influences the improvement of these characteristics. Besides, the change of one parameter can cause the change of several output characteristics. At the same time the improvement of one of the output characteristics may be accompanied by worsening of the others. That is why, optimization of complex systems, and power transformers should be referred to such systems, must be based on exact information, showing how quantitatively output characteristics are connected with the parameters of the object. Such information contains sensitivity matrix.

We will show the possibility of the solution of parametric sensitivity problem by means of differential equations (1) analysis. In this case, the algorithm of parametric sensitivity analysis is realized on the base of Neutonian iteration.

We will write the vector of steady- state parameters similarly to the vector of variable equations

$$\frac{d\mathbf{X}}{dt}=f_1(\mathbf{X},t), \quad (3)$$

where designations $\mathbf{X}=(x_1, x_2, ..., x_n)_t$, $f_1=(f_{11}, f_{12}, ..., f_{1n})_t$ are used. Here x_i – state variables (i = 1,2,..., n); t – time.

Vector of steady- state parameters:

$$\Lambda=(\lambda_1, \lambda_2, ..., \lambda_n)_t. \quad (4)$$

By analogy with matrix of sensitivity to initial condition $\Sigma(T) = \dfrac{\partial \mathbf{X}(\mathbf{X}(0), T)}{\partial \mathbf{X}(0)}$ matrix of parametric sensitivities is determined as partial derivative

$$\Xi = \frac{\partial \mathbf{X}}{\partial \Lambda}, \tag{5}$$

where $\mathbf{X}$ – vector of variable equations (3).

Any stable parameter of the investigated object could be the element of Ξ matrix.

Argument $\mathbf{X}$ is found from the equation (3), that, with the account of $\mathbf{X} = \mathbf{X}(\Lambda)$ dependence will be written in the form

$$\frac{d\mathbf{X}}{dt} = f_1(\mathbf{X}, \Lambda, t), \tag{6}$$

where f_1 – T-periodic function t.

We will differentiate (6) by Λ, taking into account (5). Linear parametric equation will be obtained

$$\frac{d\Xi}{dt} = \frac{\partial f_1(\mathbf{X}, \Lambda, t)}{\partial \mathbf{X}} \cdot \Xi + \frac{\partial f_1(\mathbf{X}, \Lambda, t)}{\partial \Lambda}. \tag{7}$$

Equation (7) also has $\Xi(t)$ periodic solution.

It is practically impossible to differentiate the right part of the equation (6) to obtain derivatives, which are in the right part (7) due to complex dependence of f_1 functions on variables. To simplify the problem matrix of additional parametric sensitivities χ relatively any other vector of $\mathbf{Y}$-argument is introduced

$$\chi = \frac{\partial \mathbf{Y}}{\partial \Lambda}. \tag{8}$$

Auxiliary equation $\dfrac{d\mathbf{Y}}{dt} = f_2(\mathbf{X}(\mathbf{Y}), t)$ regarding $\mathbf{Y}$ vector will also be parametric, i. e., dependent on vector of Λ parameters, i. e.

$$\frac{\partial \mathbf{Y}}{\partial t} = f_2(\mathbf{X}(\mathbf{Y}), \Lambda, t), \tag{9}$$

where f_2 – T-periodic function by t.

Differentiating (9) by Λ and taking into account (8) we obtain

$$\frac{d\chi}{dt} = \frac{\partial f_2(\mathbf{X}(\mathbf{Y}), \Lambda, t)}{\partial \mathbf{X}} \cdot \mathbf{A} \cdot \chi + \frac{\partial f_2(\mathbf{X}(\mathbf{Y}), \Lambda, t)}{\partial \Lambda}. \tag{10}$$

If it is taken into account, that in the model of parametric sensitivity vector of parameters corresponds to the vector of initial conditions, i. e. $\Lambda = \mathbf{X}(0)$, then the equation (10) is degenerated into homogeneous equation

$$\frac{d\chi}{dt} = \frac{\partial f_2(\mathbf{X}(\mathbf{Y}), \Lambda, t)}{\partial \mathbf{X}} \cdot \mathbf{A} \cdot \chi. \tag{11}$$

