

## FRACTAL STRUCTURE OF THE FUNDAMENTAL CONSTANTS. NUMERICAL EVALUATION OF THE VALUES OF SOME OF FUNDAMENTAL CONSTANTS WITH USE OF THE MAJOR CHARACTERISTICS OF MUON

*The structure of matter as well as the motion is a fundamental property of it. A fractal character is a distinguishing feature of a structure of matter. A similarity of spatial forms and structure levels underlies the fractality of matter structure. The fractality of matter structure manifests itself in the fractal structure of fundamental constants. The Planck length and the Planck mass are simple gold algebraic fractals of the muon Compton wavelength and of the mass of a muon. The gravitational constant, the Planck constant, the fine-structure constant, the value of the elementary charge, basic space-energy characteristics of the observable Universe are multiplicative gold algebraic fractals of the muon Compton wavelength and of the mass of a muon. Numeric evaluations of values of the gravitational constant, the Planck constant, the fine-structure constant, the value of the elementary charge can be obtained through of the muon Compton wavelength and of the mass of a muon.*

**Key words:** fractal, structure of matter, fundamental constants, muon, Dirac large numbers, observable Universe.

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### ФРАКТАЛЬНАЯ СТРУКТУРА ФУНДАМЕНТАЛЬНЫХ КОНСТАНТ. ЧИСЛЕННАЯ ОЦЕНКА ЗНАЧЕНИЙ НЕКОТОРЫХ ФУНДАМЕНТАЛЬНЫХ КОНСТАНТ НА ОСНОВЕ ОСНОВНЫХ ХАРАКТЕРИСТИК МЮОНА

*Структура материи, также как движение, является ее фундаментальным свойством. Отличительной особенностью структуры материи является ее фрактальный характер. Фрактальность структуры материи основана на подобию ее пространственных форм и структурных уровней. Фрактальность структуры материи проявляется в фрактальной структуре фундаментальных констант. Длина и масса Планка являются простыми алгебраическими золотыми фракталами комптоновской длины волны и массы мюона. Гравитационная постоянная, постоянная Планка, постоянная тонкой структуры, значение элементарного электрического заряда, основные пространственно-энергетические характеристики наблюдаемой Вселенной являются мультипликативными алгебраическими золотыми фракталами комптоновской длины волны и массы мюона. Численные оценки значений гравитационной постоянной, постоянной Планка, постоянной тонкой структуры, элементарного электрического заряда могут быть получены на основе значений комптоновской длины волны и массы мюона.*

**Ключевые слова:** фрактал, структура материи, фундаментальные константы, мюон, большие числа Дирака, наблюдаемая Вселенная.

### Introduction

The similarity of spatial forms and structure levels of matter can be represented as the geometric and algebraic fractals [1,2]. It is convenient to represent physical constants as the gold algebraic fractals (further GAF or – fractals), that underlies the golden ratio – harmonic proportion, in which the whole is to the most part as the most part to the less.

In [2] shown that any given function can be represented by an GAF function which consist of two components: mantissa function (characteristic function) and value function of structure level – scale factor function. An algebraic fractal contains his base – elementary structure component – on which a fractal is produced. In GAF such base is the number  $f_g$ , which is:

$$f_g = \frac{\sqrt{5}}{2} - 0.5 = 2 \sin(18^\circ) = 2 \cos(72^\circ) = 0.6180339887498948482045868343656 \dots \quad (1)$$

Mantissa GAF of function  $j(x)$  is a function  $M_g(x)$  that for all values  $x$  satisfy the condition:

$$\text{at } f(x) = 0: M_g(x) = 0; \text{ at } f(x) > 0: 1 \leq M_g(x) \leq \frac{1}{f_g}; \text{ at } f(x) < 0: -\frac{1}{f_g} \leq M_g(x) \leq -1 \quad (2)$$