On condition, that $\Lambda = \mathbf{X}(0)$ derivatives (8) and $\mathbf{S}(\mathbf{X}(0), t) = \dfrac{\partial \mathbf{Y}}{\partial \mathbf{X}(0)}$ – matrix of auxiliary model of sensitivity will be equal. In this case equation (10) will have the form

$$\frac{d\mathbf{S}}{dt} = \frac{\partial f_2(\mathbf{X}(\mathbf{Y}), t)}{\partial \mathbf{X}} \cdot \mathbf{A} \cdot \mathbf{S}. \tag{12}$$

Periodic solution of the equation of parametric sensitivity model (11) is found, applying the method of accelerated search. Expressions (6), (12) are liable to numerical realization, as a result, we will find periodic solution of the equations of investigated object state, and then – matrix of parametric sensitivities (5).

Therefore, in the process of searching periodic solutions of non-linear differential equations it is also possible to determine the sensitivity of these solutions to the change of constant parameters of equations or parametric sensitivity.

Results of the analysis

Let us perform the analysis of the transformer output characteristics sensitivity in no-load mode to its parameters change. For the case of no-load mode analysis the corresponding matrices of indices will have the form: $\mathbf{X} = [\psi]$, $\mathbf{\Lambda} = (L_{\sigma 1}, L_{\sigma 2}, r_1, \alpha_2'')_t$.

System of differential equations (2) will be rewritten:

$$\frac{d\Psi}{dt} = \frac{1/L_{\sigma 1}}{1/L_{\sigma 1} + 1/L_{\sigma 2} + \alpha_2''} \cdot (u_1 - r_1 \cdot i_1). \quad (13)$$

We will carry out the analysis of the sensitivity of magnetic-flux linkage change to leakage inductance change L_σ. As leakage inductance depends on winding and magnetic part of power transformer construction, then as a result of analysis characteristic of its weight-dimensions parameters can be obtained.

After performing the calculations according to the algorithm given above, it is necessary to pass to geometric parameters. We will use the dependences, obtained in [3]:

$$L_\sigma = (\mu_0 w^2 \beta s_\sigma)/h_{coil} = k_{L\sigma} \beta w^2 s^{1/2}, \quad (14)$$

where $k_{L\sigma} = (\mu_0 s_\sigma)/h_{coil}$; w – number of turns; β – coefficient of core window width usage; s_σ – efficient area of coil dissipation; s – area of core cross-section; h_{coil} – coil height; μ_0 – magnetic permeability.

With the account of transformations [3] inductance of dissipation is determined for sinusoidal voltage by the expression

$$L_\sigma = k_{L\sigma} \beta \left(\frac{U_{av}}{4{,}44 f B_{cm} k_s} \right) \frac{I}{s^{3/2}}, \quad (15)$$

where B_{cm} – maximum value of the inductance in the material of the core; U, I – acting values of voltage and current.

Area of cross-section of the core in this case is determined by the expression

$$s = \left(\frac{S}{4{,}44 f k_{iw} \sqrt{\beta} B_{cm} k_s} \right)^{4/7}, \quad (16)$$

where k_{iw} – coefficient, that takes into account the parameters of the coil, depending on the form of the window; k_s – coefficient of steel volume (steel volume $V_s = s l_s k_s$); S – full power of the transformer.

Having performed the sensitivity analysis, the data are obtained, that enabled to determine the dependence of core cross-section area and active power losses in no-load mode on the leakage inductance (see Fig. 1).