For positive constants conditions [2] are:

$$1 \leq M_g(x) \leq \frac{1}{f_g} \quad (3)$$

Wherein:  $\frac{1}{f_g} = 1.6180339887498948482045868343656\dots$

The argument  $y_x$  of the scale factor function  $S_f(y_x)$  is an integer. The algorithm of its creation as follows:

for all values  $j(x)$ , that satisfy the condition  $-1 < f(x) < 1$ , the function value  $S_f(y_x) < 0$ , and for function values  $j(x)$  that satisfy the following conditions:  $-1 < f(x) < 1$ , or  $1 \leq f(x) < \infty$ , the function value  $S_f(y_x) \geq 0$ . If  $-1 < f(x) < 1$ , then every function value  $j(x)$  divided by the value  $f_g$  as many times as necessary until the conditions

[2] is met. If  $-\infty < f(x) \leq -1$ , or  $1 \leq f(x) < \infty$ , then every function value  $j(x)$  multiplied by  $f_g$  as many times as necessary until the conditions [2] is met. In all cases the exponent  $y_x$  of  $f_g$  is the function argument  $S_f(y_x)$ . Then the function  $j(x)$  can be written as:

$$j(x) = (M_g(x))^{S_f(y_x)}. \quad (4)$$

Or for positive constants  $j$  :

$$j = M_g f_g^{y_x}. \quad (5)$$

On the basis of (5) it is possible to explore constants which have different structure scale  $y_x$ . Fractals presented as (4,5) we will call simple.

### Multiplicative GAF

Multiplicative GAF are generated as a result of a simple GAF multiplying (dividing). When multiplying (dividing) the finite number of simple fractals their mantissas are multiply (divide) and scale factors are add (subtract). If resulting mantissa does not satisfy the conditions (2) is must be corrected: divide into (multiply by)  $f_g$  as many times as necessary until the conditions (2) is met. Scale factor which is gained as a result of such a correction is takes into account with his sign in the final result of the scale factor of a multiplicative fractal.

Fractals with identical mantissas are similar fractals. Measurements errors and noise are always presented in a parameters and characteristics of physical processes. Therefore the question of fractals similarity that have close values but not identical is open. Additional information is required for it solving. For example comparison of a several mantissas of parameters and a physicals processes characteristics that have different nature (mass, length, the moment of mass, time...).

### GAF of some fundamental constants

Let's present the fundamental constants [3]: the Planck length:  $l_p = 1.616229(38) \cdot 10^{-35} m$ ; the Planck mass:  $m_p = 2.176470(51) \cdot 10^{-8} kg$ ; the muon Compton wavelength over 2pi:  $I_{C,m} = 1.867594308(042) \cdot 10^{-15} m$ ;

the mass of a muon:  $m_m = 1.883531594(048) \cdot 10^{-28} kg$ ; the fine-structure constant:  $a = 7.2973525664(0017) \cdot 10^{-3}$ , in GAF form:

GAF mantissa of the Planck length:  $l_p^m = 1.2865859866$ , then:

$$l_p = l_p^m \cdot f_g^{167} m; \quad (6)$$

GAF mantissa of the Planck mass:  $m_p^m = 1.1756969040$ , then:

$$m_p = m_p^m \cdot f_g^{37} kg; \quad (7)$$

GAF mantissa of the muon Compton wavelength over 2pi:  $I_{C,m}^m = 1.2864858073$ , then:

$$I_{C,m} = I_{C,m}^m \cdot f_g^{71} m; \quad (8)$$

GAF mantissa of the mass of a muon:  $m_m^m = 1.1757881059$ , then:

$$m_m = m_m^m \cdot f_g^{133} kg; \quad (9)$$

GAF mantissa of the fine-structure constant:  $a^m = 1.4522098299$ , then:

$$a = a^m \cdot f_g^{11}. \quad (10)$$

Let's determine the moment of the Planck mass  $I_p$  and the Compton moment of the muon mass  $I_m$  :

$$I_p = m_p \cdot l_p = 3.51767393163 \cdot 10^{-43} \text{ kgm}; \quad (11)$$

$$I_m = m_m \cdot I_{C,m} = 3.51767288389 \cdot 10^{-43} \text{ kgm}. \quad (12)$$