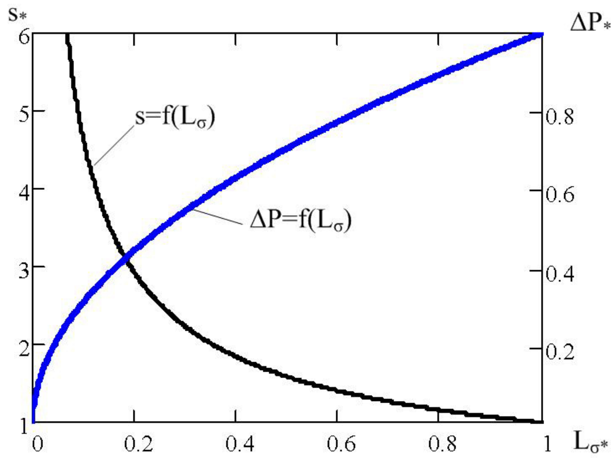

Fig. 1. Graph of the change of core cross-section area s_* and no-load losses ΔP_* on leakage inductance $L_{\sigma *}$ (in relative units)

The necessity of increasing the area of core cross-section is obviou. As a result of practical experiments spatial magnetic system was suggested. The system is shown in Fig. 2.

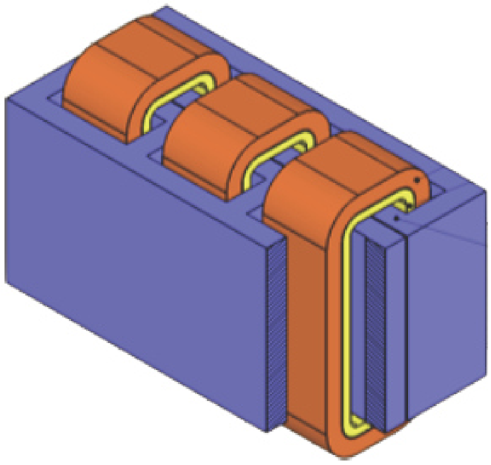

Fig. 2. View of spatial magnetic system

Patent of invention is obtained for such design of power transformer [4].

Reduction of active power losses at the expense of dissipation fluxes in the transformers with spatial magnetic system is proved by the calculations of the existing electric grids of 110/35/10 kVmodes .

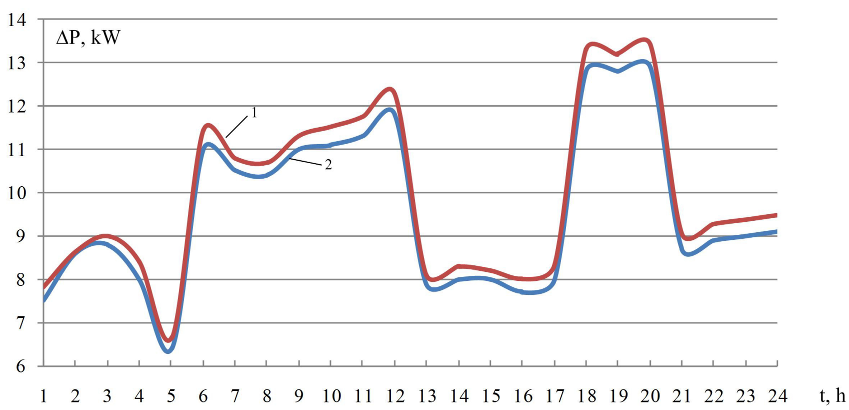

Fig. 3. Change of active power losses during the day in electric grid with power transformers with spatial magnetic system (curve 2) and flat – (curve 1)

Conclusions

Reduction of differential equations of power transformer state to Cauchy form enables to simplify not only the method of their solution but also obtain relatively simple method of parametric sensitivity assessment.

Evaluation of sensitivity of power transformer parameters change to leakage inductance allows to substantiate the transition from flat to spatial magnetic system.

Manufacturing of spatial magnetic system requires greater amount of steel as compared with the flat system that contradicts to the trend for maximal economy of active materials, in spite of considerable decrease of energy characteristics of the product. However, the reduction to minimum the dissipation fields by spatial magnetic system leads to considerable increase of working field and reactive power of the transformer, reduction of losses from these fields and excitation current and increase of single-unit active power.

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