Let's present the mass moment  $I_p$  and  $I_m$  in the form of multiplicative GAF:

multiplicative GAF mantissa of the Planck mass moment:  $I_p^m = l_p^m \cdot m_p^m = 1.5126351612$ , then:

$$I_p = I_p^m \cdot f_g^{204} \text{ kgm}; \quad (13)$$

multiplicative AGF mantissa of the muon Compton mass moment:

$$I_m^m = m_m^m \cdot I_{C,m}^m = 1.5126347107, \text{ then:}$$

$$I_m = I_m^m \cdot f_g^{204} \text{ kgm}. \quad (14)$$

The relative errors of nonidentity of the mantissas values of the length fractals  $d_{l,l}$ , of the mass  $d_m$ , and the moment of mass  $d_I$  are:

$$d_{l,l} = \frac{2|l_p^m - I_{C,m}^m|}{l_p^m + I_{C,m}^m} \cdot 100\% = 0.00779\%; \quad (15)$$

$$d_m = \frac{2|m_p^m - m_m^m|}{m_p^m + m_m^m} \cdot 100\% = 0.00777\%; \quad (16)$$

$$d_I = \frac{2|I_p^m - I_m^m|}{I_p^m + I_m^m} \cdot 100\% = 2.9785 \cdot 10^{-5}\%. \quad (17)$$

Taking into account the small values of the relative errors of nonidentity of the mantissas values of the length fractals  $d_{l,l}$  (15), of the mass  $d_m$  (16), the fact of significant reduction of the relative error of nonidentity of the mantissas values of the moments of mass  $d_I$  (17) (in relation to their constituents), the fact of different physical nature of length fractals, mass and moment of mass, then the Planck length fractals and muon Compton wavelength, the Planck mass fractals and muon mass, multiplicative fractals of the Planck mass moment and of the muon Compton mass moment can be considered as pair wise similar.

**Conclusions:**

- The Planck length and the Planck mass are simple GAF of the muon Compton wavelength and of the muon mass.
- The Planck mass moment and the Compton moment of the muon mass are equal and are multiplicative GAF.

**Evaluation of numerical values of some fundamental constants on the basis of numerical values of the muon Compton wavelength and mass of a muon.**

On the basis of pairwise fractal connections of the Planck length and the muon Compton wavelength and also the Planck mass and a muon mass will make replacement in formula (6) of the mantissa  $l_p^m$  to the mantissa  $I_{C,m}^m$  and in formula (7) the mantissa  $m_p^m$  to the mantissa  $m_m^m$  and will obtain the new value estimations of the length and Planck mass based on appropriate muon characteristics:

$$l_{p,m} = I_{C,m}^m \cdot f_g^{167} = 1.6161031532318473 \cdot 10^{-35} \text{ m}; \quad (18)$$

$$m_{p,m} = m_m^m \cdot f_g^{37} = 2.1766388345063263 \cdot 10^{-8} \text{ kg}. \quad (19)$$

According to National Institute of Standards and Technology (NIST) [3] for 2015: the gravitational constant  $G$ , the speed of light in vacuum  $c$ , the Planck constant  $h$ , the reduced Planck constant  $\hbar$ , the Planck time  $t_p$ , the elementary charge  $e$ , mass of electron  $m_e$ , the Bohr radius  $a_0$  are:

$$\begin{aligned} G &= 6.67408 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}; \quad c = 2.99792458 \cdot 10^8 \text{ m s}^{-1}; \quad h = 6.62607004 \cdot 10^{-34} \text{ Js}; \\ \hbar &= 1.054571800 \cdot 10^{-34} \text{ Js}; \quad t_p = 5.39116 \cdot 10^{-44} \text{ s}; \quad m_e = 9.10938356 \cdot 10^{-31} \text{ kg}; \\ a_0 &= 5.2917721067 \cdot 10^{-11} \text{ m}. \end{aligned} \quad (20)$$

Then with respect to (18,19) and also (7,10) in [4] on a Planck units:

$$t_{p,m} = \frac{l_{p,m}}{c} = 5.3907398606 \cdot 10^{-44} \text{ s}; \quad (21)$$

$$G_m = \frac{l_{p,m}^3}{m_{p,m} t_{p,m}^2} = 6.673045869 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}; \quad (22)$$

$$h_m = \frac{m_{p,m} l_{p,m}^2}{t_{p,m}} = 1.054571800 \cdot 10^{-34} \text{ kg}^1 \text{ m}^2 \text{ s}^{-2}; \quad (23)$$

$$h_m = 2p h_m = 6.62607004 \cdot 10^{-34} \text{ kg}^1 \text{ m}^2 \text{ s}^{-1}; \quad (24)$$

$$a_m = \frac{m_{p,m} l_{p,m}}{a_0 m_e} = 0.0072973525659; \quad (25)$$

$$e_m = \sqrt{10^7 a_m m_{p,m} l_{p,m}} = 1,6021766208 \cdot 10^{-19} \text{ C}, [m^{\frac{1}{2}} \text{ kg}^{\frac{1}{2}}]. \quad (26)$$

Evaluations of values of electrical charge (26) and the Planck constant (23,24) on base of the muon characteristics are equal to it standard values (20). Evaluation of the gravitational constant (22) on base of the muon characteristics differ from it standard value (20) by relative value  $d_G$ :

$$d_G = \frac{2|G - G_m|}{G + G_m} \cdot 100\% = 0.0155\%. \quad (27)$$

Taking into account the fact of values identity of the electrical charge and Planck constant with their standard values and also the fact that evaluation  $G_m$  is based on the standard values of the muon characteristics which are obtained on base of experimental data it is possible to make a suggestion that evaluation  $G_m$  is closer to the true value than standard value  $G$ .

#### Conclusions:

- The main fundamental constants have fractal structure.
- Evaluation of numerical values of the fundamental constants can be obtained on base of muon Compton wavelength and mass of a muon, which are based on experimental data.

#### GAF of some Dirac large numbers

It was shown in [5] that the energy of any body of the observable Universe, which has a mass  $m$ , including the Universe itself, can be presented as:

$$E = mc^2 = F_p S = h_e t_{dm}, \quad (28)$$

where  $F_p$  - is Planck force,  $S$  - is the radius of curvature of deformed space under the impact of a body with mass

$m$ ,  $h_e = \frac{E_p}{t_p}$  is the Planck energy quantum,  $t_{dm}$  is a delay in light signal propagation on the distance equal to

$S$ . Then

for observable Universe:  $E = E_U, m = M_U, S = R_U, t_{dm} = T_U$ , for a hypothetical Planck particle:

$E = E_p, m = m_p, S = l_p, t_{dm} = t_p$ , then:

$$E_U = M_U c^2 = F_p R_U = h_e T_U, \quad (29)$$

$$E_p = m_p c^2 = F_p l_p = E_p. \quad (30)$$

Dirac large energetic number  $N_{De}$ :

$$N_{De} = \frac{E_U}{E_p} = \frac{M_U c^2}{m_p c^2} = \frac{F_p R_U}{F_p l_p} = \frac{h_e T_U}{E_p} = \frac{M_U}{m_p} = \frac{R_U}{l_p} = \frac{T_U}{t_p}. \quad (31)$$

According to Planck Collaboration [6] the Hubble Constant for 2016 is:

$$H_0 = 66.93 \pm 0.62 (\text{km} / \text{s}) / \text{Mpc}.$$

For this value of the Hubble Constant, values  $M_U, R_U, T_U$ , are [7]:

$$T_U = \frac{1}{H_0} = 4.431534683326 \cdot 10^{17} s, \quad (32)$$

$$R_U = cT_U = 1.328540675427 \cdot 10^{26} m, \quad (33)$$

$$M_U = \frac{c^3 T_U}{G_m} = 1.789337036792 \cdot 10^{53} kg. \quad (34)$$

Then  $N_{De}$  taking into account (32,33,34) and (18,19,21) is:

$$N_{De} = \frac{M_U}{m_{p,m}} = \frac{R_U}{l_{p,m}} = \frac{T_U}{t_{p,m}} = 8.2206428022 \cdot 10^{60}. \quad (35)$$

In [2] is shown that number  $A_a$  is the fractal of the fine-structure constant:

$$A_a = f_g^5 \left( \frac{1}{a} \right)^{29} = 8.3804482103 \times 10^{60}. \quad (36)$$

From (35,36) follows that:  $N_{De} \approx A_a$ .

By analogy with the elementary quantum of action:

$$\mathbf{h}_m = \frac{m_{p,m} l_{p,m}^2}{t_{p,m}} = c m_{p,m} l_{p,m} = 1.054571800 \cdot 10^{-34} kg^1 m^2 s^{-1}, \quad (37)$$

let's look at the hypothetical elementary quantum of action of observable Universe:

$$\mathbf{h}_U = \frac{M_U R_U^2}{T_U} = c M_U R_U = 7.12668740 \cdot 10^{87} kg^1 m^2 s^{-1}. \quad (38)$$

Then taking into account (37,38) the Dirac large number of the moment of mass  $N_{DI}$ , is:

$$N_{DI} = \frac{\mathbf{h}_U}{\mathbf{h}_m} = \frac{M_U R_U}{m_{p,m} l_{p,m}} = N_{De}^2 = 6.75789680 \cdot 10^{121}. \quad (39)$$

The formula (31) is based on “Plank Universal proportions” law [5]. From formulas (31) and (35) follows that:

$$M_U = m_{p,m} \frac{R_U}{l_{p,m}} = m_{p,m} \frac{T_U}{t_{p,m}}; \quad (40)$$

$$R_U = l_{p,m} \frac{M_U}{m_{p,m}} = l_{p,m} \frac{T_U}{t_{p,m}}; \quad (41)$$

$$T_U = t_{p,m} \frac{M_U}{m_{p,m}} = t_{p,m} \frac{R_U}{l_{p,m}}. \quad (42)$$

From (40,41,42) follows that the main characteristics of observable Universe:  $M_U, R_U, T_U$  are multiplicative GAF of appropriate muon characteristics.

From “Plank Universal proportions” law [5] also follows that for any body of observable Universe with mass  $m$ , including the Universe itself, for the gravitational constant  $G_m$  and the Planck force  $F_{p,m}$  is true next:

$$G_m = \frac{l_{p,m}^3}{m_{p,m} t_{p,m}^2} = \frac{R_U^3}{M_U R_U^2} = \frac{S^3}{m t_{dm}^2} = 6.673045869 \cdot 10^{-11} m^3 kg^{-1} s^{-2}; \quad (43)$$

$$F_{p,m} = m_{p,m} \frac{l_{p,m}}{t_{p,m}^2} = M_U \frac{R_U}{T_U^2} = m \frac{S}{t_{dm}^2} = 1.21048301950 \cdot 10^{44} kg^1 m^1 s^{-2}. \quad (44)$$

i.e. the gravitational constant  $G_m$  and Planck force  $F_{p,m}$  are multiplicative GAF of the main muon characteristics.

#### Conclusion:

- The main characteristics of observable Universe and fundamental constants are GAF of muon characteristics.

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Отримана/Received : 27.9.2017 р. Надрукована/Printed : 9.10.2017 р.  
Стаття рецензована редакційною колегією